Heavy Ion Pub
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Heavy quarks in heavy ion collisions

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Outline

1. Quark-gluon plasma and heavy ion collision
2. Heavy quark diffusion in heavy ion collision
3. Heavy quark diffusion from spectral function
4. Summary

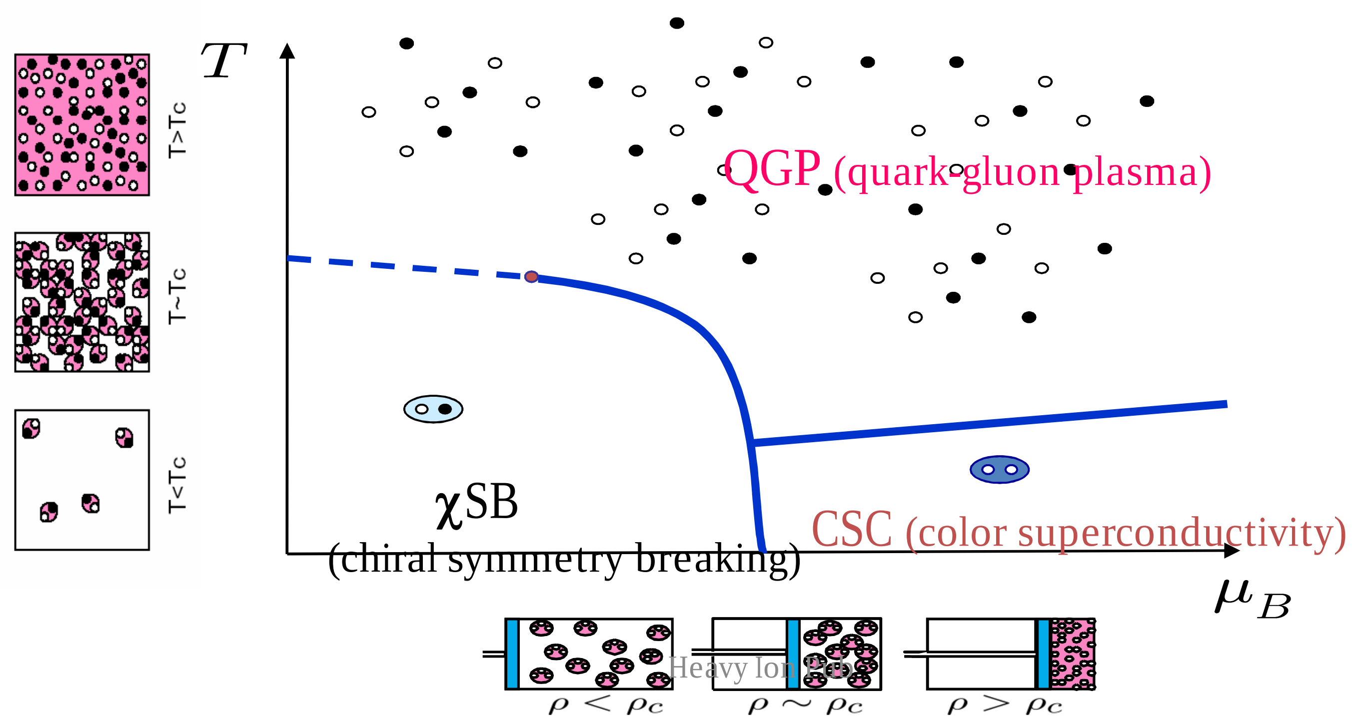
1. Quark-gluon plasma and heavy ion collision

Quantum Chromo Dynamics (QCD)

$$L = \overline{\psi}_i \left(i\gamma_\mu D_{ij}^\mu - m\delta_{ij} \right) \psi_j - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

- Asymptotic freedom
- Confinement

QCD Phase Diagram



1. Quark-gluon plasma and heavy ion collision

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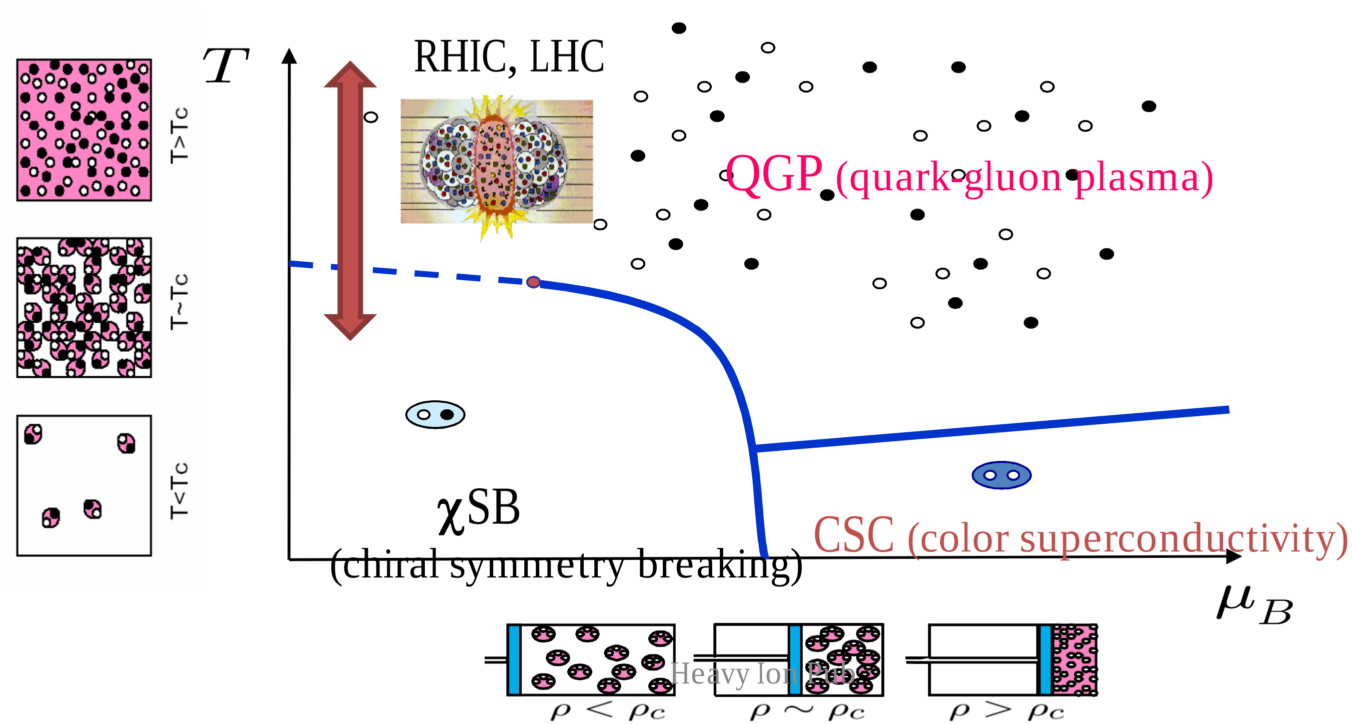
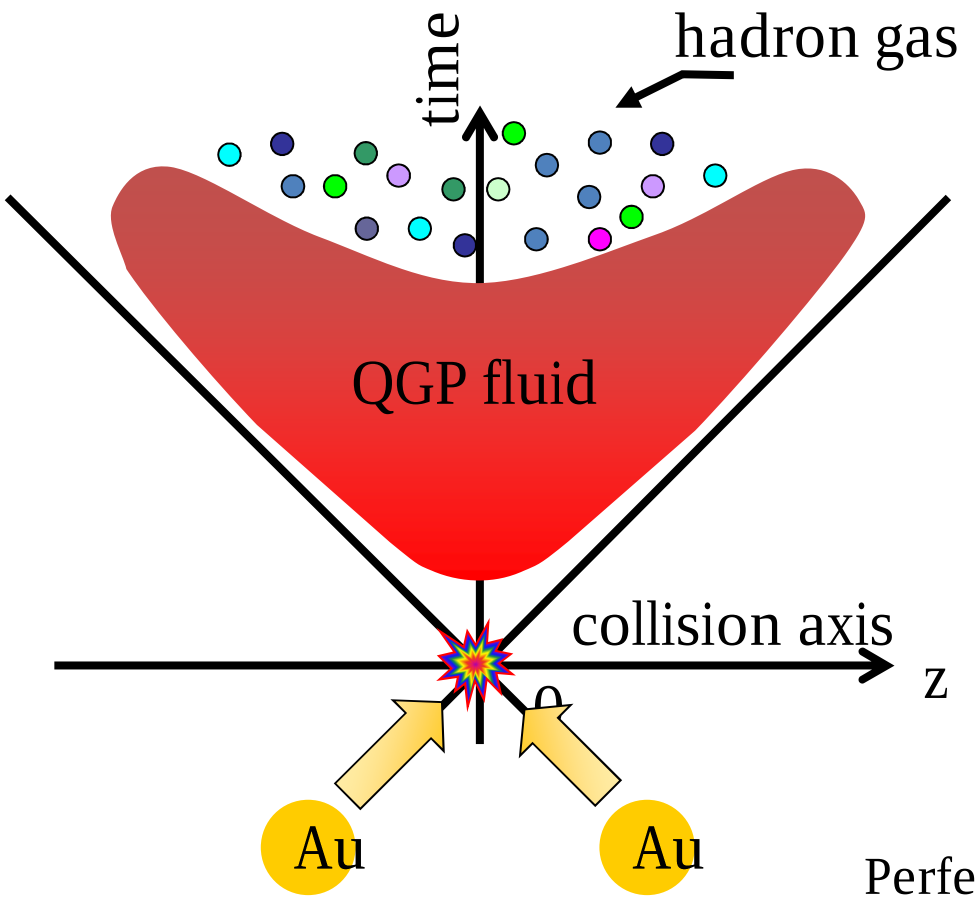


