




# *Chiral Magnetic Effect*



Kenji Fukushima

(Department of Physics, Keio University)

# Talk Plan

- 
- Chiral Magnetic Effect ~ conventional explanation
  - Current from infinity – *What really flows?*
  - Chiral Magnetic Effect in a hadronic phase
  - Is there a current... or not? *Signature of what??*
  - Experimental challenges and theories

# Chiral Magnetic Effect

~ conventional explanation ~

# Basic Formula



## Chiral Magnetic Effect

$$\mathbf{j} = N_c \sum_{\text{flavor}} \frac{q_f^2 \mu_5}{2\pi^2} \mathbf{B}$$

# Basic Formula



## Chiral Magnetic Effect

Hard to understand

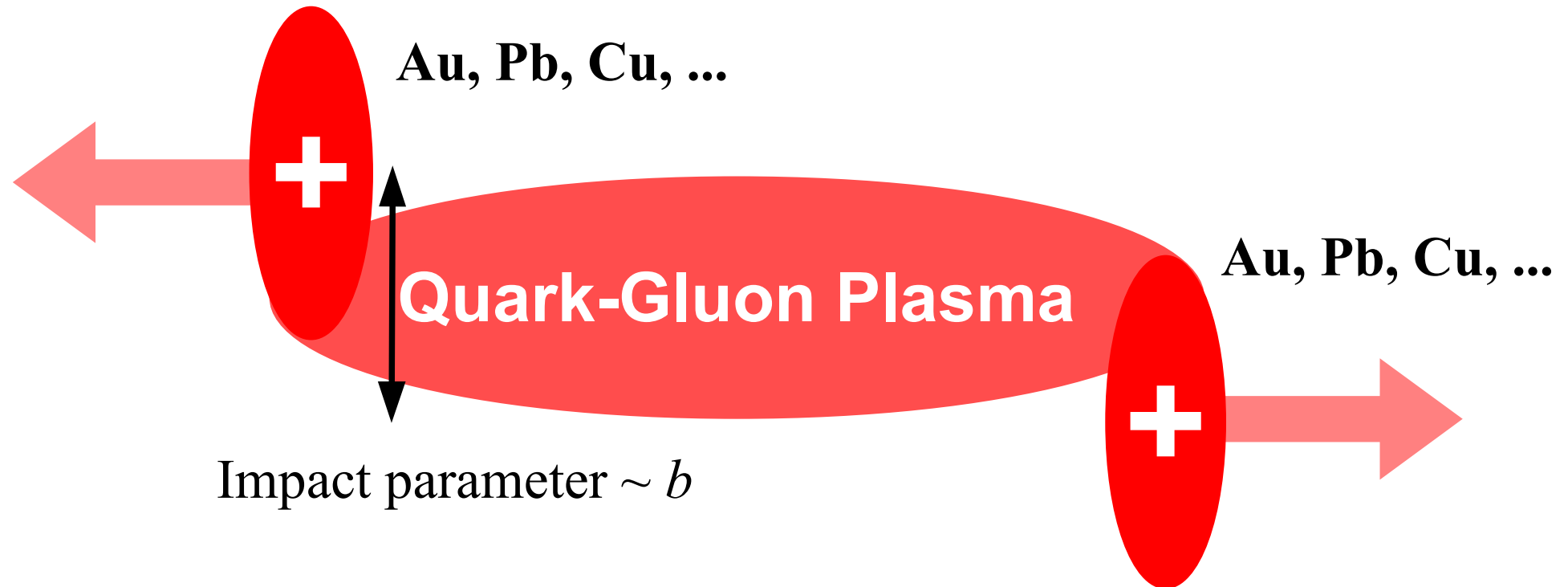
$$\mathbf{j} = N_c \sum_{\text{flavor}} \frac{q_f^2 \mu_5}{2\pi^2} \mathbf{B}$$

Easy to understand

Very hard to understand

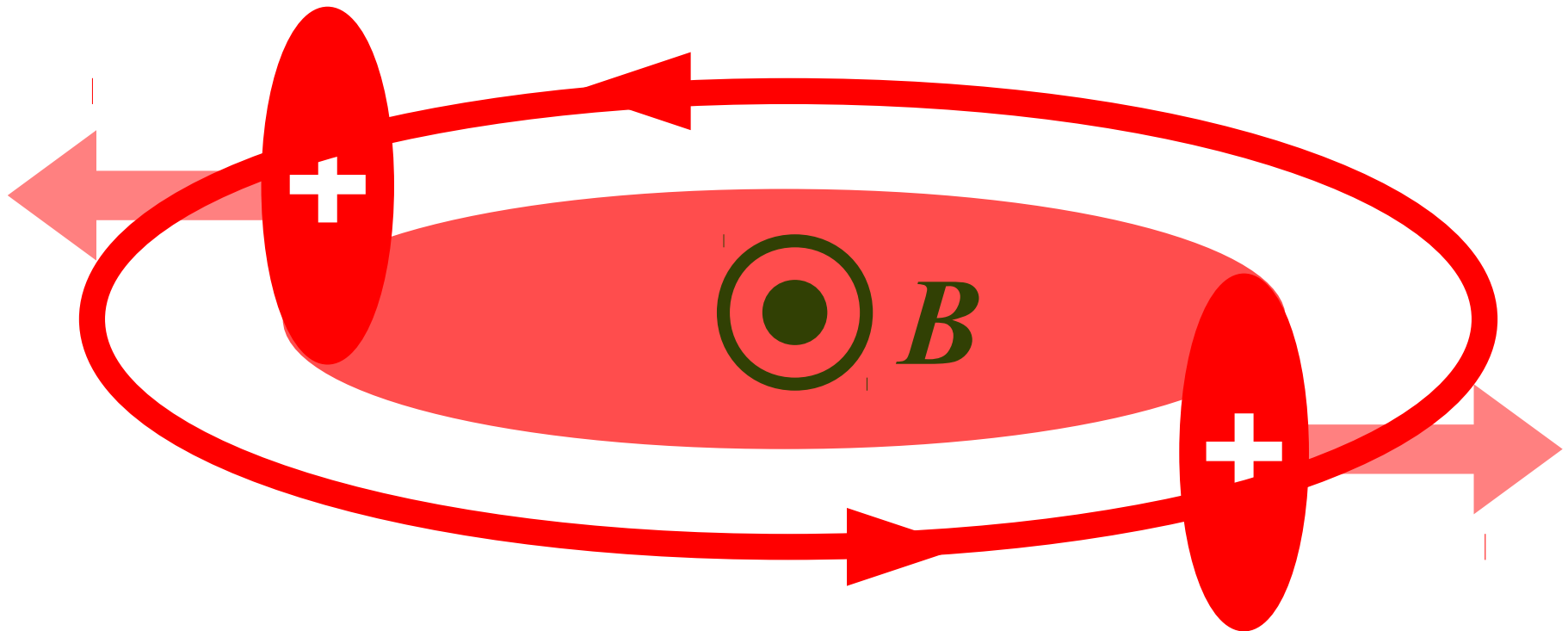
# Origin of the Magnetic Field

Moving almost at the speed of light



# Origin of the Magnetic Field

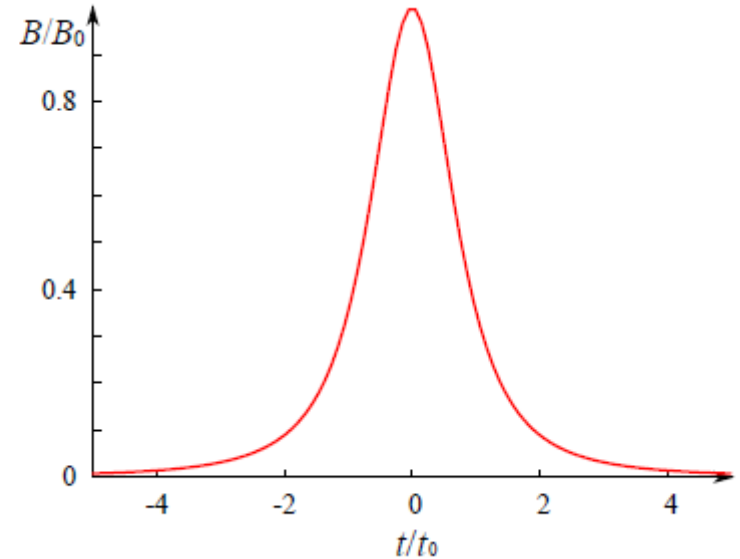
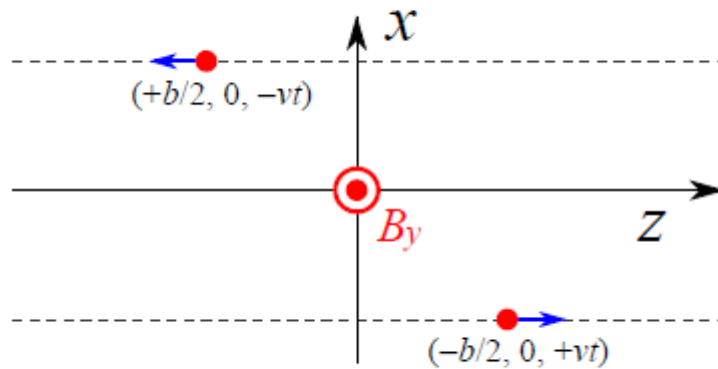
**Strong  $B$  generated due to Electrodynamics**



**on top of the Quark-Gluon Plasma**

# Point-charge Approximation

## Lienard-Wiechert potential



$$eB(t) = \frac{eB_0}{[1 + (t/t_0)^2]^{3/2}}$$

$$eB_0 = (47.6 \text{ MeV})^2 \left( \frac{1 \text{ fm}}{b} \right)^2 Z \sinh(Y) \sim \gamma, \quad t_0 = \frac{b}{2 \sinh(Y)}$$

Discussed by Rafelski, Mueller, ... (~1976)

May 10, 2012 @ HIP



# *Physics of the Strong Magnetic Field*



**Magnetic field looks like a “medium”**

A photon becomes massive.

*Generally, the dispersion relations of photons with respective polarizations are distorted (**Birefringence**)*

A photon on top of  $B$  decays into pair particles.

Chiral symmetry breaking is enhanced.