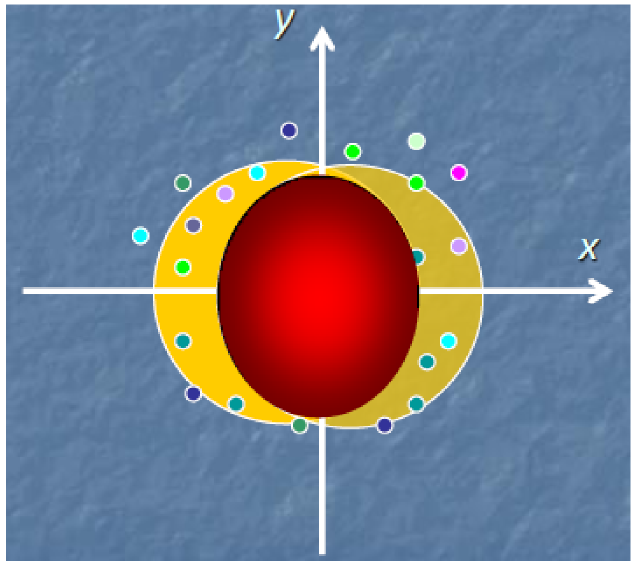
Heavy ion collision and hydrodynamic model

Dynamics of light particles (u,d,s,g)

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu}$$





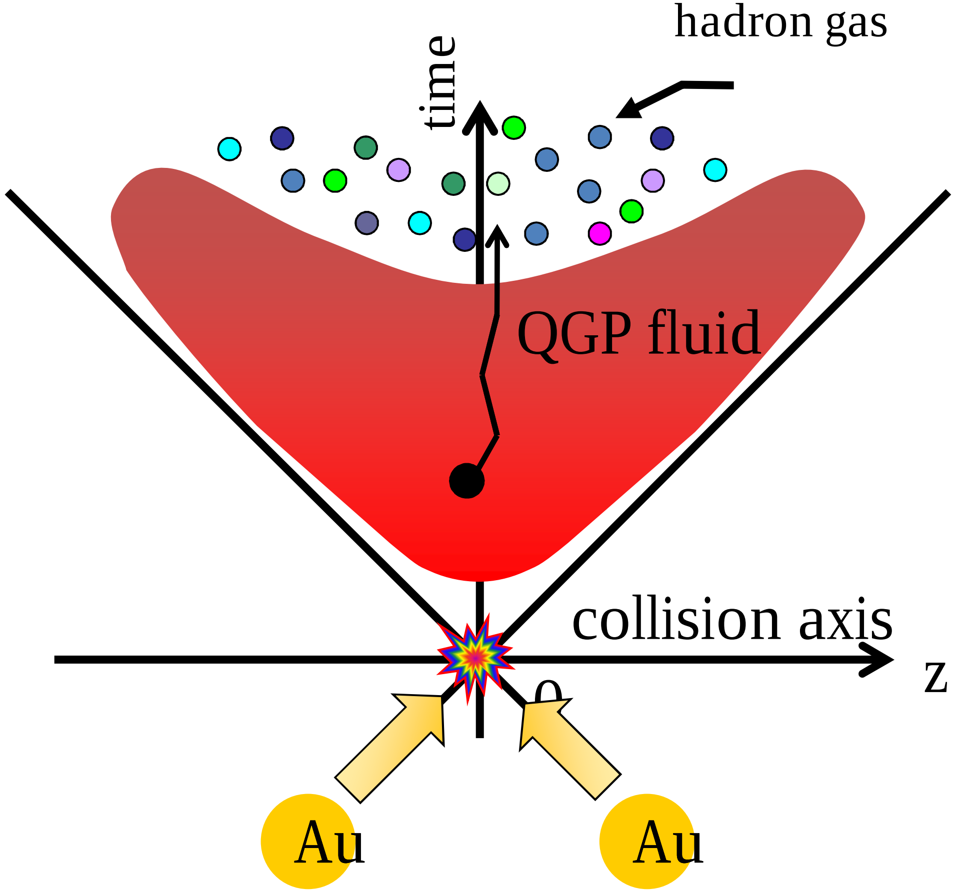
Perfect fluidity, Local thermal equilibrium
Early thermalization, Hadronization

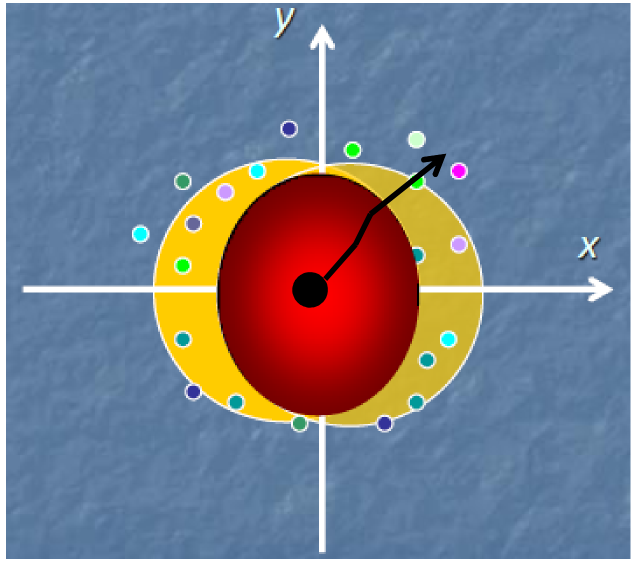
2. Heavy quark diffusion in heavy ion collision

Ref : Akamatsu, Hatsuda, Hirano, PRC79('09),054907, PRC80('09), 031901(R)

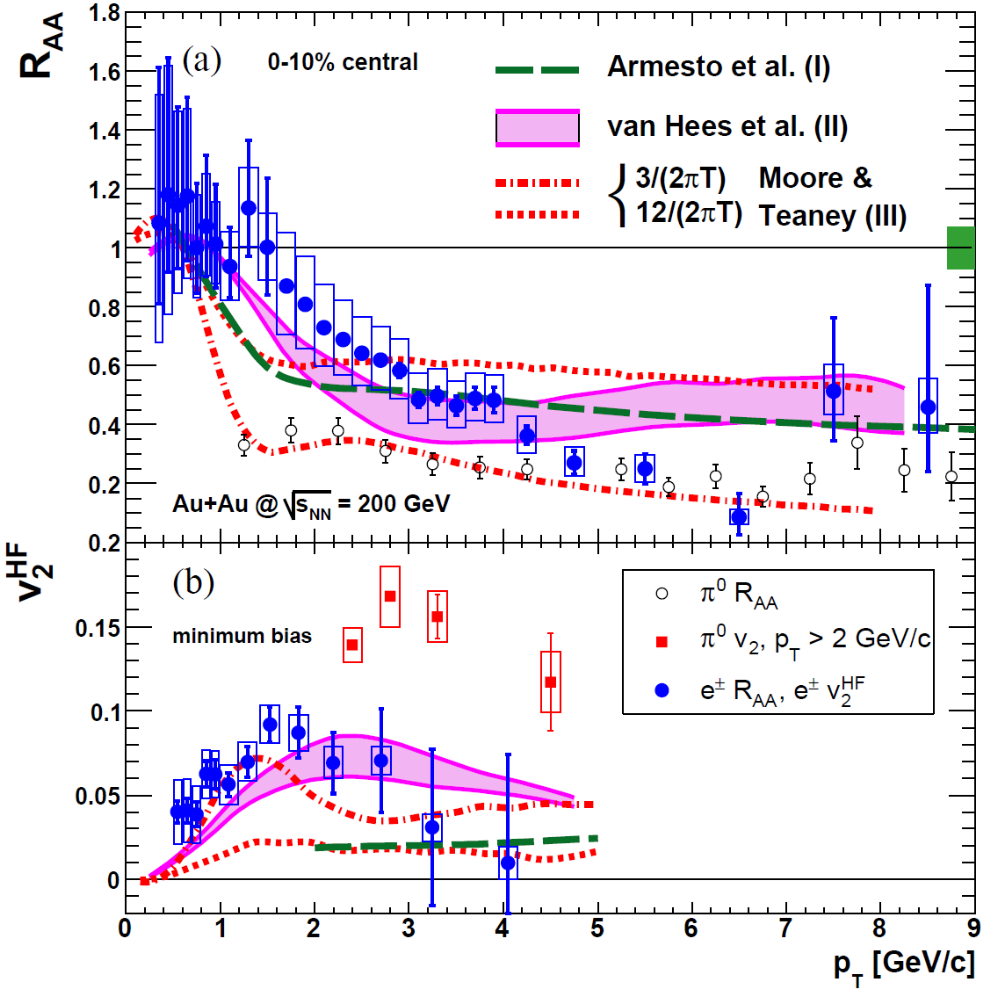
Charm and bottom in QGP

$T/M_{c,b} << 1$ → impurity probe





Previous works on charm and bottom in QGP



Radiative energy loss
Resonance scattering
Collisional energy loss

Based on weak coupling
or model calculation

How about
strong coupling "model"?