*Not known how the phase diagram is affected.*

*Pion structural change is hardly understood...*

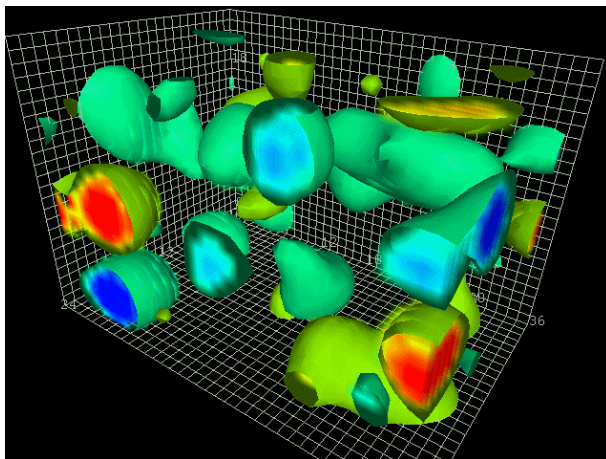
# Origin of the Chiral Chemical Potential

## Generation of the chiral charge

$$\frac{dN_5}{dt} = -\frac{g^2 N_f}{8\pi^2} \int d^3x \operatorname{tr} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

**Chirality** ← **Topological charge density**

Topologically non-trivial gauge configuration



$$\Rightarrow N_5 \Rightarrow \mu_5$$

# Origin of the Topological Charge $Q_w$



## Topologically non-trivial gauge configurations

### Instanton (tunneling under barrier)

Lives only in Euclidean space-time

Makes the  $\eta'$  meson heavy

**Suppressed at high temperature**  $\chi \sim \exp[-c \rho^2 T^2]$

### Sphaleron (excitations over barrier)

Lives in Minkowski space-time

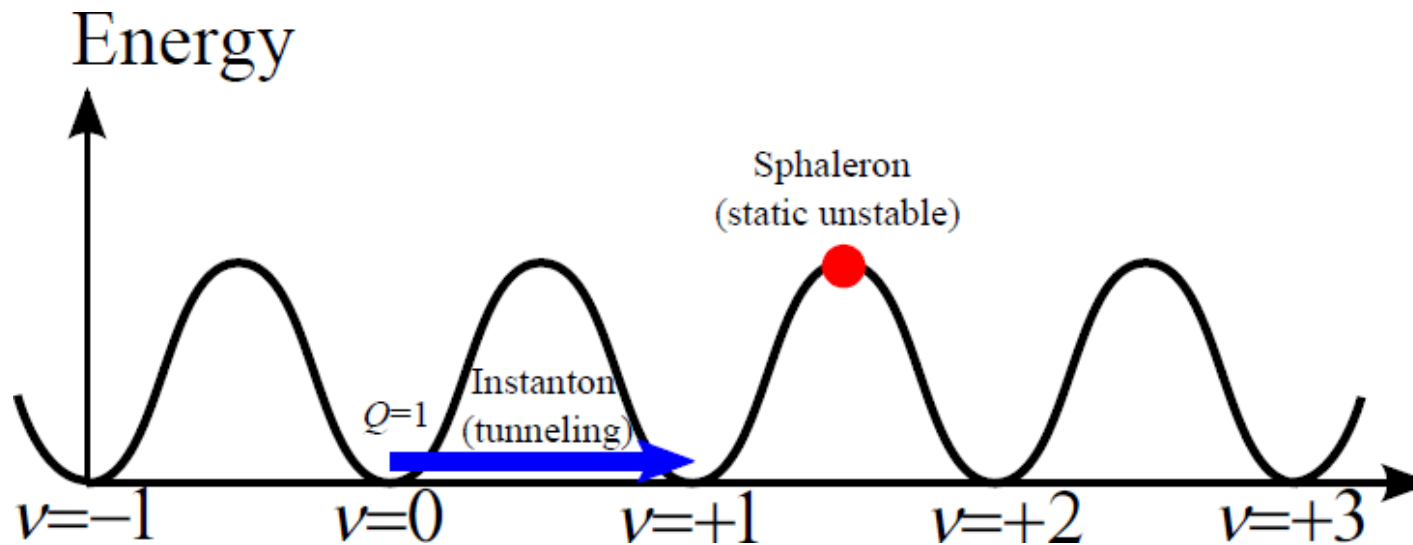
Breaks the baryon and lepton numbers (in the weak int.)

**Enhanced at high temperature**  $\Gamma \sim T^4$  Arnold-McLerran  
Kharzeev-McLerran-  
-Warringa

**Sphalerons get excited ( $\sim 1\text{fm}/c$ ) only after  $B$  decays**

# Origin of the Topological Charge

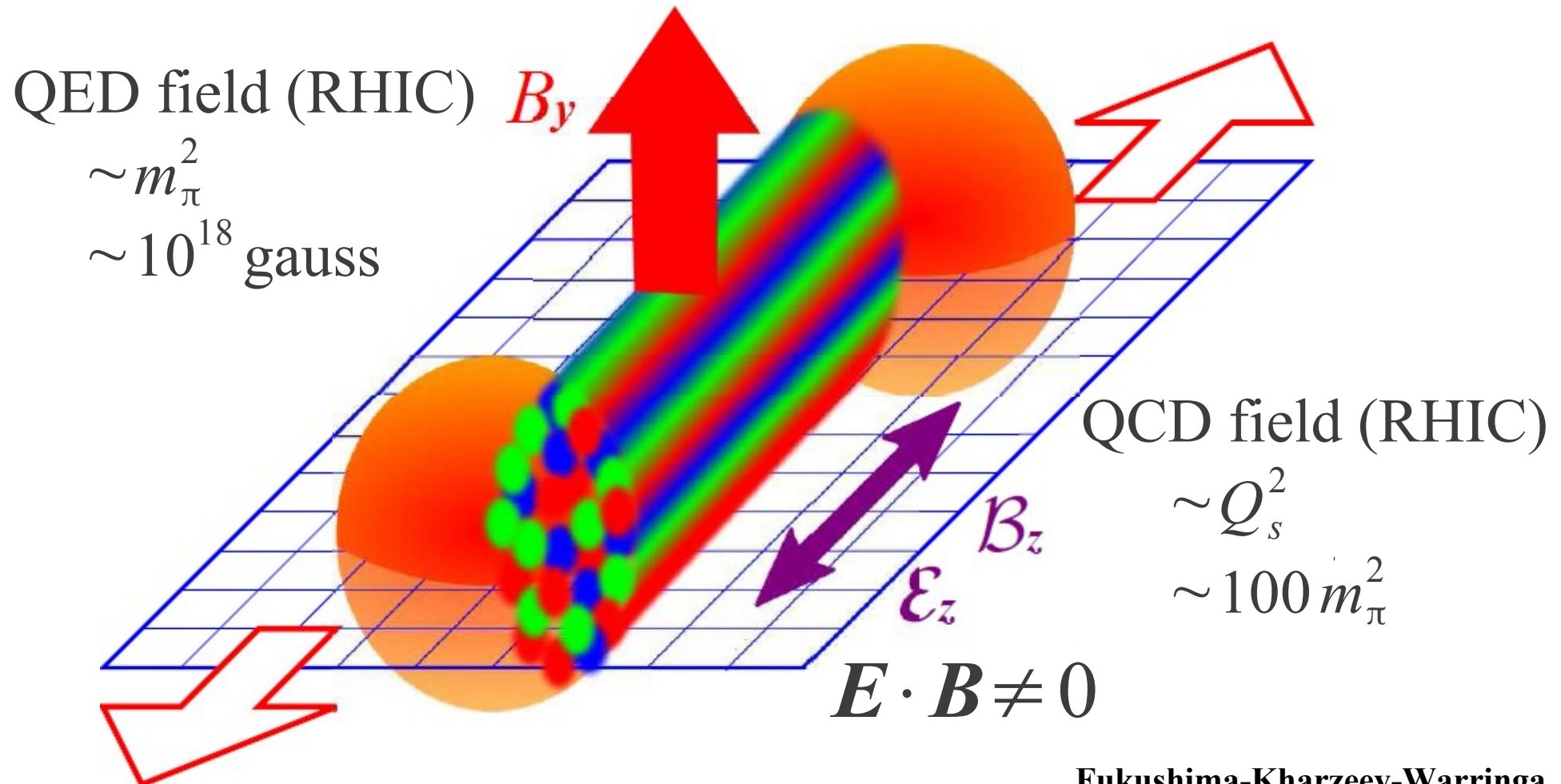
## Difference between instanton and sphaleron



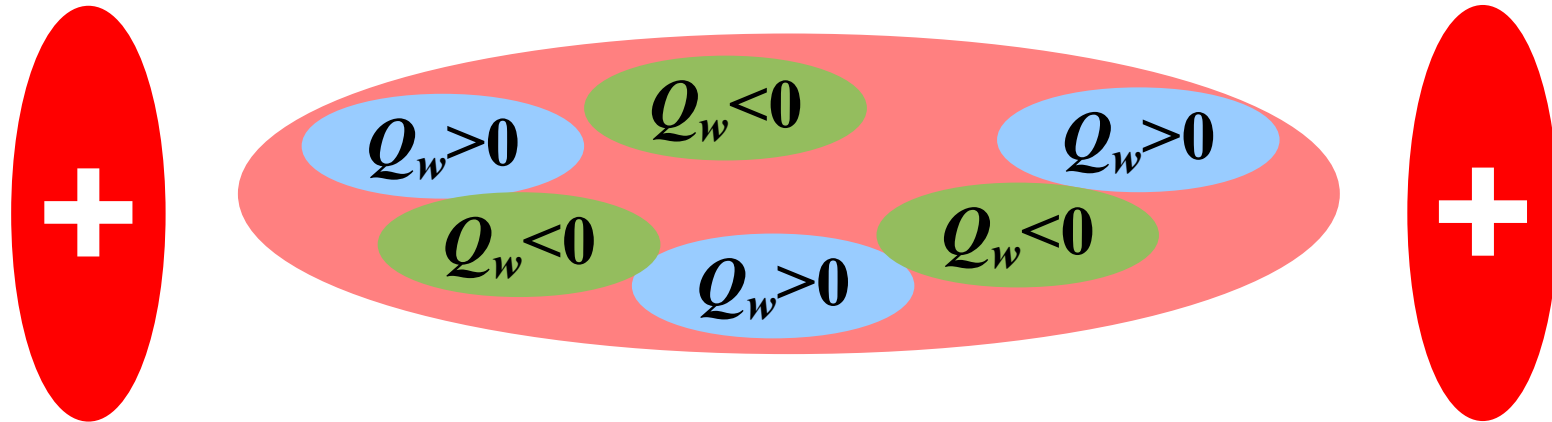
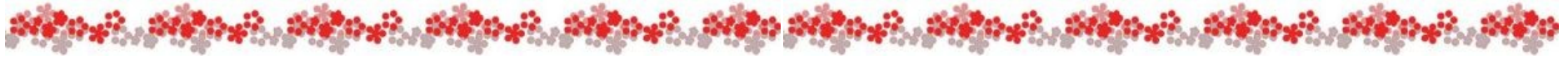
**Instantons (Euclidean windings) are suppressed at high  $T$  but communications in real time are not and dominated by the contribution from the zero-winding sector.**

# Origin of the Topological Charge

## Color Glass Condensate



# Local Parity Violation (LPV)



## Physics Impact

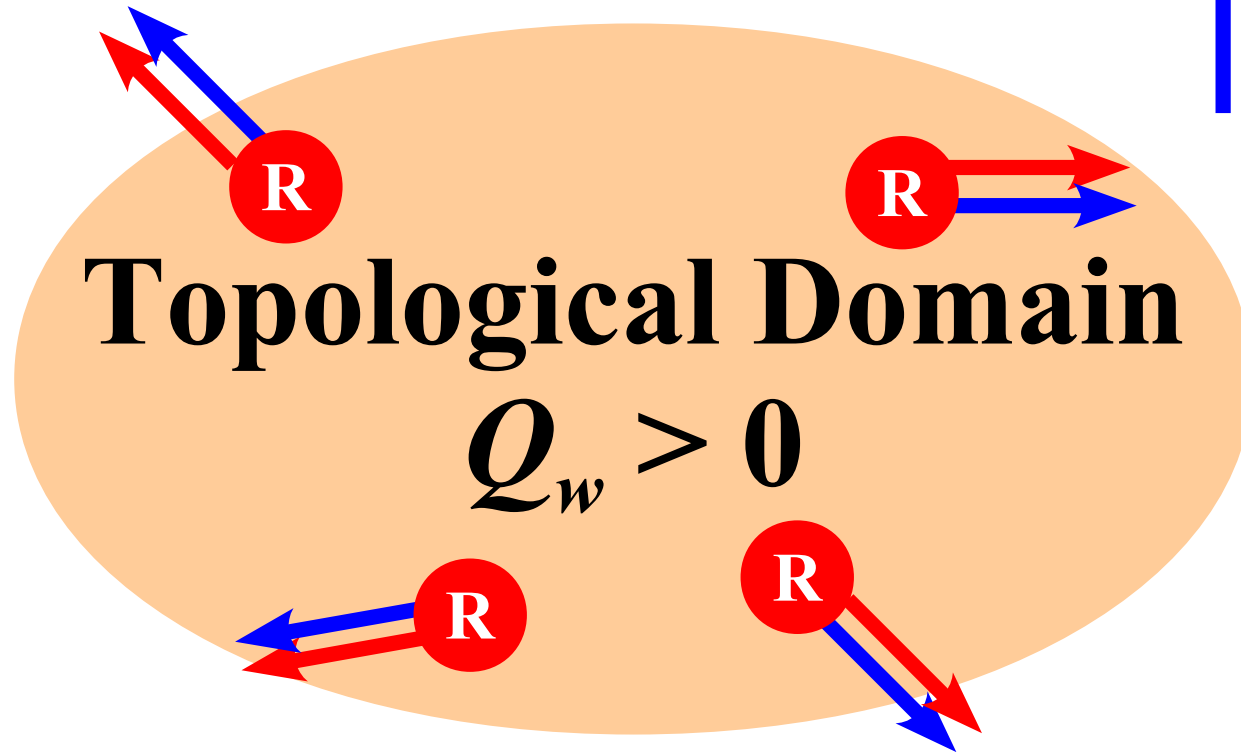
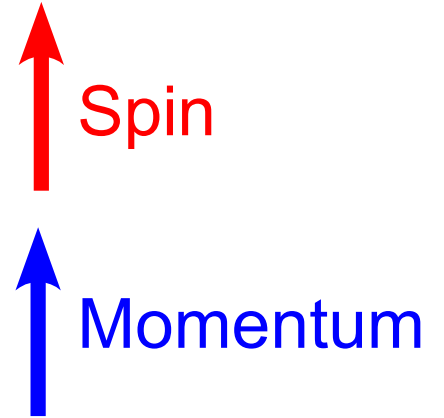
Experimental evidence for the quantum anomaly  
Origin of the mass – QCD instanton never detected  
Particle production from the CGC initial condition

# Charge Separation



## Without $B$ Fields

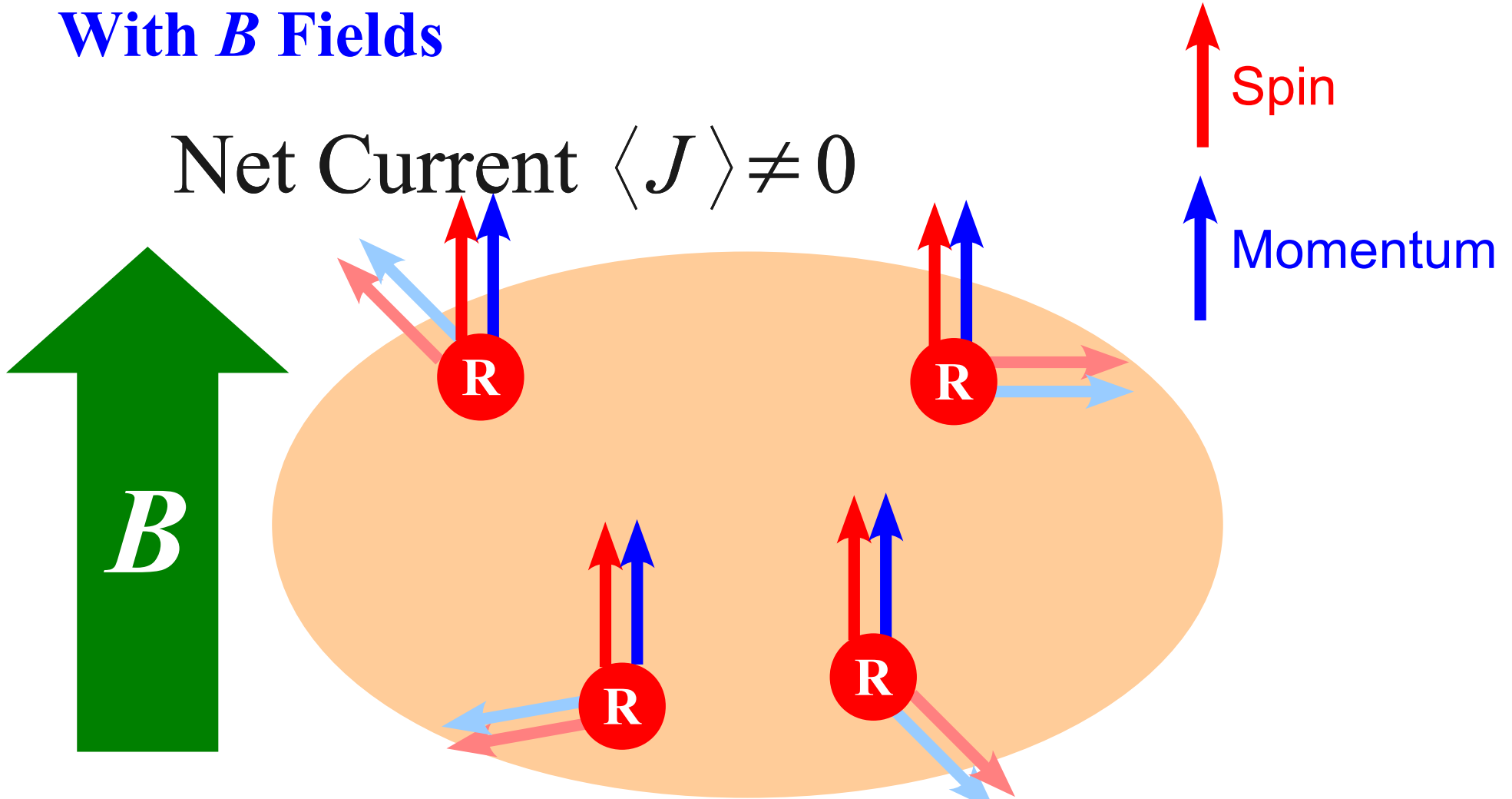
Net Current  $\langle J \rangle = 0$



# Charge Separation

With  $B$  Fields

Net Current  $\langle J \rangle \neq 0$



Kharzeev-McLerran-Warringa



# *Caution!*

**Such a classical picture is not correct!**

**With the *B*-effect many phenomena (related to the Landau quantization) look classical, but their origin is purely quantum!**

## *Caution!*

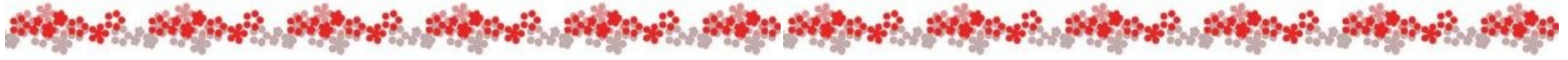
**Such a classical picture may easily lead people to a wrong interpretation of the chiral magnetic effect!**

**Deconfined and light (chiral-symmetric) quarks are seemingly necessary... but really so?**

# Current from infinity

– What really flows?

# Landau Quantization



## Energy dispersion relation in $B$

$$\omega^2 = p_z^2 + \underline{2|g B|(n + 1/2)} + m^2 - 2s g B$$

Transverse motion = Harmonic Oscillator

- **Light fermions ( $s=1/2$ ) have zero mode.**  
(Nearly) massless quarks at high  $T$  and/or  $\mu$
  - **Light vector bosons have (Nielese-Olesen) instability.**  
Gluons in the chromo- $B$  /  $\rho$  in a superstrong  $B$  (Chernodub)
  - **Charged scalar bosons are all massive.**  
 $\pi^+$ ,  $\pi^-$ , ... Explicit breaking of isospin symmetry
- Etc, etc...**

# Quantum Anomaly



**Formula insensitive to  $M, T, \mu \leftarrow$  IR scales**

**Zero-point Oscillation**  $\Gamma = -N_c \sum_{\text{flavor}} \frac{|q_f B|}{2\pi^2} \sum_s \sum_{n=0}^{\infty} \alpha_{n,s} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \omega_{n,s}$

Landau  
degeneracy

spin

Landau  
level

Chirality

$$\omega_{n,s}^2 = \left( \sqrt{p_z^2 + 2|q_f B|n} + \text{sgn}(p_z) s \mu_5 \right)^2 + M^2$$

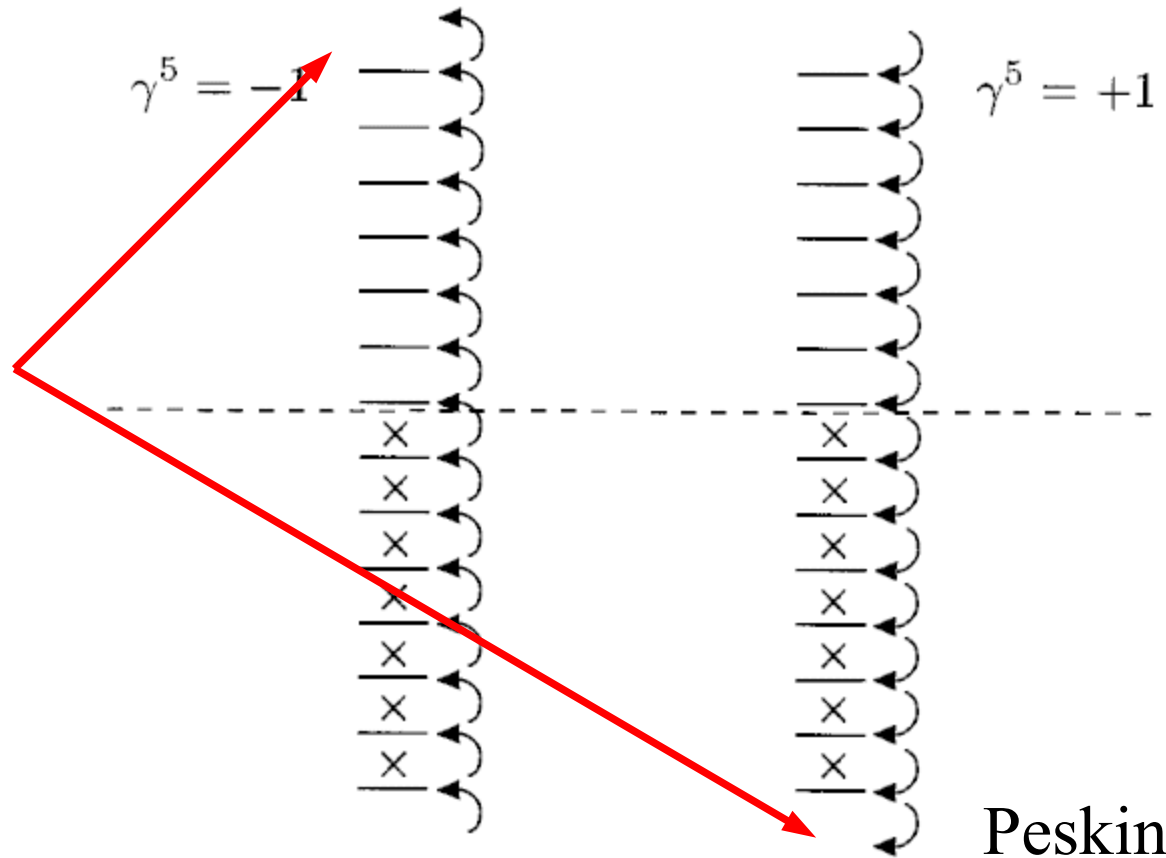
$$j_z = \frac{d\Gamma}{dA_z} = q_f \frac{d\Gamma}{dp_z} = \# \left[ \omega_{n,s}(p_z = \infty) - \omega_{n,s}(p_z = -\infty) \right]$$

$$= N_c \sum_{\text{flavor}} q_f \frac{|q_f B|}{2\pi^2} \sum_{n,s} \alpha_{n,s} s \mu_5 = N_c \sum_{\text{flavor}} \frac{q_f^2 B \mu_5}{2\pi^2}$$

# *(1+1)-dimensional Anomaly*

Anomaly is a “surface” effect

Particles  
from momenta  
at infinity!



# *Current from the Chiral Magnetic Effect*



Particles from infinitely large momentum components in virtual excitations of quarks!?