Model of heavy quark in medium

Relativistic Langevin equation

$$\Delta\vec{p} = -\gamma\frac{T^2}{M}\vec{p}\Delta t + \vec{\xi}(t)$$

$$\Delta\vec{x} = \frac{\vec{p}}{E}\Delta t$$

$$P(\vec{\xi}) \propto \exp\left[-\frac{\vec{\xi}^{\,2}}{2\kappa(p)\Delta t}\right]$$

⟺

Fokker-Planck equation by Ito discretization

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{E}\cdot\frac{\partial}{\partial\vec{x}}\right)P(\vec{p},\vec{x},t)$$

$$= \frac{\partial}{\partial\vec{p}}\left(\Gamma(p)\vec{p} + \frac{1}{2}\cdot\frac{\partial}{\partial\vec{p}}\kappa(p)\right)P(\vec{p},\vec{x},t)$$

$$\left(\Gamma(p) = \gamma T^2/M \Leftarrow \Delta\vec{p} = -\Gamma(p)\vec{p}\Delta t + \vec{\xi}\right)$$

Generalized fluctuation-dissipation theorem

$$P_{eq} \propto \exp\left(-\sqrt{p^2+M^2}/T\right)$$

$$\Gamma(p) + \frac{d\kappa(p)}{d(\vec{p}^{\,2})} = \frac{\kappa(p)}{2ET} \rightarrow \boxed{\kappa(p) = \gamma\frac{2T^3}{M}(E+T)}$$

Drag coefficient γ

Weak coupling (pQCD)

$\gamma \sim 0.2$ (leading order) ⟶ Poor convergence (Caron-Huot '08)

Strong coupling (SYM by AdS/CFT → sQGP)

N=4 SYM theory [$\propto g_{YM}^4$ for naïve perturbation]

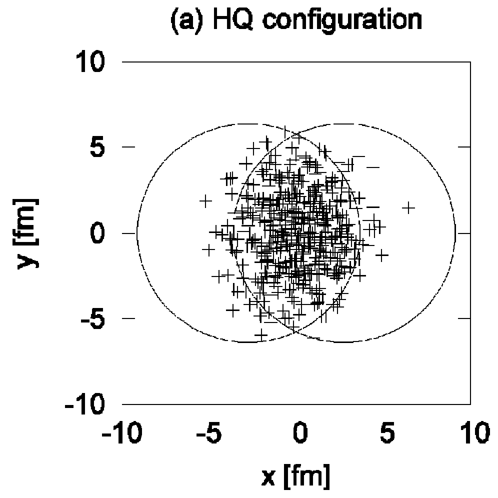
$$\frac{d\vec{p}}{dt} = -\frac{\pi\sqrt{g_{YM}^2 N}}{2} T^2 \frac{\vec{v}}{\sqrt{1-v^2}} = -\gamma \frac{T^2}{M}\vec{p} \qquad (g_{YM}^2 N, N \to \infty)$$

(Gubser '06, Herzog et al. '06, Teaney '06)

"Translation" to sQGP

$\gamma = 2.1 \pm 0.5$ (Gubser '07)

Heavy Quark Langevin + Hydro Model



0 fm…. — Little Bang

0.6 fm… — Initial Condition ← (pp + Glauber), generated by PYTHIA

QGP

Brownian Motion ← Full 3D hydrodynamics (Hirano ’06)

Local temperature and flow

T(x), u(x)

Heavy Quark Spectra

$c(b) \rightarrow D(B) \rightarrow e^- + \bar{\nu}_e + \pi$ etc

(independent fragmentation)

O(10)fm…

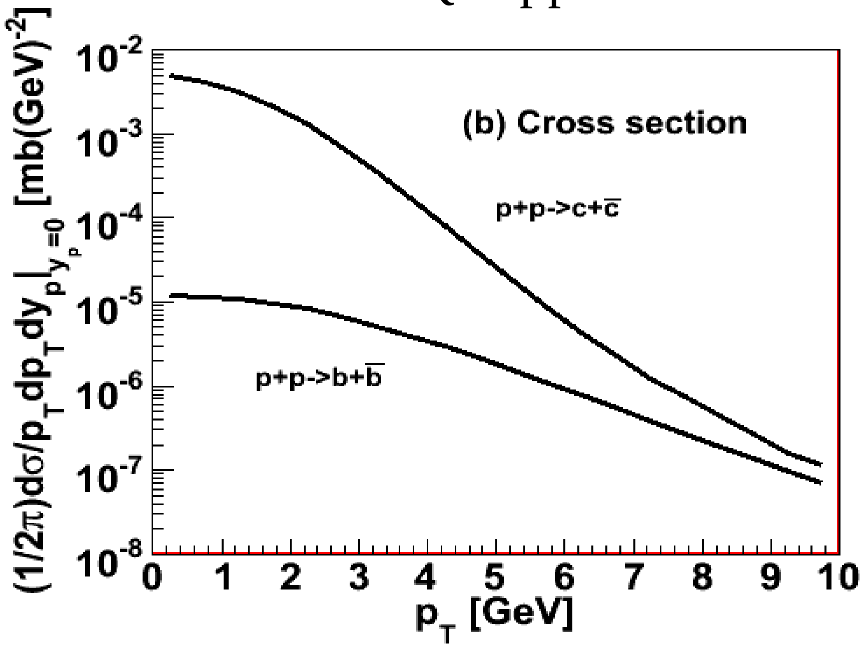
Electron Spectra + …. ⟷ Experiment (PHENIX, STAR ’07)

time

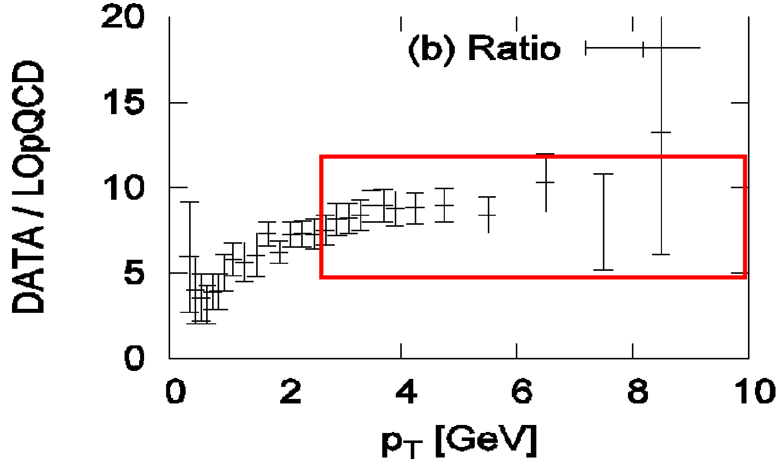
Notes in our model

✓Initial condition

<HQ in pp>



<decayed electron in pp>



available only spectral shape above p_T ~ 3GeV

Reliable at high pT

✓No nuclear matter effects in initial condition

✓No quark coalescence effects in hadronization

✓Where to stop in mixed phase at 1st order P.T.

→ 3 choices (no/half/full mixed phase)