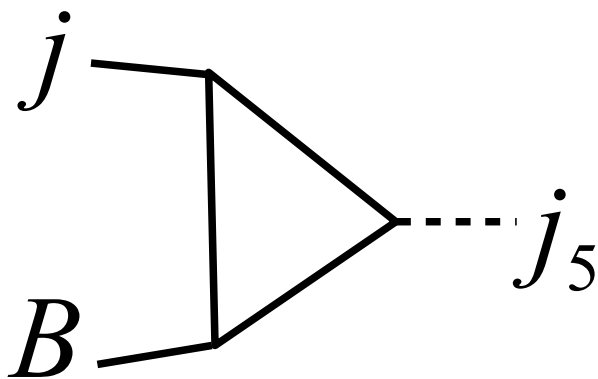
All soft scales ( $M$ ,  $T$ ,  $\mu$ ) are irrelevant!?

**What would be seen in real experiments?**

Let's take another example of quantum anomaly for a better intuition!

# Triangle Anomaly

## Anomaly Matching Condition ('t Hooft)

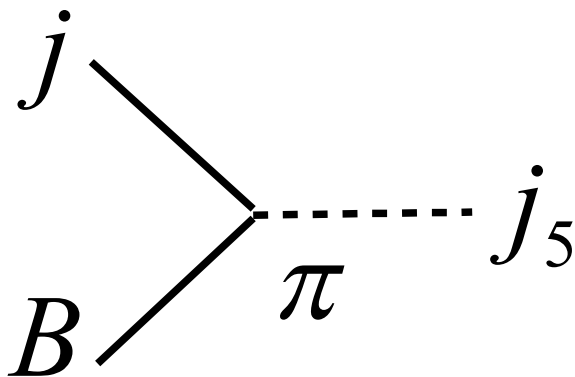


$$K_{\mu\nu}^{AB}(q) = i \int d^4x e^{iq \cdot x} \langle j_{5\mu}^A(x) j_\nu^B(0) \rangle_B$$

$$\sim -N_c \frac{eB}{2\pi^2} \frac{q_\mu \tilde{q}_\nu}{q^2} \frac{1}{2} \delta^{AB}$$

Equivalent to the chiral magnetic effect

If quarks are confined, how can be this singularity saturated?



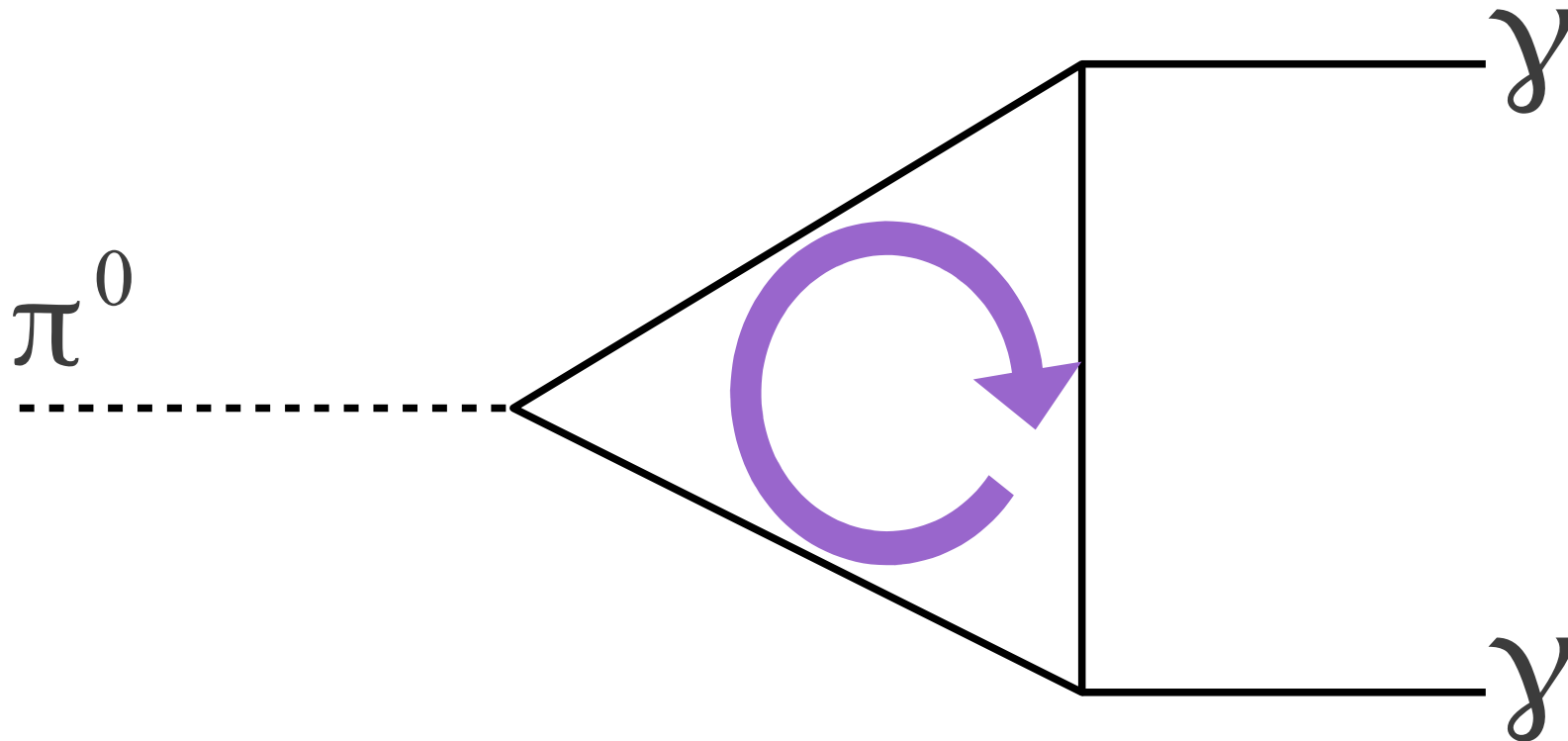
**In the confined world there must be massless NG bosons (chiral broken).**

For  $N_f = 2$ , massless proton and neutron could saturate the IR singularity...



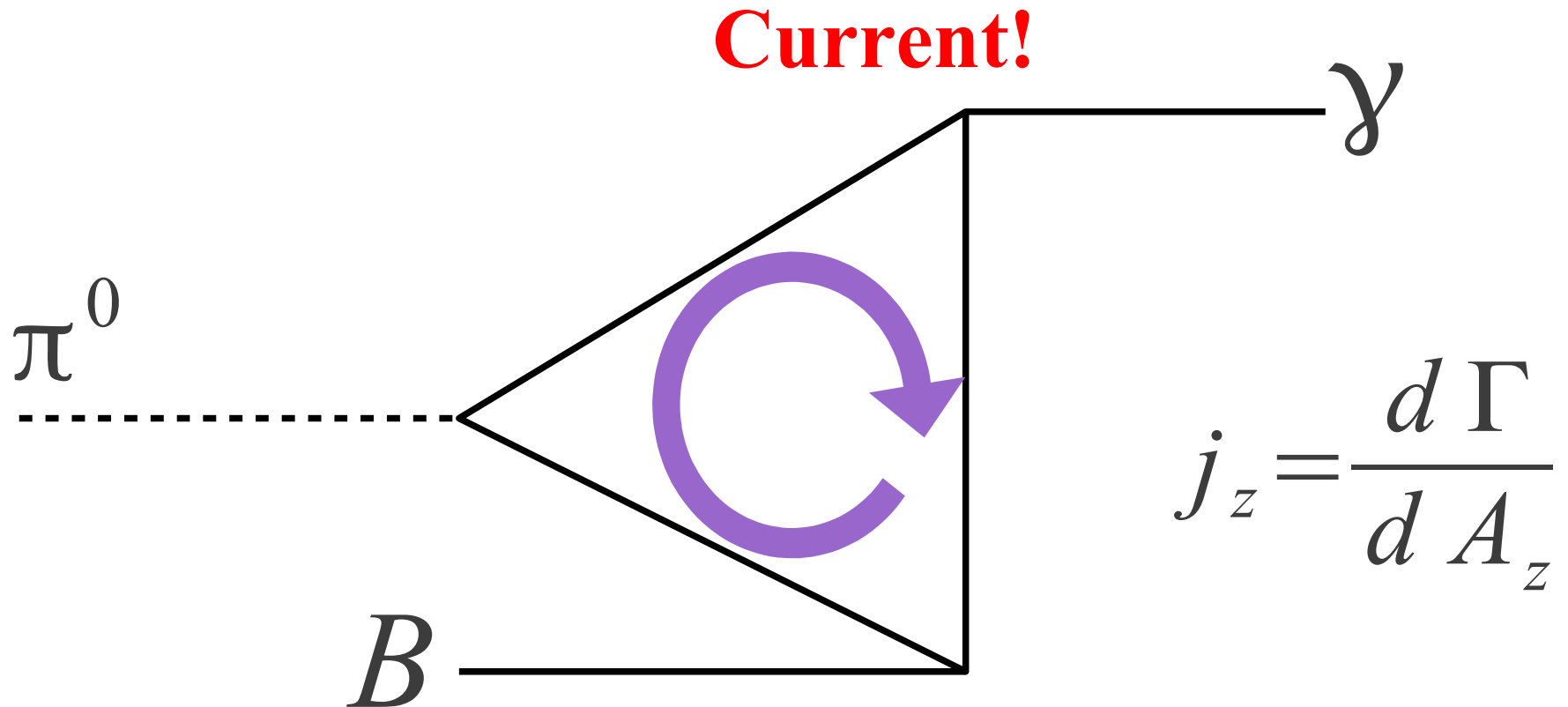
$$\pi^0 \rightarrow 2\gamma$$

Typical example of the anomalous process





Typical example of the anomalous process



Asakawa-Majumder-Mueller

# *Current on top of pions*



**If  $\pi^0$  distributes coherently (with condensation), microscopic currents in  $\pi^0$  make a macroscopic current (similar to the Josephson current in SC).**

**\*  $\pi^0$  domain wall and the current**

**(Son-Stephanov)**

**\* Skyrmion and induced charge**

**(Eto-Hashimoto-Iida-Ishii-Maezawa)**

# Chiral Magnetic Effect

## in a hadronic phase

# Chiral Lagrangian

$$L = \frac{f^2}{4} \text{tr} \left[ \partial_\mu U \partial^\mu U^\dagger + U^\dagger \chi + \chi^\dagger U \right] - \frac{N_f \chi_{\text{top}}}{2} (-i \ln \det U - \theta)^2$$

$$\text{Condensate: } U = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix} \quad \text{Mass: } \chi = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}$$

$$\text{Potential: } V(\phi) = -\frac{f^2}{2} \sum_i m_i^2 \cos \phi_i + \frac{N_f \chi_{\text{top}}}{2} \left( \sum_i \phi_i - \theta \right)^2$$

**Physics is periodic in terms of  $\theta$  (Dashen's phenomena)**

**Witten**

# $\theta = \text{Pseudo-scalar Condensate}$



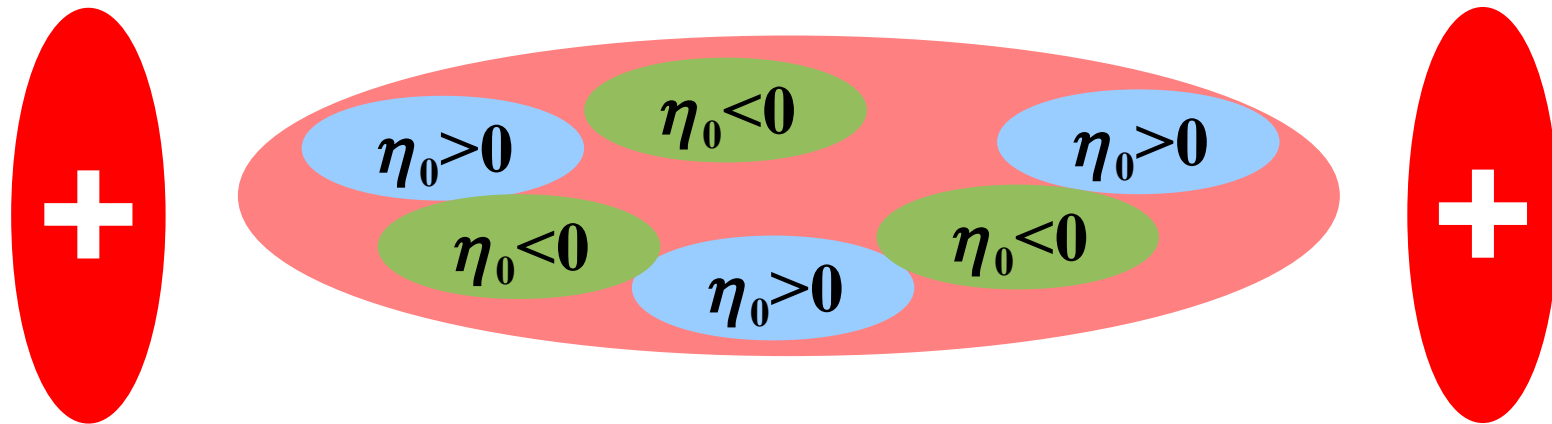
$\phi_1, \phi_2, \phi_3 = -\phi_1 - \phi_2$  describe  $\eta_0$  and  $\eta_8$

Potential: 
$$V(\phi) = -\frac{f_\pi^2}{2} \sum_i m_i^2 \cos \phi_i + \frac{N_f \chi_{\text{top}}}{2} \left( \sum_i \phi_i - \theta \right)^2$$

**If any of flavors is massless,  $\theta$ -dependence is gone;  
Absorbed by the pseudo-scalar condensate**

**If any of flavors is massless,  $\theta$  and the pseudo-scalar  
condensate are simply identifiable.**

# Local Parity Violation (LPV) again



Disoriented Chiral Condensate in not only the pion direction but also the iso-singlet direction.

Pionic DCC would also exhibit the LPV.

Spatially and temporally fluctuating  $\eta_0$  condensates

→ Inhomogeneous (and non-zero)  $\theta$

# *Question*

**Calculate a current with  $B$  and  $\theta(t)$**

**Do not use deconfined and massless quarks  
but just stick to the Chiral Lagrangian!**



# Anomaly in the Chiral Lagrangian



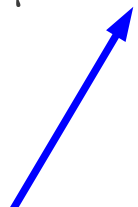
## Wess-Zumino-Witten Action

$$\pi^- \rightarrow e^- \bar{\nu}_e \gamma$$

$$L_{\text{WZW}} = -\frac{N_c}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \left\{ \text{tr} \left[ U^\dagger \hat{r}_\mu U \hat{l}_\nu - \hat{r}_\mu \hat{l}_\nu \right. \right. \\ \left. \left. + i \Sigma_\mu (U^\dagger \hat{r}_\nu U + \hat{l}_\nu) \right] \text{tr} (v_{\rho\sigma}) + \frac{2}{3} \text{tr} (\Sigma_\mu \Sigma_\nu \Sigma_\rho) \text{tr} (v_\sigma) \right\}$$

$$\pi^0 \rightarrow 2 \gamma \qquad \qquad \qquad \gamma \pi^0 \rightarrow \pi^+ \pi^-$$

$$\hat{r}_\mu = \hat{v}_\mu + \hat{a}_\mu, \quad \hat{l}_\mu = \hat{v}_\mu - \hat{a}_\mu, \quad \Sigma_\mu = U^\dagger \partial_\mu U$$



Vector field (traceless)

Axial vector field (traceless)

**Kaiser-Leutwyler**

# Another WZW (Contact Term)



## WZW term without dynamical $U$ fields

$$L_P = \frac{N_c}{8 N_f \pi^2} \epsilon^{\mu\nu\rho\sigma} \left\{ \text{tr} \left[ v_\mu \left( \partial_\nu v_\rho - \frac{i}{3} [v_\nu, v_\rho] \right) \right] \partial_\sigma \theta \right. \\ \left. + \text{tr} (a_\mu D_\nu a_\rho) \left[ \frac{4}{3} \text{tr} (a_\sigma) + \partial_\sigma \theta \right] \right\} - \frac{N_c}{12 N_f^2 \pi^2} \text{tr} (a_\mu) \text{tr} (\partial_\nu a_\rho) \partial_\sigma \theta$$

$$\text{QED fields: } v_\mu = eQ A_\mu = e \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} A_\mu$$

**Kaiser-Leutwyler**

# Current from the WZW Action

$$\Gamma_p = \int \frac{N_c}{8 N_f \pi^2} \epsilon^{\mu\nu\sigma\rho} \text{tr} [v_\mu \partial_\nu v_\rho] \partial_\sigma \theta + \dots$$

$$j_z = \frac{d \Gamma_p}{d A_z} = \frac{N_c}{4 N_f \pi^2} \epsilon^{zxyt} \text{tr} (Q^2) B_z \partial_t \theta$$

$$= N_c \sum_f \frac{q_f^2 B_z}{2 \pi^2} \left( \frac{\partial_t \theta}{2 N_f} \right) \mu_5$$

Time-dependent  
DCC in the singlet channel

# *Full Computation*



## **Non-anomalous**

Currents from  $\pi^+$ ,  $\pi^-$  flows

## **Non-singlet anomalous (conventional DCC)**

Currents from inhomogeneous  $\pi$  condensation

## **Singlet anomalous (CME)**

Currents from inhomogeneous  $\eta_0$  condensation

**In completion (Fukushima-Mameda)**

# Similar Effects

$$j_{\mu} = \epsilon_{\mu\nu\sigma\rho} (\partial^{\nu} \theta) F^{\sigma\rho}$$

Derivative of a scalar quantity

$\eta'$  condensate

Pseudo-scalar condensate

Strong  $\theta$  angle

2nd-rank tensor

Field strength tensor

Angular momentum

Angular velocity

**Chiral separation / vortical / ... / effect**

**Is there a current... or not?**

Signature of what?

# *What is necessary for the CME?*



**Is “quark deconfinement” necessary?**

# *What is necessary for the CME?*



**Is “quark deconfinement” necessary?**

**NO!?**

**Anomaly should be saturated with quarks and hadrons**

**This is realized by the Wess-Zumino-Witten action.  
WZW is used for similar phenomena (Son-Stephanov)  
and can be for the CME.**

*What flows microscopically in a hadronic phase?  
Through high-momentum quark component in  
the  $\eta_0$  condensate??*



# *What is necessary for the CME?*



**Is “quark deconfinement” necessary?**

**NO!?**

**Anomaly should be saturated with quarks and hadrons**

**This is realized by the Wess-Zumino-Witten action.**

**WZW is used for similar phenomena (Son-Stephanov)  
and can be for the CME.**

**Dima says sphaleron transitions enhanced  
in the deconfined phase at high temperature...**

# *What is necessary for the CME?*



**Is “chiral symmetry restoration” necessary?**

# *What is necessary for the CME?*



**Is “chiral symmetry restoration” necessary?**

**Probably Yes!?**

**WZW action does not require the chiral limit.  
CME current is insensitive to the quark mass.**

**To form the DCC, far off-equilibrium matter from  
chiral symmetric (and  $U(1)_A$  symmetric) state is  
implicitly assumed.**

**Picture in a hadronic phase**

# *What is necessary for the CME?*



**Is “chiral symmetry restoration” necessary?**

**Probably Yes!?**

**IF, topological gauge configurations (sphaleron) are given, not  $\mu_5$  but  $N_5$  is generated.  
 $N_5$  decays with the quark mass term.  
(Chirality makes sense only for massless quarks)**

**Picture in a deconfined phase**

# **Experimental Challenges and Theories**

# *Difficulty*

**P-odd quantity is zero on average**

**What we can observe in physics  
of the strong interaction is:**

**NOT P (and CP) Violation**

**BUT *Local* P (and CP) Violation**

**Not *qualitative* but *quantitative***

# *Difficulty*

$$\langle (\text{P-odd Observable}) \rangle = 0$$

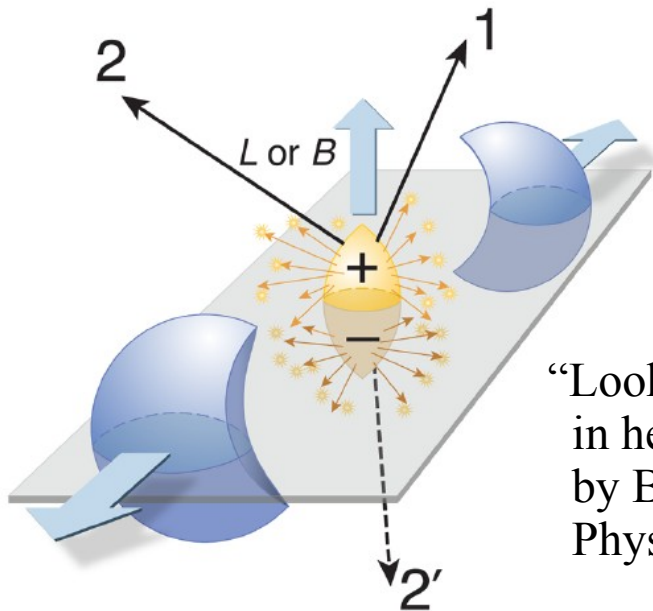
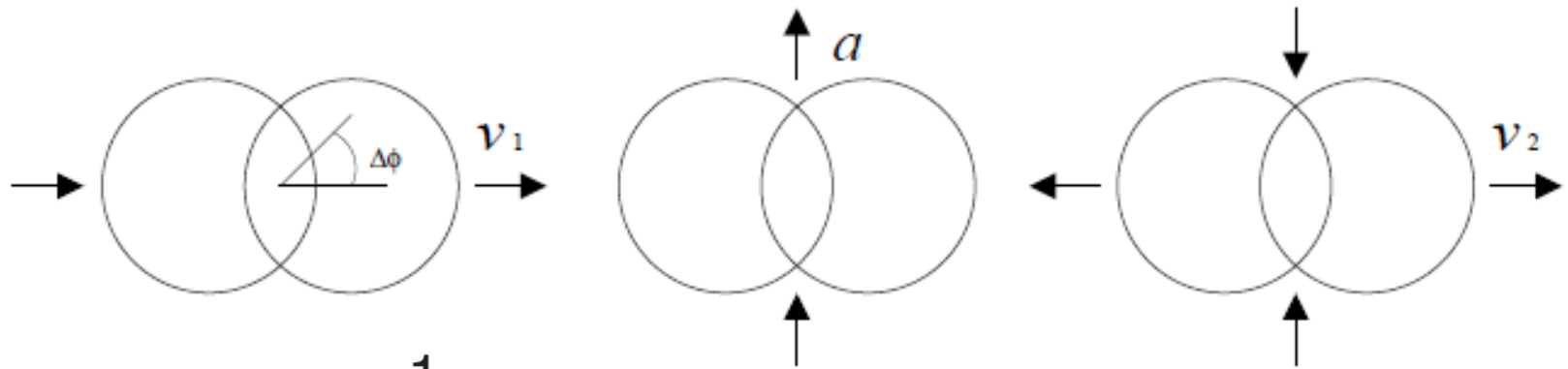
$$\langle (\text{P-odd Observable})^2 \rangle = \text{Large or Small?}$$

**Even if we find the latter large,  
can we really say that we find  
the local parity violation?**

***Theoretical prediction needed...***

# Fluctuation of Charge Separation

$$\frac{dN_{\pm}}{d\phi} \propto 1 + 2v_{1\pm} \cos(\Delta\phi) + 2a_{\pm} \sin(\Delta\phi) + 2v_{2\pm} \cos(2\Delta\phi) + \dots$$