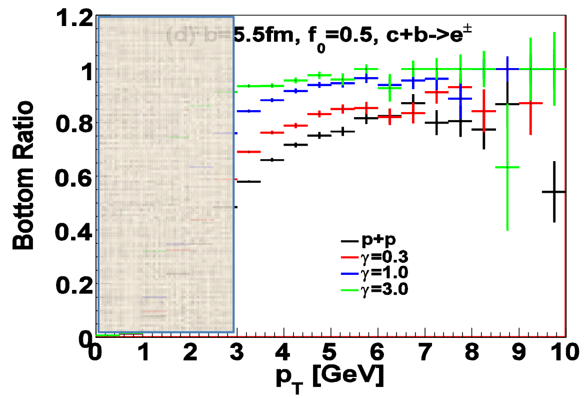
Numerical Results

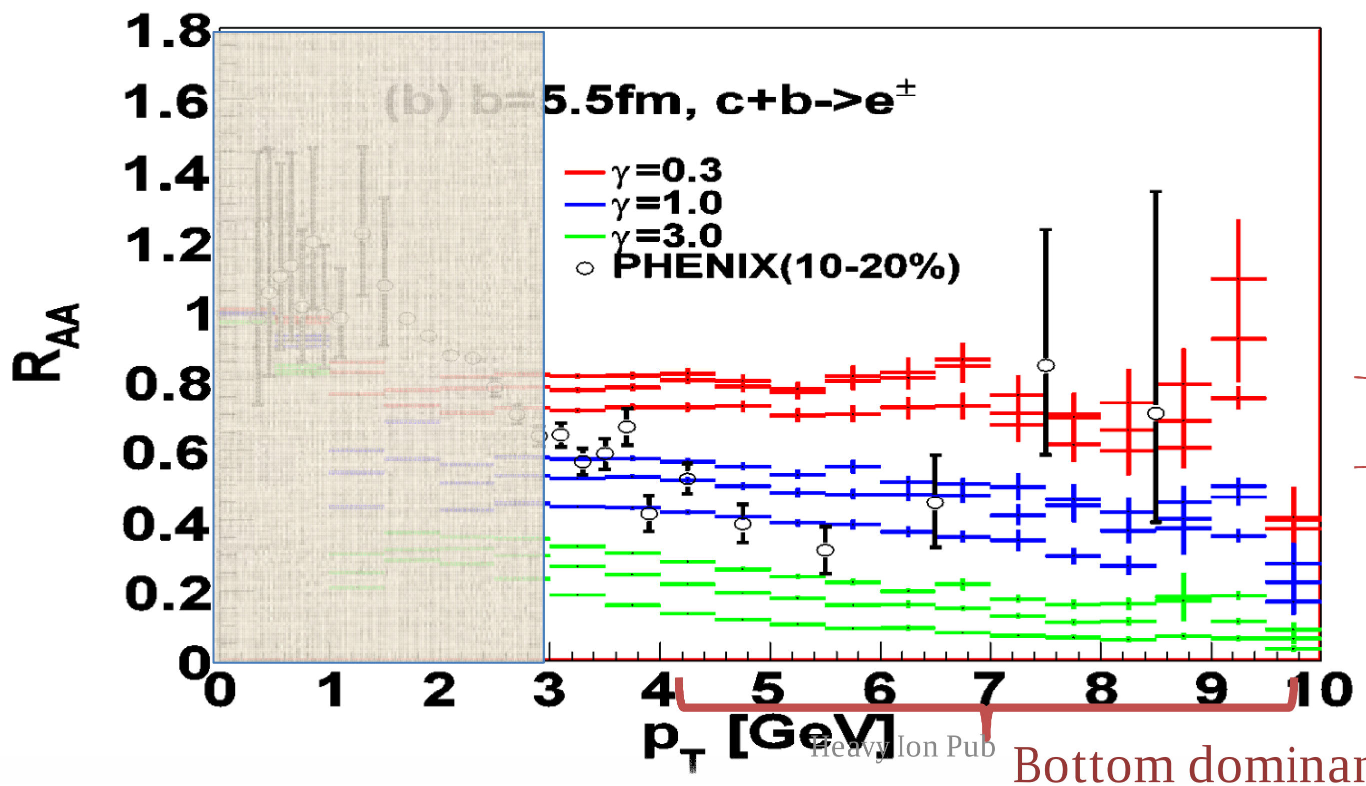
Nuclear Modification Factor

$$R_{AA} = \frac{1}{N_{coll}} \frac{dN_{A+A} / dp_T}{dN_{p+p} / dp_T}$$

Experimental result ➔ γ=1-3

⟺ AdS/CFT γ=2.1±0.5





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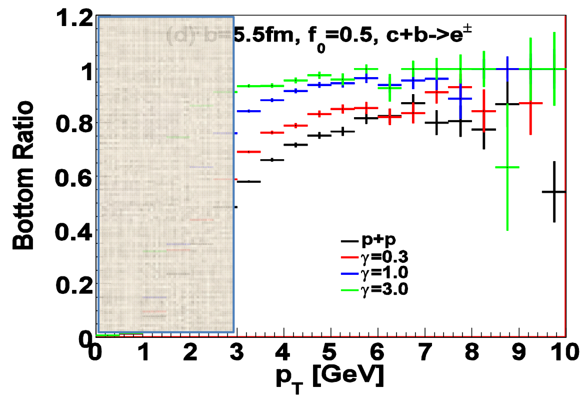
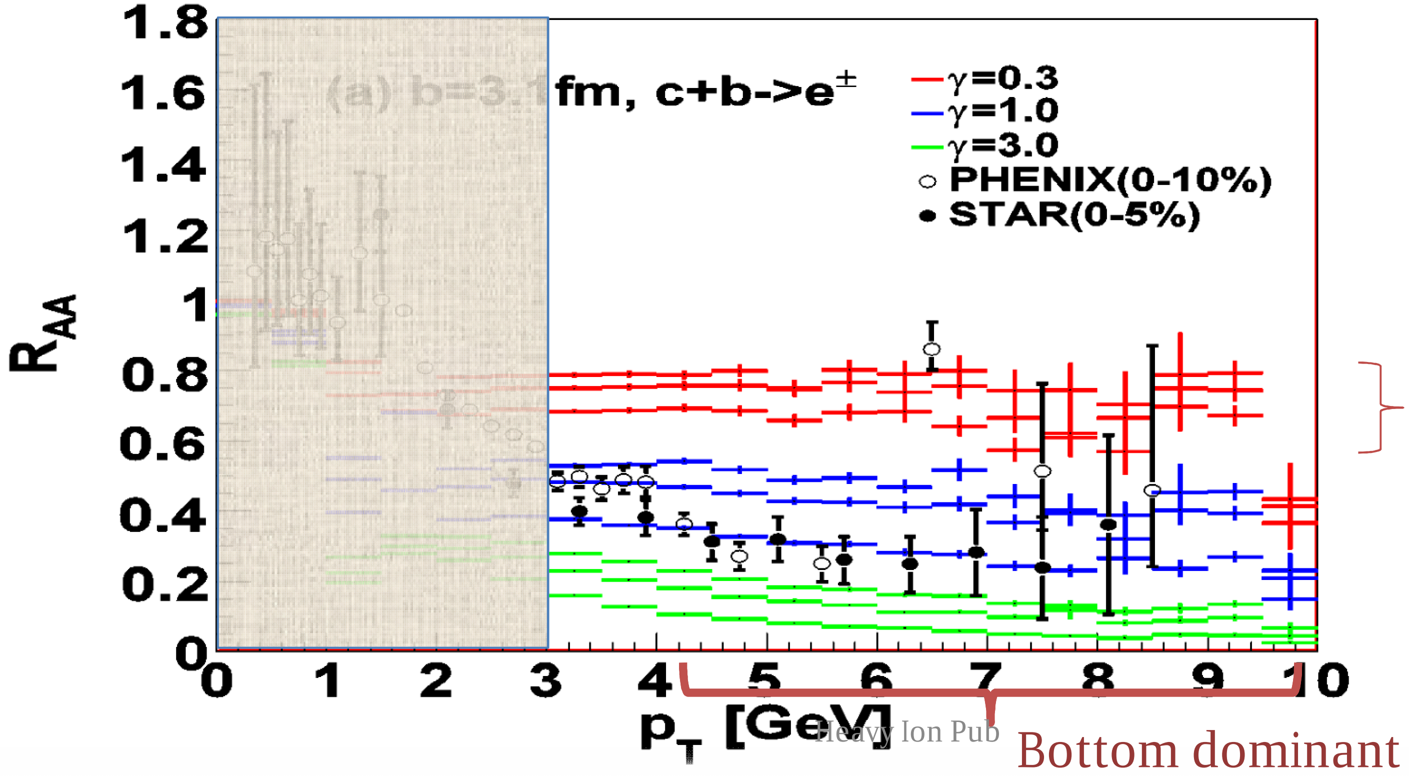


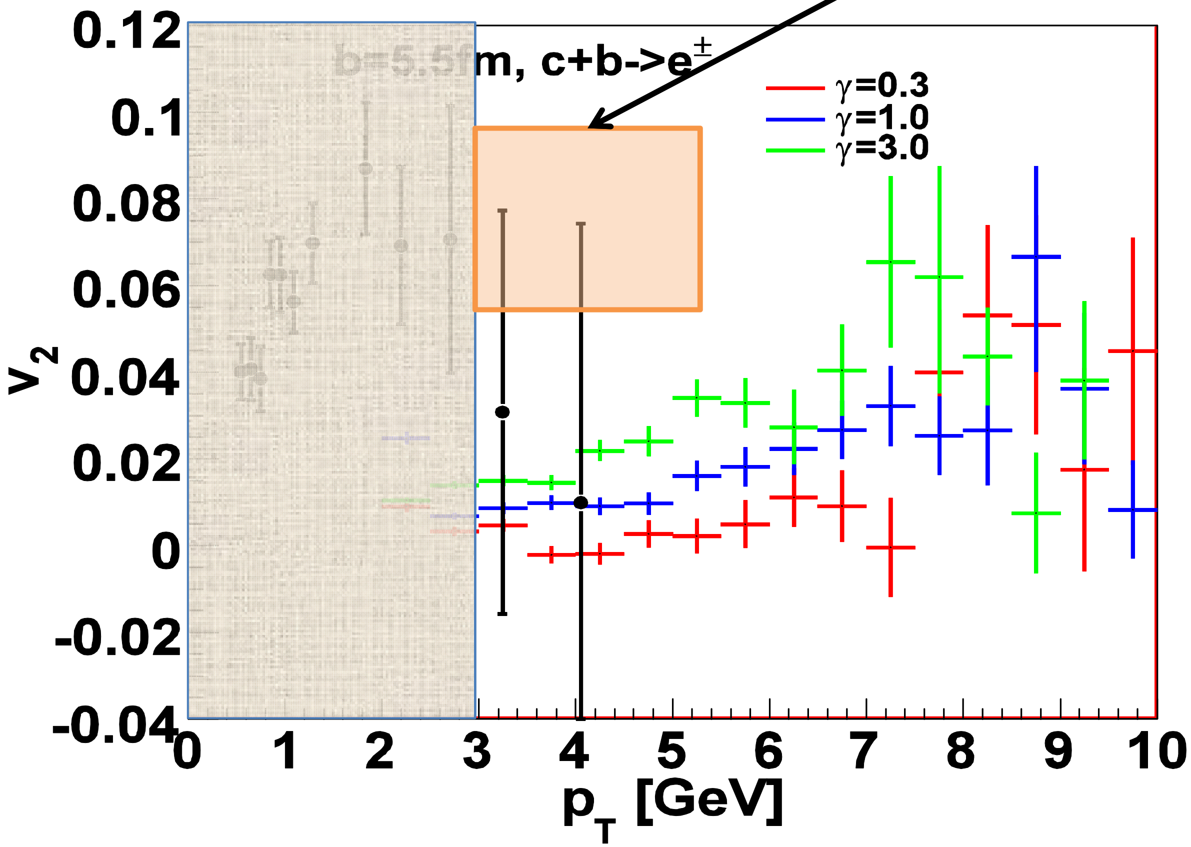


Different freezeouts at 1st order P.T.

Elliptic Flow

$$v_2(p_T) = \langle \cos 2\phi \rangle$$

Poor statistics, but at least consistent with γ=1-3.
(Still preliminary, PHENIX at Run7: v_2~0.05-0.1 for p_T~3-5GeV)



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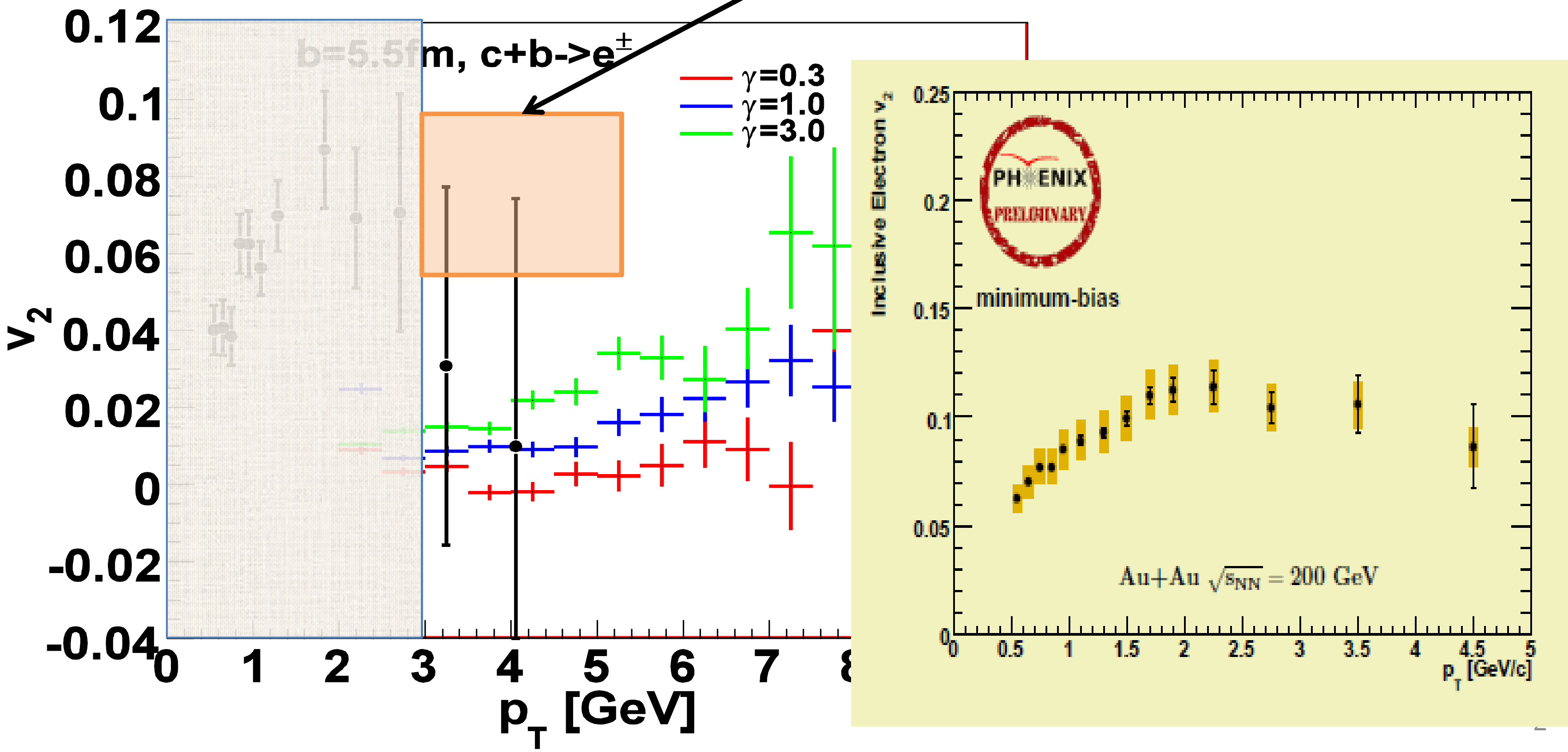
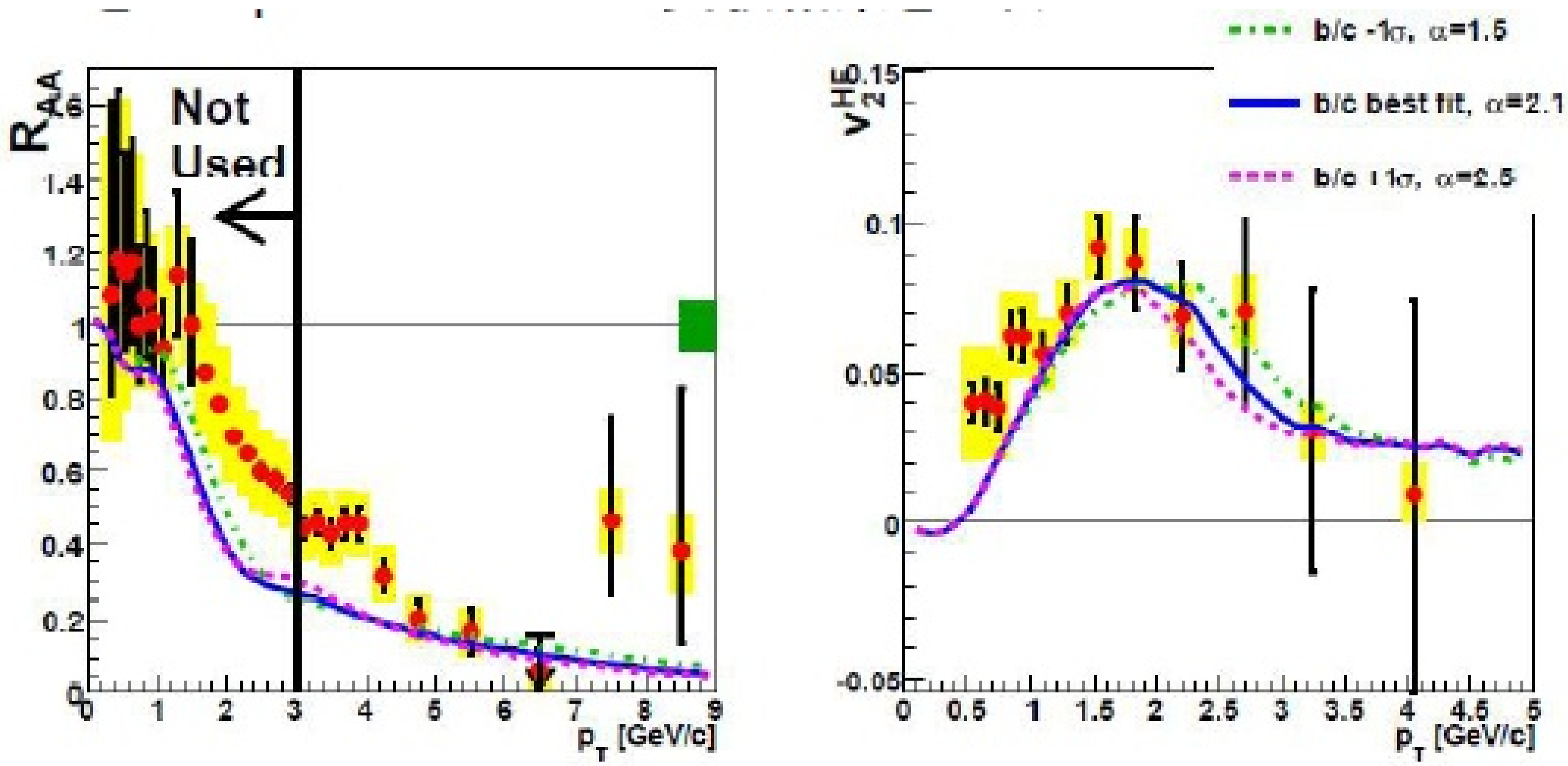


Including Recombination Model



Strong "medium" effect at freezeout can explain large v2.

Degree of HQ Thermalization

- Stay time $t_S \sim 3-4[\mathrm{fm}]$
- Relaxation time $\tau_{HQ} \equiv \frac{M}{\gamma T^2}$

	$\gamma = 0.3$	$\gamma = 1$	$\gamma = 3$
$\tau_c[\mathrm{fm}]$	22	6.7	2.2
$\tau_b[\mathrm{fm}]$	72	21	7.2

thermalized

not thermalized

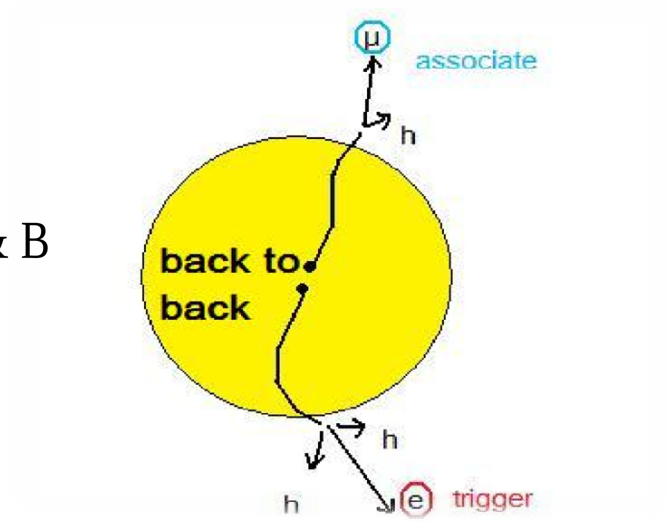
Experimental result **γ**=1-3
→ charm : nearly thermalized,
bottom : not thermalized

Azimuthal Correlation

Back to back correlation of a heavy quark pair

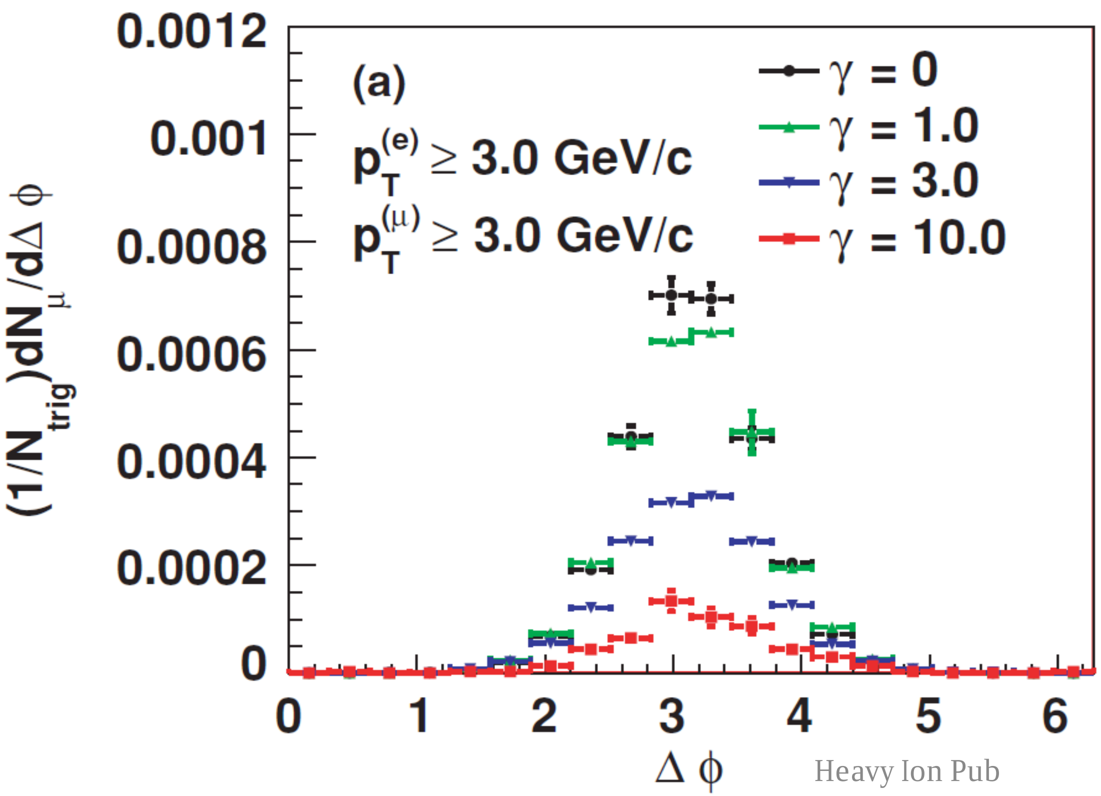
⬇ diffusion

Loss of correlation in decay products from D & B



e(mid)-μ(fwd) correlation : one peak

no contribution from vector meson decay



IAA : quantitative measure

$$\Sigma_{AA} = \int_{\phi_{min}}^{\phi_{max}} d(\Delta\phi) \left[\frac{1}{N_{trig}} \frac{dN_{assoc}}{d\Delta\phi} \right]_{ZYAM}$$