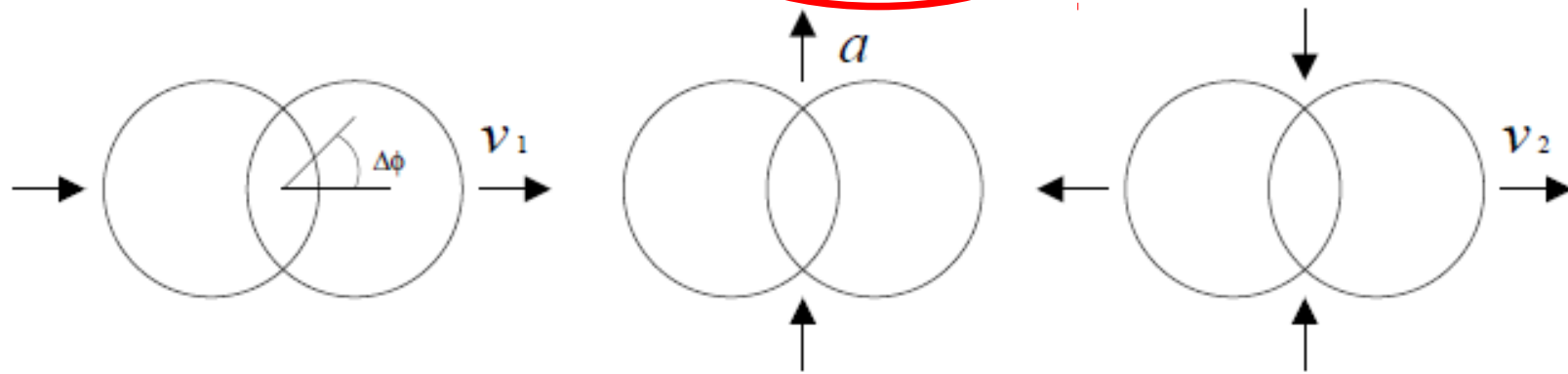
$\phi$  : Azimuthal angle  
 $v_1$  : Directed flow  
 $v_2$  : Elliptic flow

“Looking for parity violation  
in heavy-ion collisions”  
by Berndt Müller  
Physics 2, 104 (2009)



# Interpretation

$$\frac{dN_{\pm}}{d\phi} \propto 1 + 2v_{1\pm} \cos(\Delta\phi) + \underbrace{2a_{\pm} \sin(\Delta\phi)}_{\text{Parity-Odd (Charge Separation)}} + 2v_{2\pm} \cos(2\Delta\phi) + \dots$$



$$\begin{aligned} \langle\langle \cos(\Delta\phi_{\alpha} + \Delta\phi_{\beta}) \rangle\rangle &\equiv \left\langle\left\langle \frac{1}{N_{\alpha}N_{\beta}} \sum_{i=1}^{N_{\alpha}} \sum_{j=1}^{N_{\beta}} \cos(\Delta\phi_{\alpha,i} + \Delta\phi_{\beta,j}) \right\rangle\right\rangle \\ (\alpha, \beta = +, -) &= \langle\langle \cos \Delta\phi_{\alpha} \cos \Delta\phi_{\beta} \rangle\rangle - \langle\langle \sin \Delta\phi_{\alpha} \sin \Delta\phi_{\beta} \rangle\rangle \\ &= \left( \langle\langle v_{1,\alpha} v_{1,\beta} \rangle\rangle + \underline{B_{\alpha\beta}^{\text{in}}} \right) - \left( \langle\langle a_{\alpha} a_{\beta} \rangle\rangle + \underline{B_{\alpha\beta}^{\text{out}}} \right). \end{aligned}$$

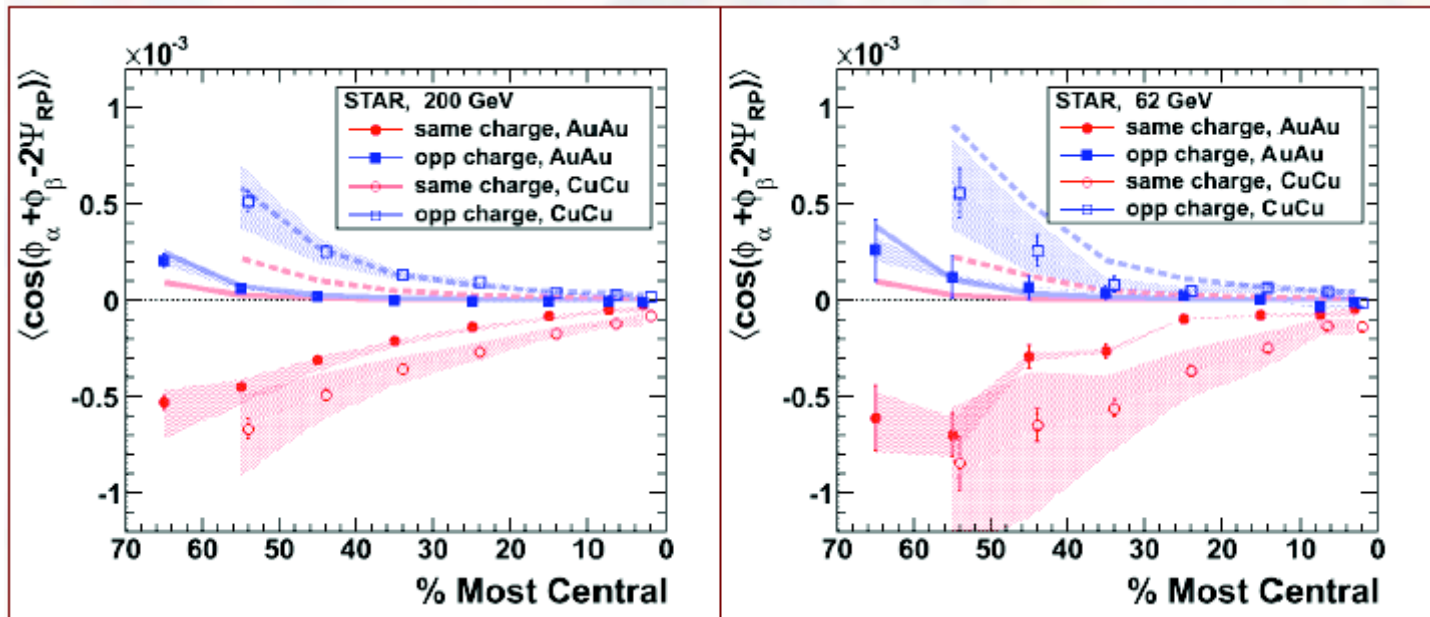
**Cancel? NO!**

# Famous (Infamous?) Results

## 3-Particle Correlation (fluctuation measurement)

$$\begin{aligned} \langle\langle \cos(\Delta\phi_\alpha + \Delta\phi_\beta) \rangle\rangle &\equiv \left\langle\left\langle \frac{1}{N_\alpha N_\beta} \sum_{i=1}^{N_\alpha} \sum_{j=1}^{N_\beta} \cos(\Delta\phi_{\alpha,i} + \Delta\phi_{\beta,j}) \right\rangle\right\rangle \\ &= \langle\langle \cos \Delta\phi_\alpha \cos \Delta\phi_\beta \rangle\rangle - \langle\langle \sin \Delta\phi_\alpha \sin \Delta\phi_\beta \rangle\rangle \\ &= \left( \langle\langle v_{1,\alpha} v_{1,\beta} \rangle\rangle + B_{\alpha\beta}^{\text{in}} \right) - \left( \langle\langle a_\alpha a_\beta \rangle\rangle + B_{\alpha\beta}^{\text{out}} \right). \end{aligned}$$