$$I_{AA} = \Sigma_{AA} / \Sigma_{pp}$$

e-μ azimuthal correlation: sensitive probe for heavy quark thermalization rate

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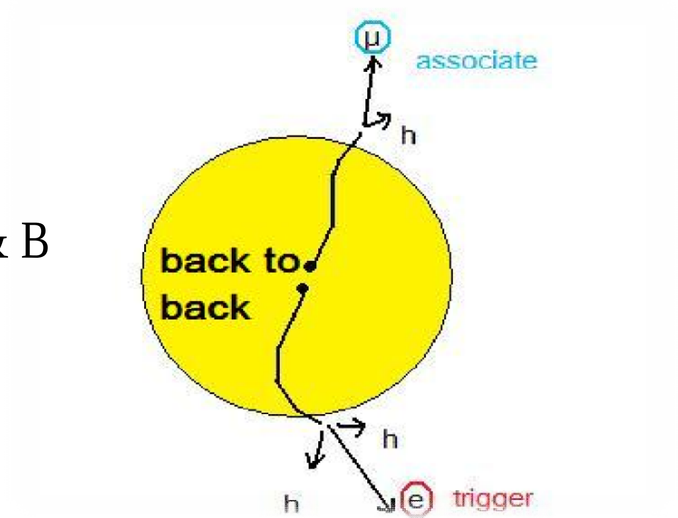

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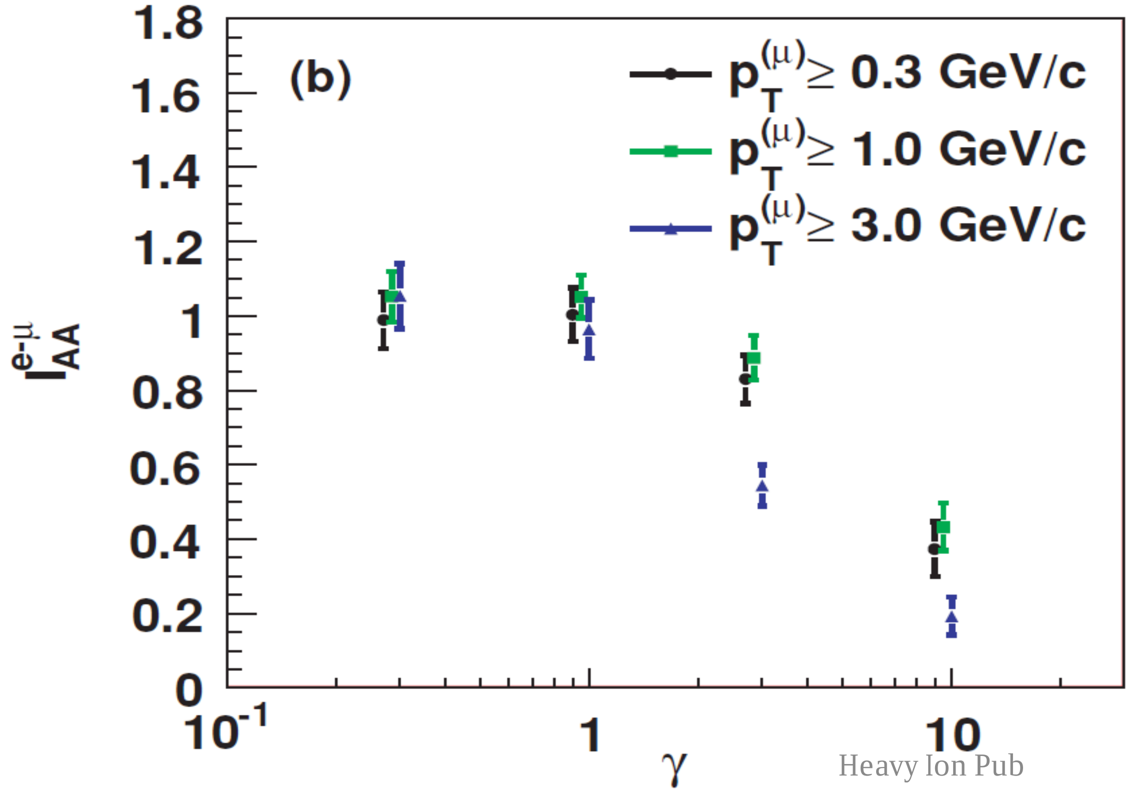

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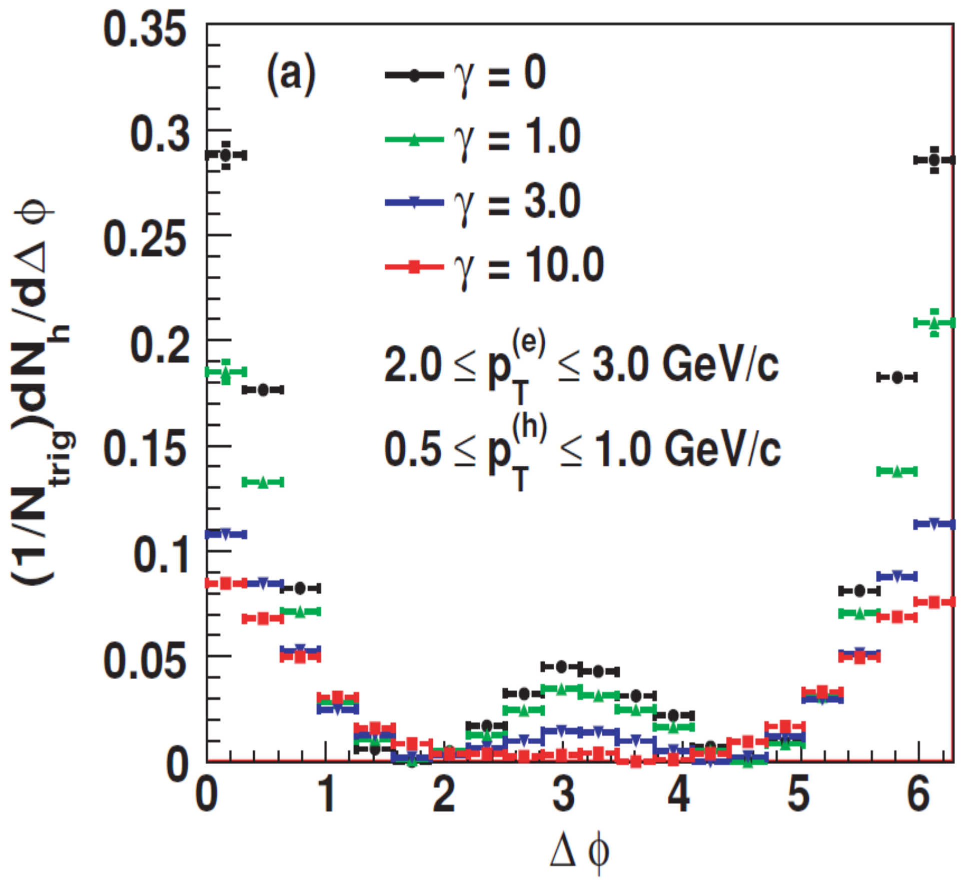
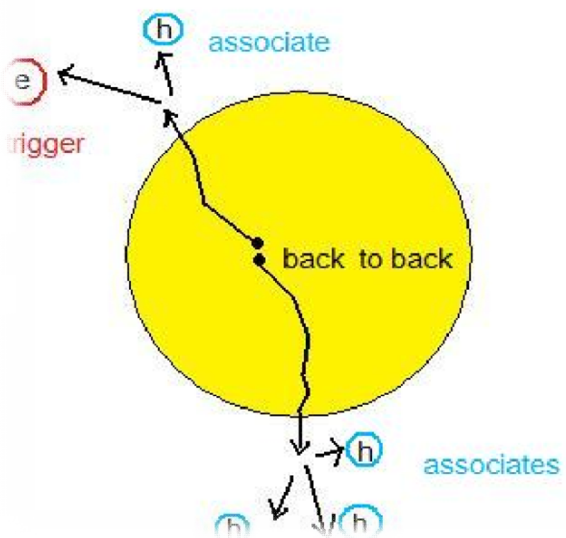
e-μ azimuthal correlation: sensitive probe for heavy quark thermalization rate e-h correlation (mid) : two peaks





A sensitive probe but not clean ...

Effects we ignore :

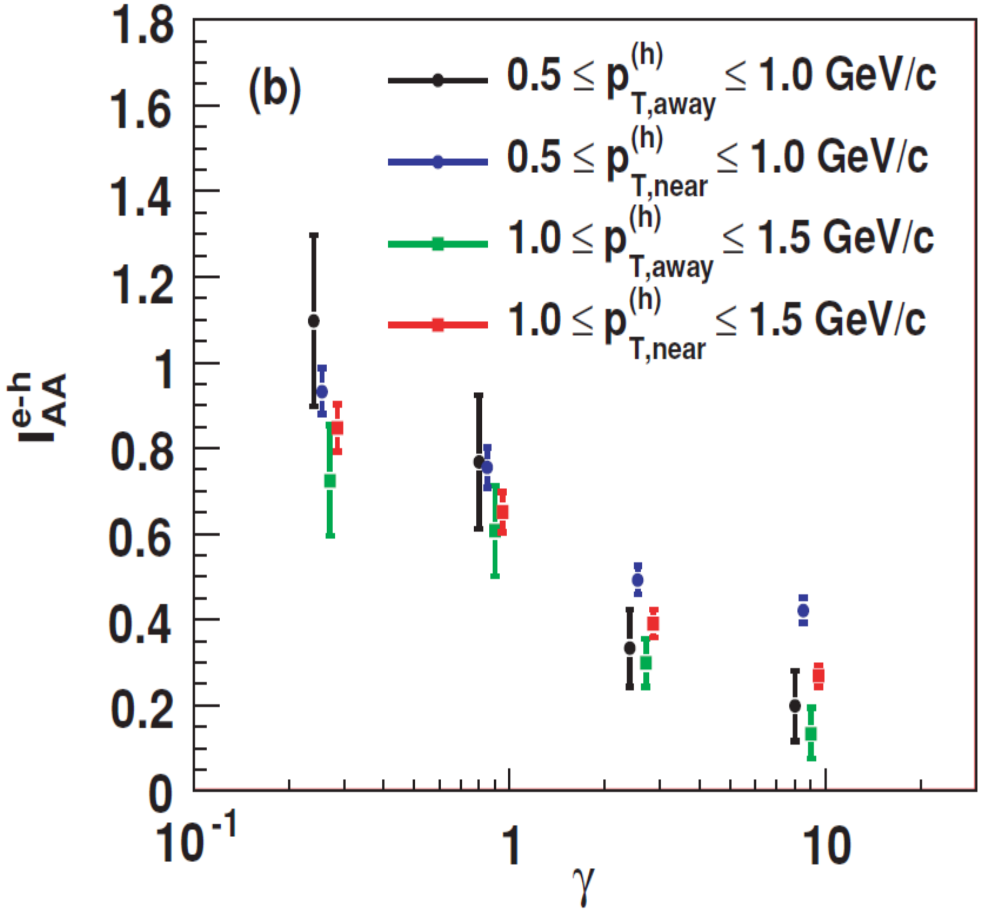
- Hadronic interaction of associates
- Medium response to HQ propagation
- Fictitious correlation due to bulk v2

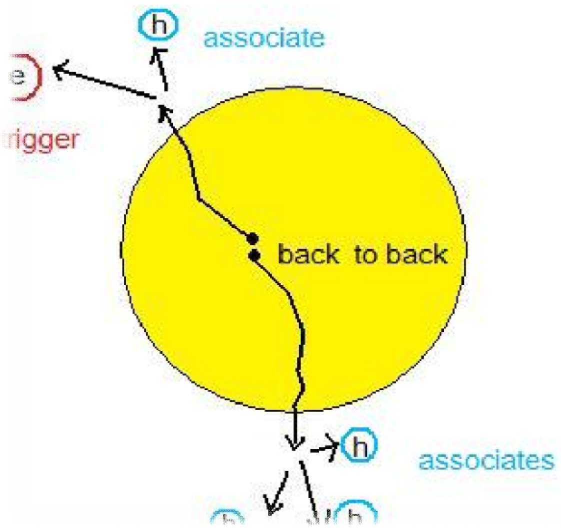
Relative angle range for IAA

Near side : $-0.5\pi \leqq \Delta\phi \leqq 0.5\pi$

Away side : $0.5\pi \leqq \Delta\phi \leqq 1.5\pi$

e-h correlation (mid) : two peaks





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Outlook for updates

- FONLL initial condition
- Systematic study using Bag EoS and Lattice EoS for hydro

3. Heavy quark diffusion from spectral function

Goal

- Non-perturbative definition of drag parameter

Ref :

Petreczky and Teaney '06
Caron-Huot and Moore '08
Caron-Huot, Laine and Moore '09
Burnier, Laine, Langelage, Mether '10

Response and spectral function

$$\rho^{00}(k) \equiv \int d^4x \frac{1}{2}\left\langle [J^0(x), J^0(0)]\right\rangle \exp(ik\cdot x)$$

Linear response theory

$$\left\langle J^0(x)\right\rangle = i\int_{-\infty}^{\infty} dt' \int d^3r\,\theta(t-t')\left\langle [J^0(x), J^0(x')]\right\rangle A_0(x')$$

Switch off external field A0 at t=0

$$A_0(x) = \theta(t) A_0(\vec{r})\mathrm{e}^{\varepsilon t}$$

Response at t>0

$$J^0(\vec{k}, z) \equiv \int d^3r \int_0^{\infty} dt \left\langle J^0(x)\right\rangle \mathrm{e}^{izt - i\vec{k}\cdot\vec{r}} \qquad (\mathrm{Im}\, z > 0)$$

$$\Rightarrow \mathrm{Re}\left[\frac{J^0(\vec{k}, z = \omega + i\varepsilon)}{A_0(\vec{k})}\right] = \frac{\rho^{00}(\vec{k}, \omega)}{\omega}$$

"hydrodynamics"

Spectral function from "hydrodynamics"

Conservation law

$$\partial_\mu J^\mu = 0$$

Constitutive equation

$$\partial_t \vec{J} = -\frac{1}{\tau}\left[\vec{J} + D\vec{\nabla} J^0\right]$$

(Introduction of relaxation time τ)

Response at t>0

$$J^0(\vec{k}, z) = \frac{\chi(1 - iz\tau)A_0(\vec{k})}{-iz + Dk^2 - \tau z^2}$$

susceptibility

$$\chi \equiv \frac{1}{T}\int d^3r \left\langle J^0(\vec{r}) J^0(\vec{0})\right\rangle$$

$$\rho^{00}(\vec{k}, \omega) = \frac{\chi D k^2 \omega}{\omega^2 + (Dk^2 - \tau\omega^2)^2}$$

$$\rightarrow \sum_i \rho^{ii}(\vec{k} = \vec{0}, \omega) = \frac{3\chi D\omega}{1 + \tau^2\omega^2}$$

Spectral function from Fokker-Planck equation

(Non-relativistic) Fokker-Planck equation

$$\left(\partial_t + \frac{\vec{p}}{M}\cdot\vec{\nabla}\right)P(\vec{p},\vec{r},t)$$

$$= \frac{\partial}{\partial\vec{p}}\left(\Gamma\vec{p} + \frac{1}{2}\cdot\frac{\partial}{\partial\vec{p}}\kappa\right)P(\vec{p},\vec{r},t)$$

$$\left(\Delta\vec{p} = -\Gamma\vec{p}\Delta t + \vec{\xi},\ \kappa = 2\Gamma MT\right)$$

Definition of current

$$J^0(x) \equiv \int d^3r P(\vec{p},\vec{r},t)$$

$$\vec{J}(x) \equiv \int d^3r \frac{\vec{p}}{M} P(\vec{p},\vec{r},t)$$

Conservation law

$$\partial_\mu J^\mu = 0$$

Constitutive equation

$$\partial_t \vec{J}(x) = -\nabla_i \int d^3p \frac{p_i p_j}{M} P(\vec{p},\vec{r},t) - \Gamma \vec{J}(x)$$

$$\left\langle \frac{p_i p_j}{M} \right\rangle_{\substack{(local)\\ equilibrium}} = J^0(x)\frac{T(x)}{M}\delta_{ij} \cong J^0(x)\frac{T}{M}\delta_{ij}$$

$$\therefore \partial_t \vec{J}(x) = -\frac{T}{M}\vec{\nabla}J^0(x) - \Gamma\vec{J}(x) = -\Gamma\left(\vec{J}(x) + \frac{T}{M\Gamma}J^0(x)\right)$$

$$\rho^{00}(\vec{k},\omega) = \frac{\chi\left(k^2T/M\Gamma\right)\omega}{\omega^2 + \left(k^2T/M\Gamma - \omega^2/\Gamma^2\right)^2}$$

$$\sum_i \rho^{ii}(\vec{k}=\vec{0},\omega) = \frac{3\chi\left(T/M\Gamma\right)\omega}{1+\left(\omega/\Gamma\right)^2}$$

$$\Gamma \Leftrightarrow 1/\tau,\ T/M\Gamma \Leftrightarrow D$$

$$\partial_t \vec{J} = -\frac{1}{\tau}\left[\vec{J} + D\vec{\nabla}J^0\right]$$

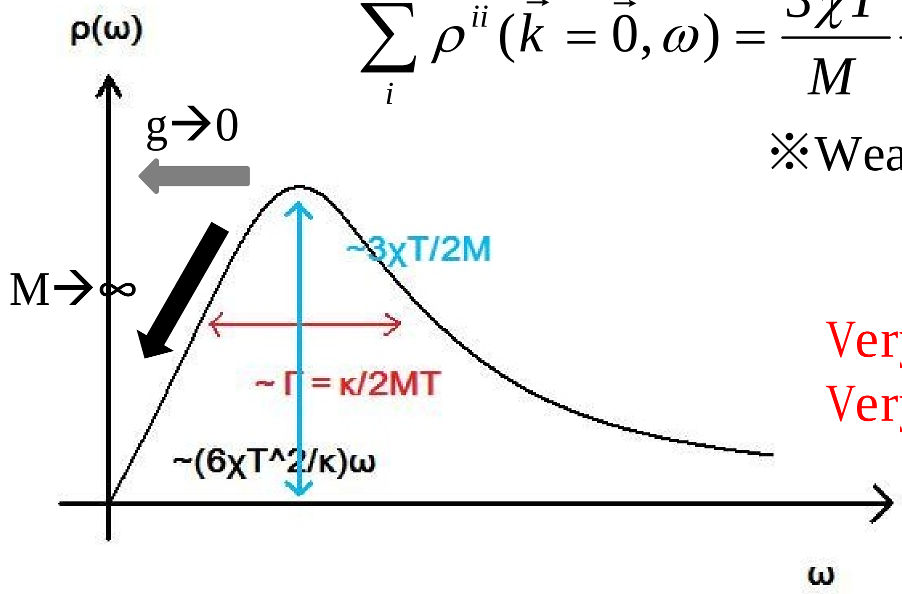
Kubo formula for the drag parameter

"normal" Kubo formula

$$\lim_{\omega \to 0} \sum_i \frac{1}{\omega} \rho^{ii}(\vec{k} = \vec{0}, \omega) = \frac{6\chi T^2}{\kappa}$$

$$\sum_i \rho^{ii}(\vec{k} = \vec{0}, \omega) = \frac{3\chi T}{M} \frac{(\kappa / 2MT)\omega}{\omega^2 + (\kappa / 2MT)^2} \underset{M \to \infty}{\to} \frac{3\chi T}{M} \pi\omega\delta(\omega)$$