## STAR Results

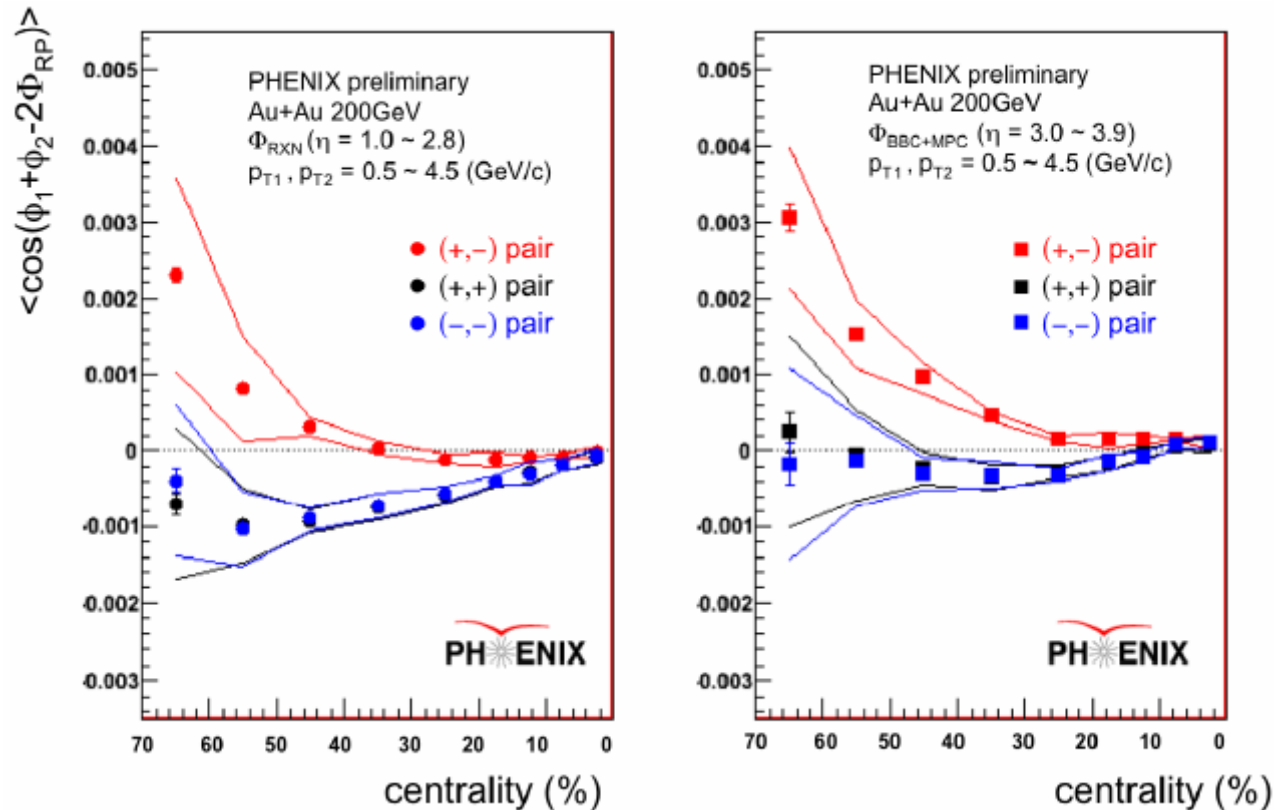


Voloshin  
STAR

# Followed up by RHIC (PHENIX)

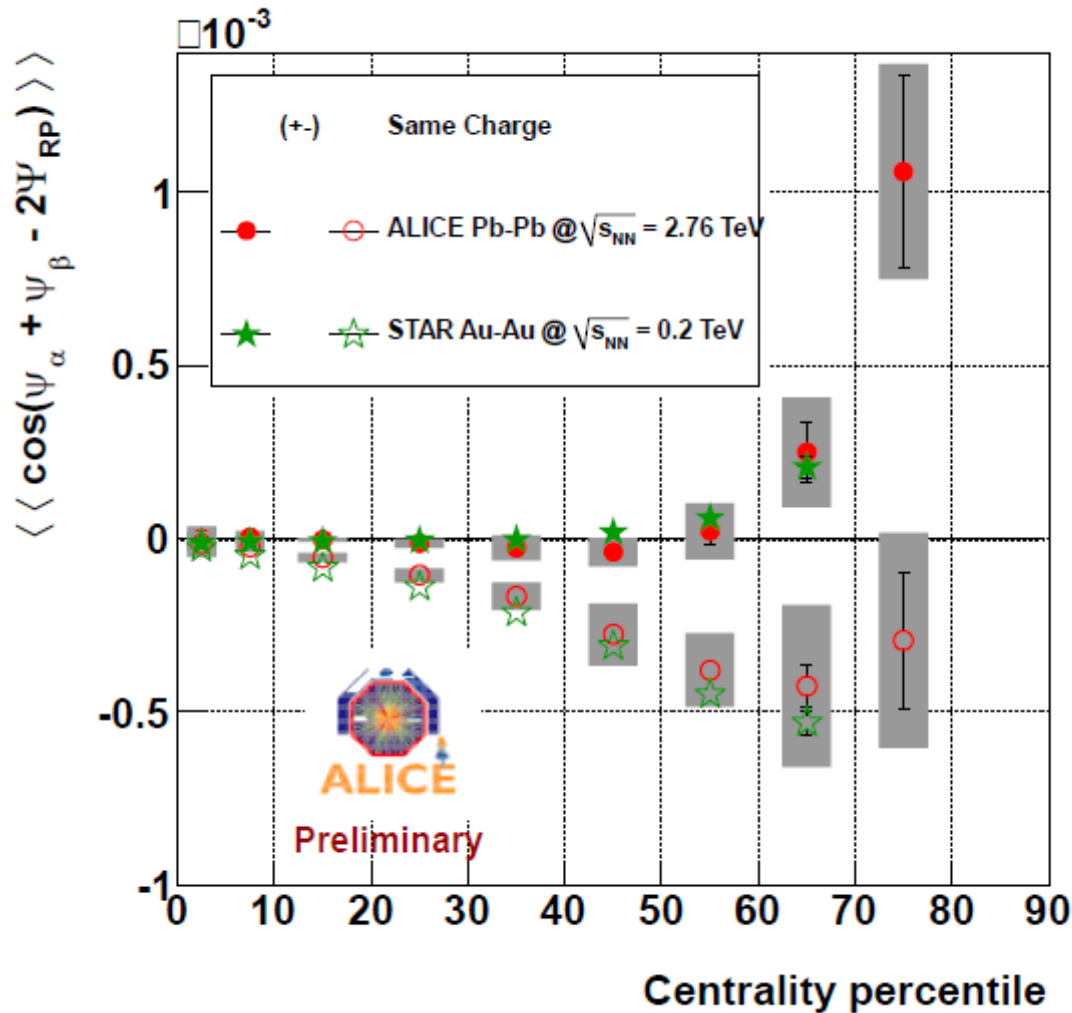


## Qualitative agreement with STAR



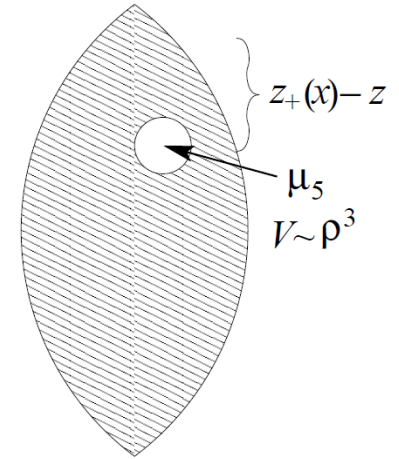
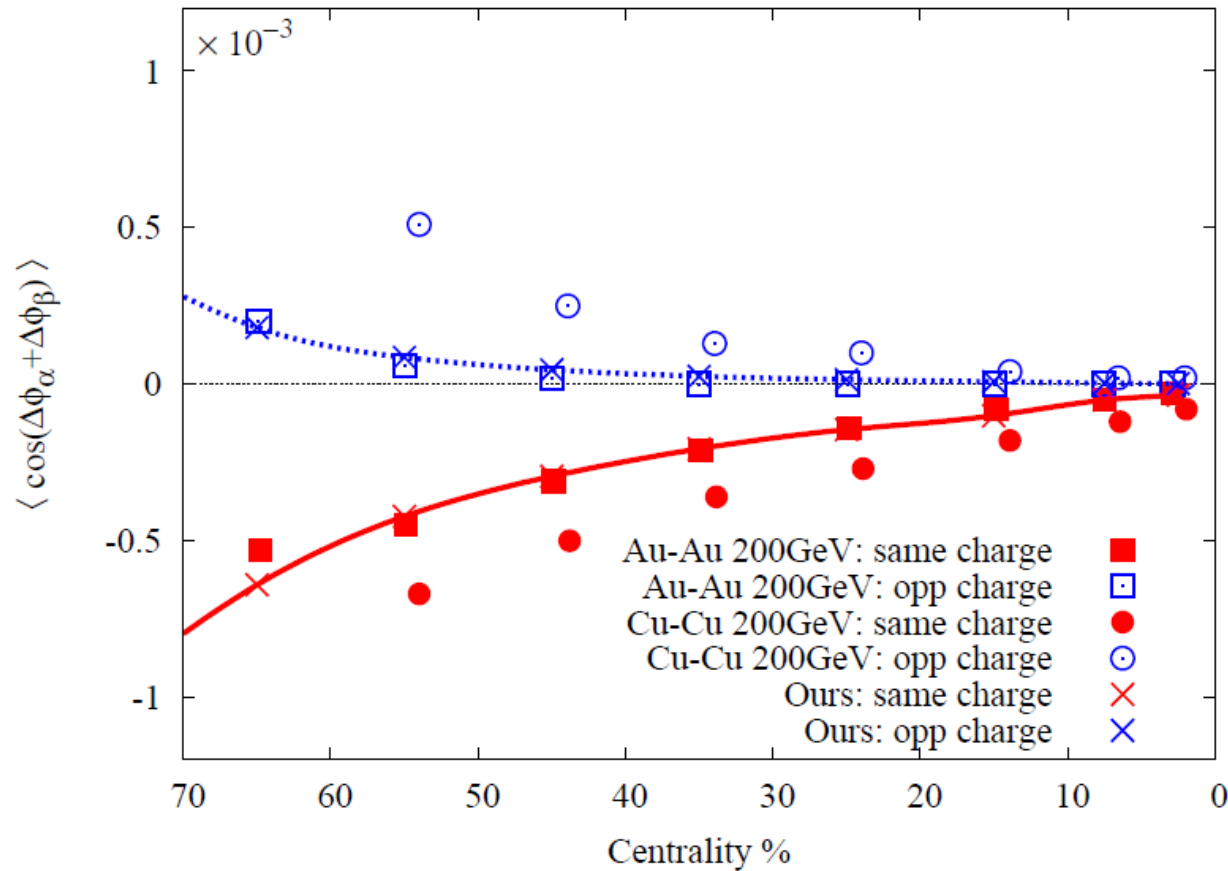
Figures from  
a talk by Lacey  
in CPODD