※Weak coupling : $\kappa \propto g^4$



Very tiny peak for very heavy quark
Very narrow and steep for weak coupling

Euclidean correlator → SPF

(Petreczky and Teaney '06)

Lattice calculation for Euclidean correlator
is found to be insensitive to diffusion constant κ.
→cannot extract κ

Alternative definition of the drag parameter

$$\sum_i M^2 \omega \rho^{ii}(\vec{k}=\vec{0},\omega) = 3\chi MT \frac{(\kappa/2MT)\omega^2}{\omega^2+(\kappa/2MT)^2} \xrightarrow[M\to\infty]{} \frac{3\chi}{2}\kappa$$

⬇

$$\kappa = \frac{2}{3\chi}\lim_{\omega\to 0}\frac{1}{\omega}\sum_i \rho_{E_Q^i E_Q^i}(\omega,\vec{k}=\vec{0}) = \frac{1}{6T\chi}\int_{-\infty}^{\infty} d^4x \left\langle \left\{ \vec{E}_Q(x), \vec{E}_Q(0) \right\} \right\rangle_Q$$

$$\vec{E}_Q \equiv Q^+ g\vec{E}Q - Q_c^+ g\vec{E}Q_c$$

No sharp or low peak expected
because EQ is not even an approximately conserved quantity.
M→∞: too heavy to change velocity, therefore spatial current is conserved.
g→0: spatial current will not undergo scattering, therefore conserved.

Leading order perturbation

→ Wilson line=1

Hard Thermal Loop

$$\kappa = \frac{16\pi\alpha_s}{9}\int \frac{d^3p}{(2\pi)^3} p^2 G_{00}^{>,\mathrm{HTL}}(\omega=0,p)$$

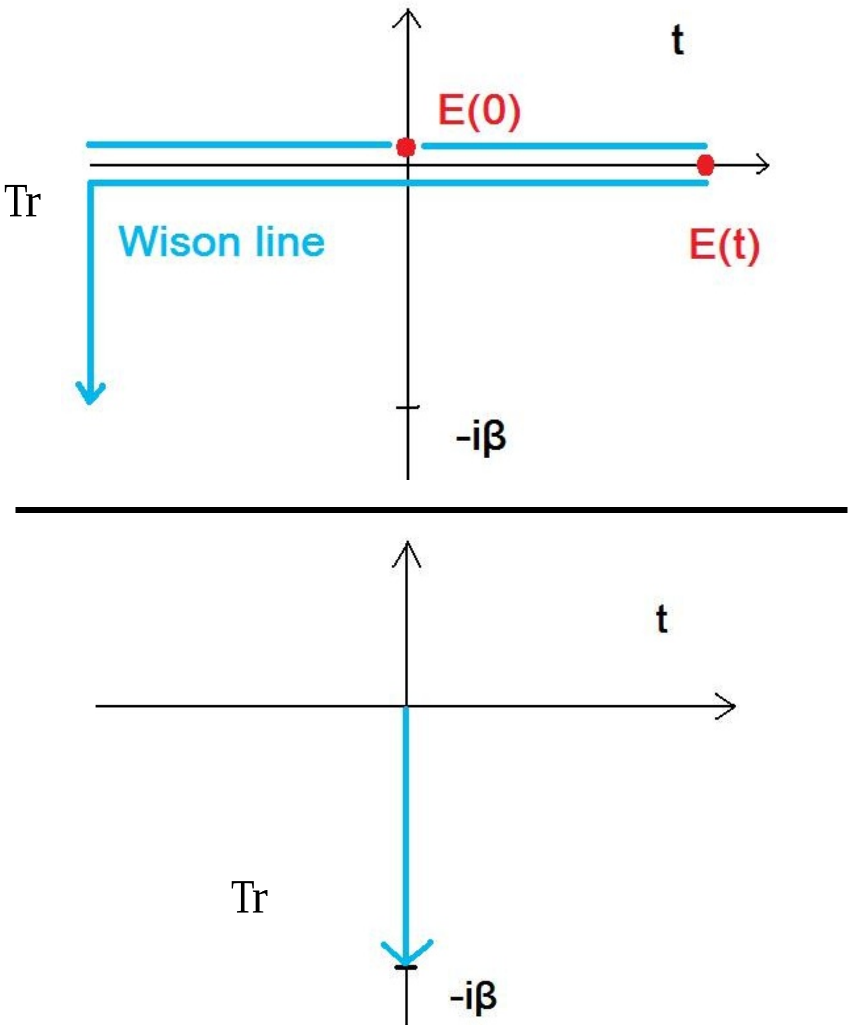
$$\kappa = \frac{C_H g^4 T^3}{18\pi}\left[N_c(\ln\frac{2T}{m_D}+\xi) + \frac{N_f}{2}(\ln\frac{4T}{m_D}+\xi)\right]$$

$$C_H = 4/3, \xi = -0.64718$$

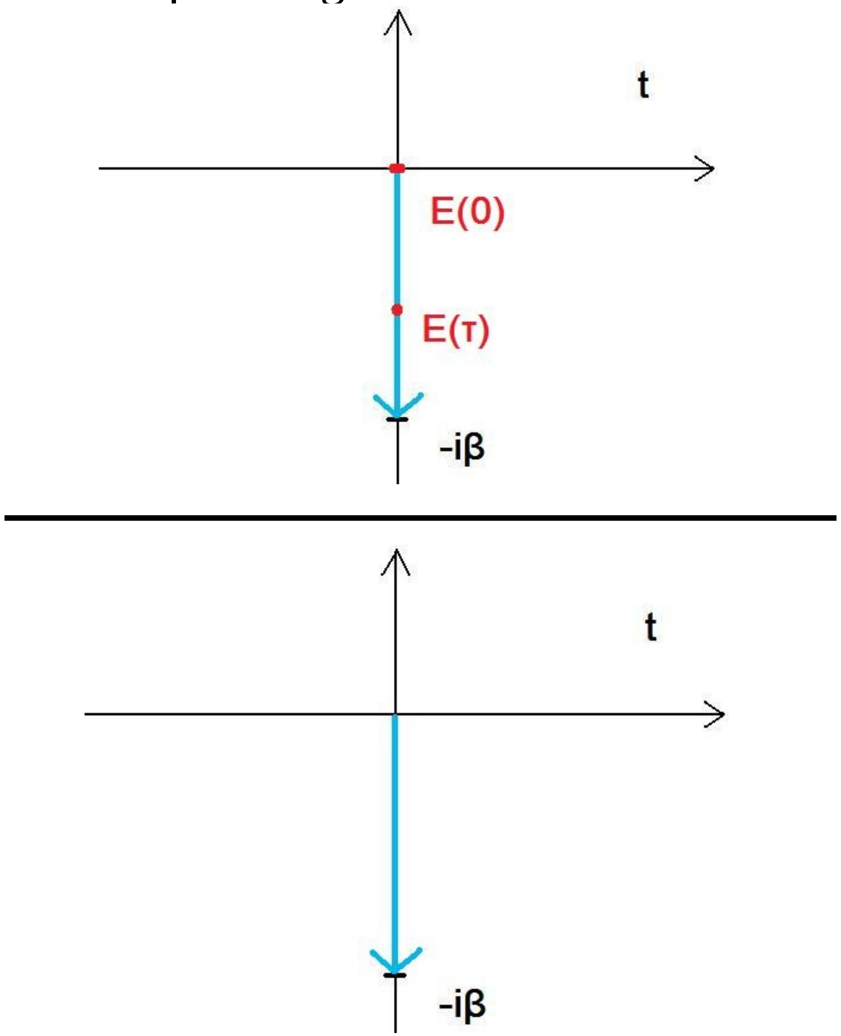
Caron-Huot and Moore '08

Integrating out heavy quark field

Real time contour



Corresponding Euclidean correlator



A way to lattice QCD

$$G(\tau) \equiv \frac{2T}{3} \frac{\left\langle \mathrm{Tr} W(-i\beta;-i\tau)\vec{E}_{eucl}(-i\tau)W(-i\tau;0)\vec{E}_{eucl}(0)\right\rangle}{\left\langle \mathrm{Tr} W(-i\beta;0)\right\rangle}$$

$$= -\int_0^\infty \frac{d\omega}{\pi} \frac{\cosh(\beta/2-\tau)\omega}{\sinh \beta\omega/2}\rho(\omega) \quad (\vec{\mathrm{E}}_{eucl} = \mathrm{i}\vec{\mathrm{E}}_{\mathrm{mink}})$$

$$\kappa = \lim_{\omega\to 0}\frac{1}{\omega}\rho(\omega)$$

Caron-Huot, Laine and Moore '09

Lattice calculation around Tc may be a noisy measurement because of the Polyakov in the denominator of G(τ).

4. Summary

- Heavy quark drag parameter $\Gamma \sim 2T^2/M$ is extracted from phenomenological study.

- Drag parameter near Tc is interesting and may be relevant to heavy quark hadronization.

- Heavy quark (momentum) diffusion constant κ is defined non-perturbatively.
  In near future, lattice determination may be possible.