# Further confirmation from ALICE



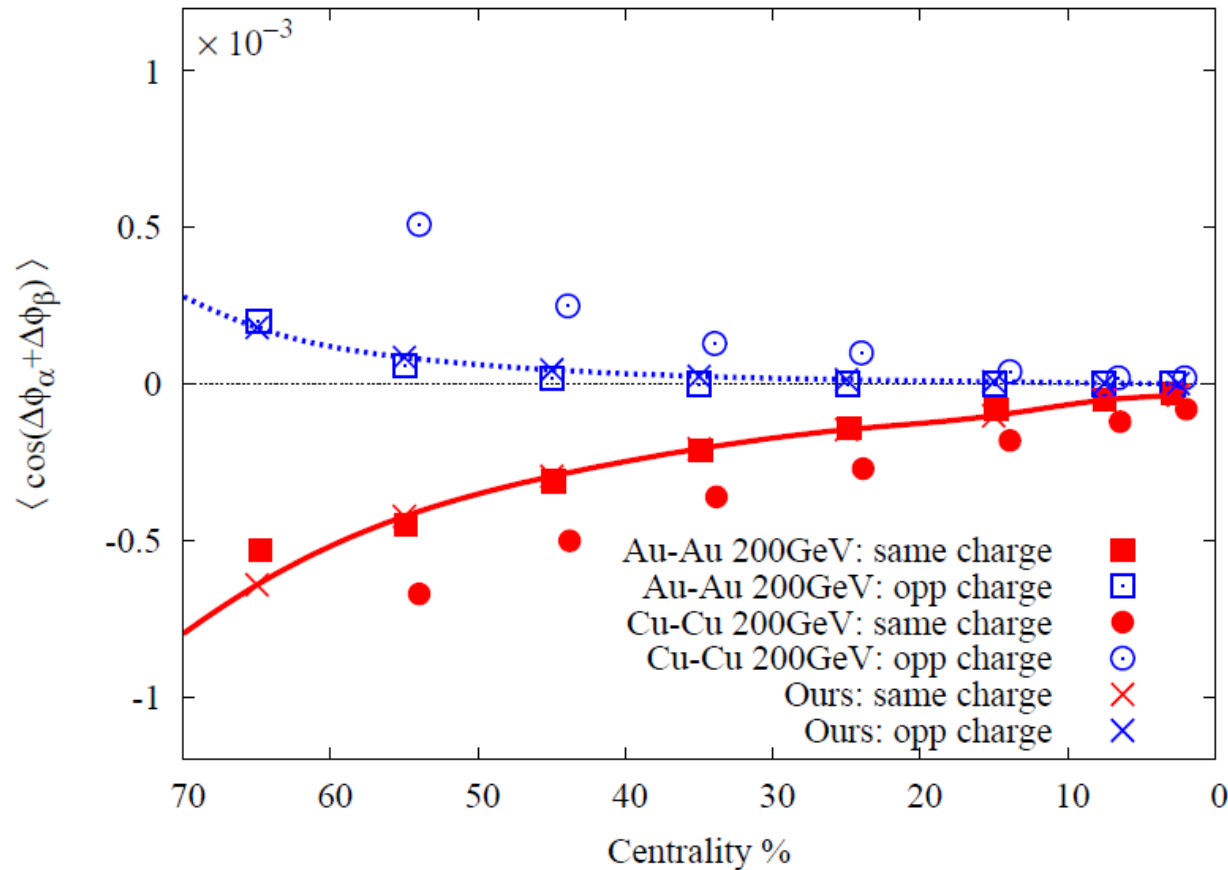
Data from ALICE

# What really happens?



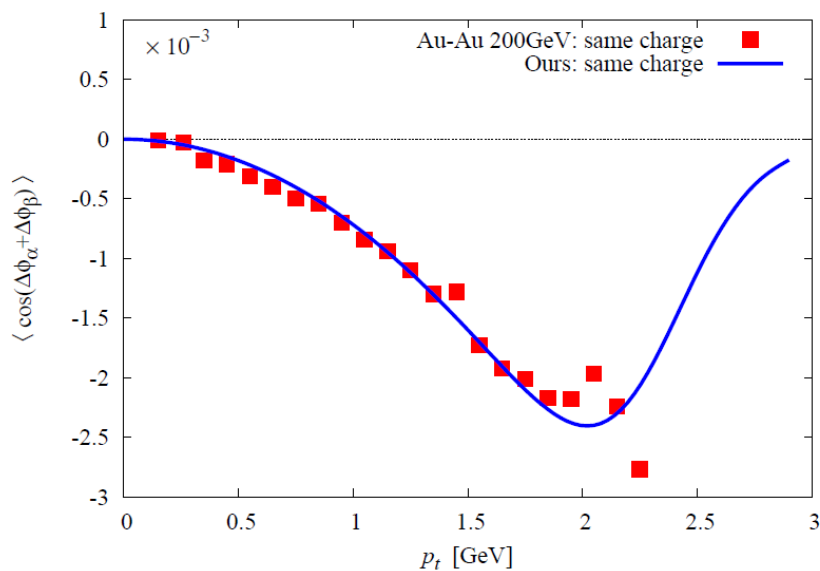
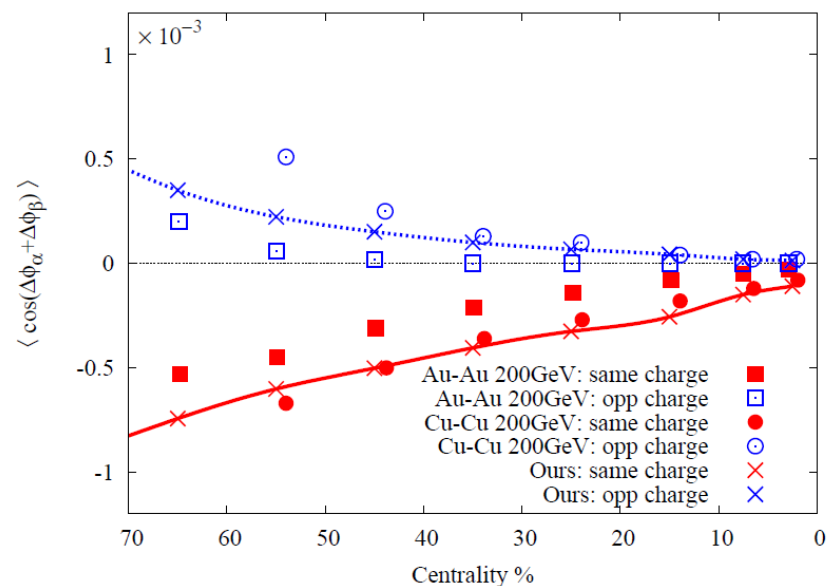
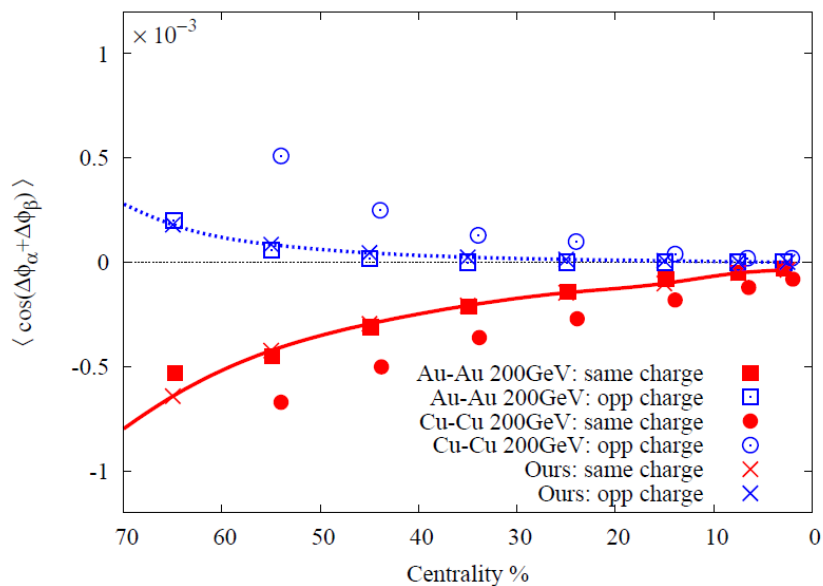
**Theoretical fit I made three years ago  
but didn't publish and just threw away!**

# What really happens?



**To fit the data, the CME is not necessary.  
If going to peripheral, less particles and  
larger fluctuations.... that's all!**

# Systematic fit possible?



**Species dependence**

**$P_t$  dependence**

# *Difficulty*

**Theoretical fit is always possible.  
But... so what?**

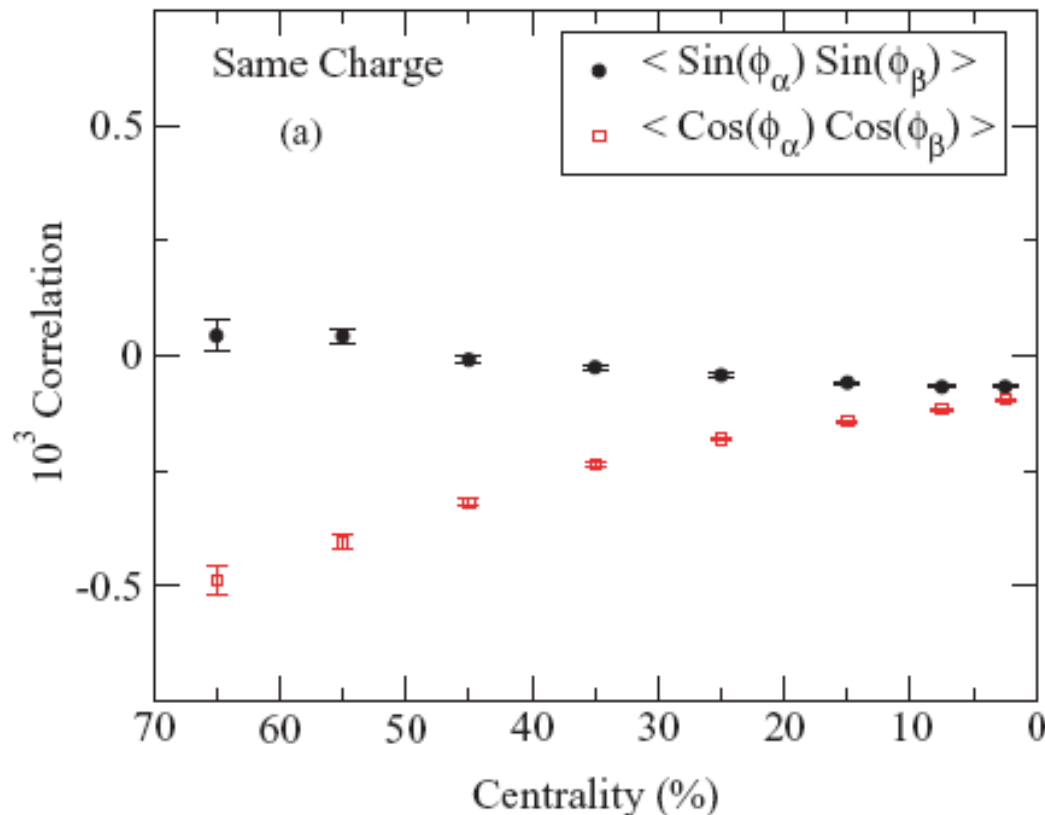
**We cannot conclude anything!**



# Decomposition (Background)

$$\langle \cos(\Delta\phi_\alpha + \Delta\phi_\beta) \rangle = \langle \cos\Delta\phi_\alpha \cos\Delta\phi_\beta \rangle - \langle \sin\Delta\phi_\alpha \sin\Delta\phi_\beta \rangle$$

$$\langle \cos(\Delta\phi_\alpha - \Delta\phi_\beta) \rangle = \langle \cos\Delta\phi_\alpha \cos\Delta\phi_\beta \rangle + \langle \sin\Delta\phi_\alpha \sin\Delta\phi_\beta \rangle$$



**Bzdak-Koch-Liao (2009)**

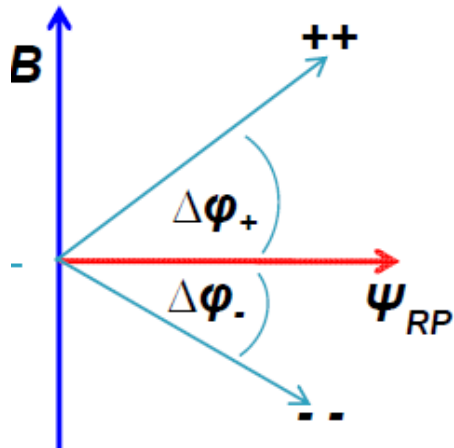
Not ruled out (yet?)

No longer considered as  
an evidence of anything...

# Multi-Particle Correlation



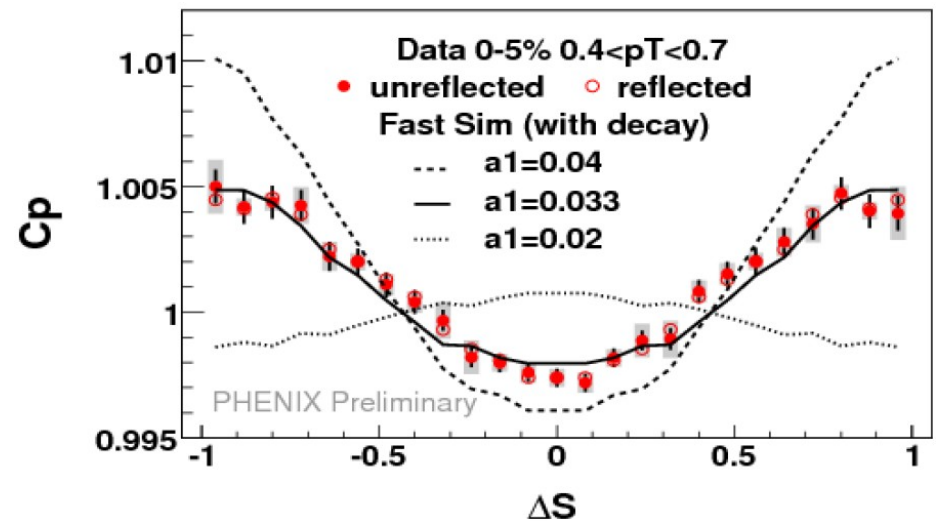
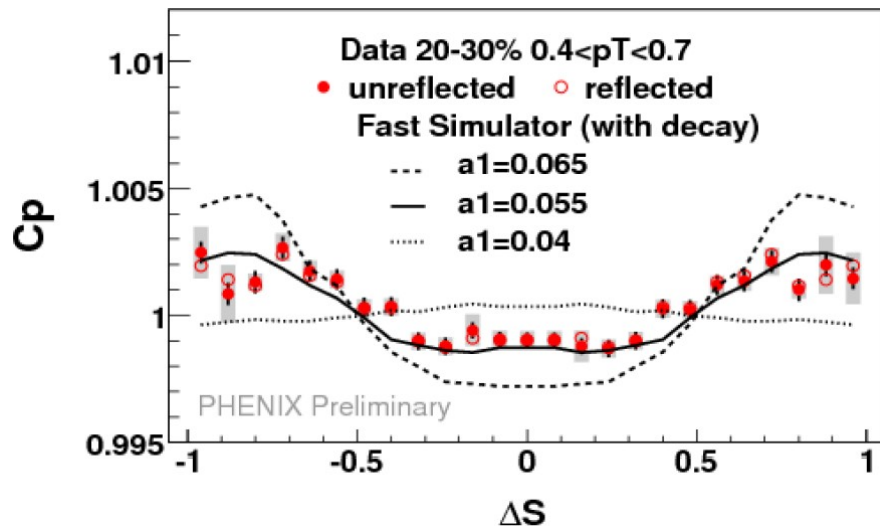
## Two Distribution Functions




$$\langle S_p^{h+} \rangle = \sin \Delta \phi_{p+} \quad \langle S_n^{h-} \rangle \quad \langle S_p^{h\mp} \rangle \quad \langle S_n^{h\mp} \rangle$$

$$C_p(\Delta S) = \frac{N(\langle S_p^{h+} \rangle - \langle S_n^{h-} \rangle)}{N(\langle S_p^{h\mp} \rangle - \langle S_n^{h\mp} \rangle)}$$

**PHENIX**  
Talk by Lacey  
in CPODD



# Summary

- 
- Theory of the Chiral Magnetic Effect is robust, but phenomenological inputs have uncertainties...
  - Parity Violation itself cannot be seen, and only its fluctuations are observed which are parity even! Subtraction of backgrounds is very hard...
  - Important to look at physics observables with which flow effects are distinguishable (multi-particle corr.)
  - More from forthcoming conferences:
    - CPODD (BNL – June)    QM (Washington D.C. – Aug.)
    - QCD in a Strong B (ECT\* – Nov.)