相対論的流体力学の理論的進展
(傾向と対策)

本郷 優 (新潟大学)

2022/04/30, 名古屋大学 & Zoom (ハイブリッド)

第39回Heavy Ion Cafe & 第35回Heavy Ion Pub 合同研究会「ポストQM2022」

注意 (言い訳)

New developments in relativistic hydrodynamics

Nora Weickgenannt

Quark Matter 22, Krakow, Poland | April 9, 2022

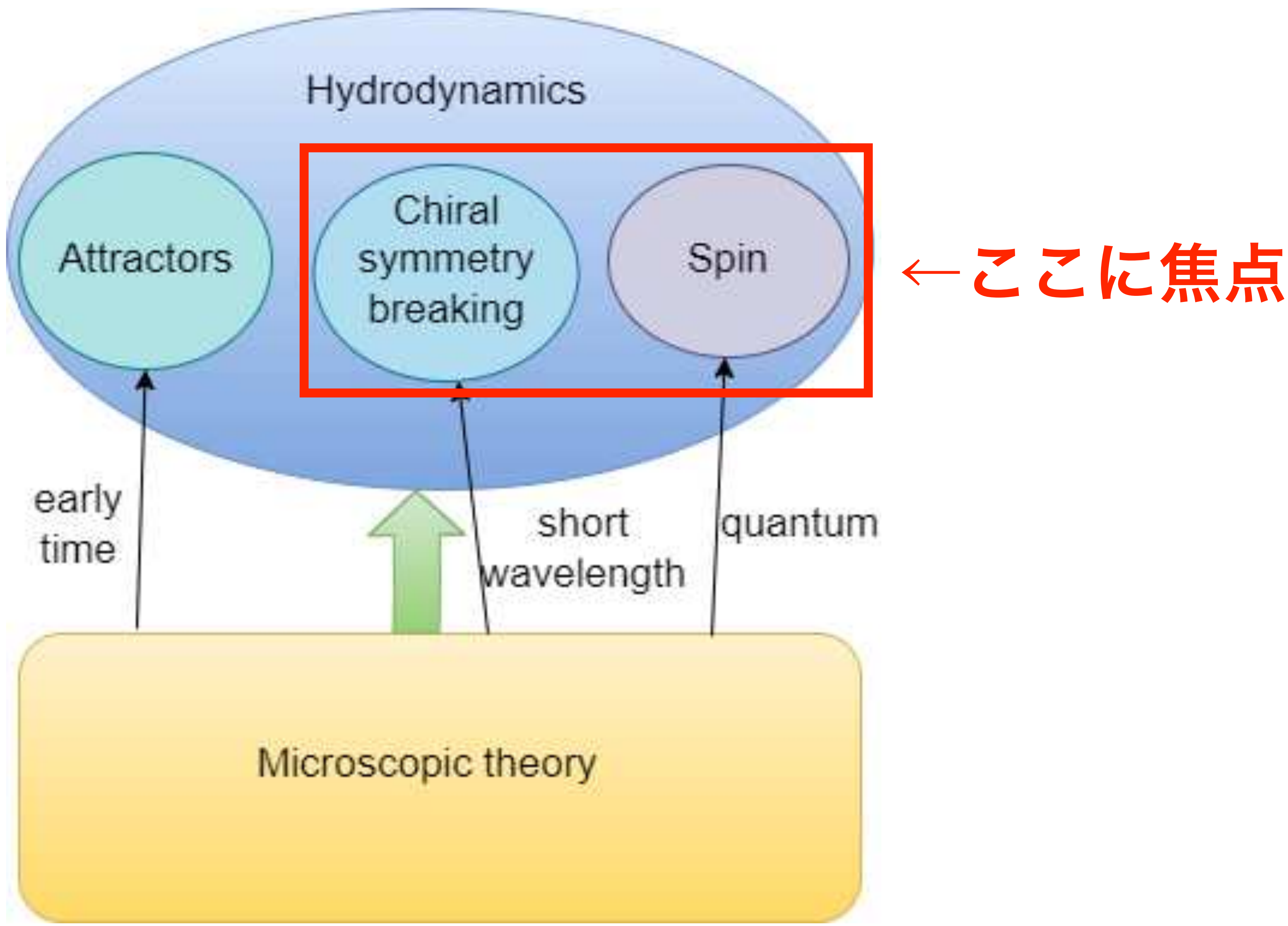
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC–TR 211 Strong-interaction matter under extreme conditions

EL EM EN TS

DFG

What is relativistic hydrodynamics *really*?



Hydrodynamics successful even in regimes where not expected to be applicable, in particular far from local equilibrium

流体モデルを拡張する

1. 新しい自由度を導入する

臨界モード，パイオン，スピン

2. 新しい項を見つける

Thermal shear

(3. 上の2つを現象に適用する)

スピン偏極

流体モデルを拡張する

1. 新しい自由度を導入する

臨界モード，パイオン，スピン

2. 新しい項を見つける

Thermal shear

(3. 上の2つを現象に適用する)

スピン偏極

臨界ダイナミクス その1

Talk by Eduardo Grossi

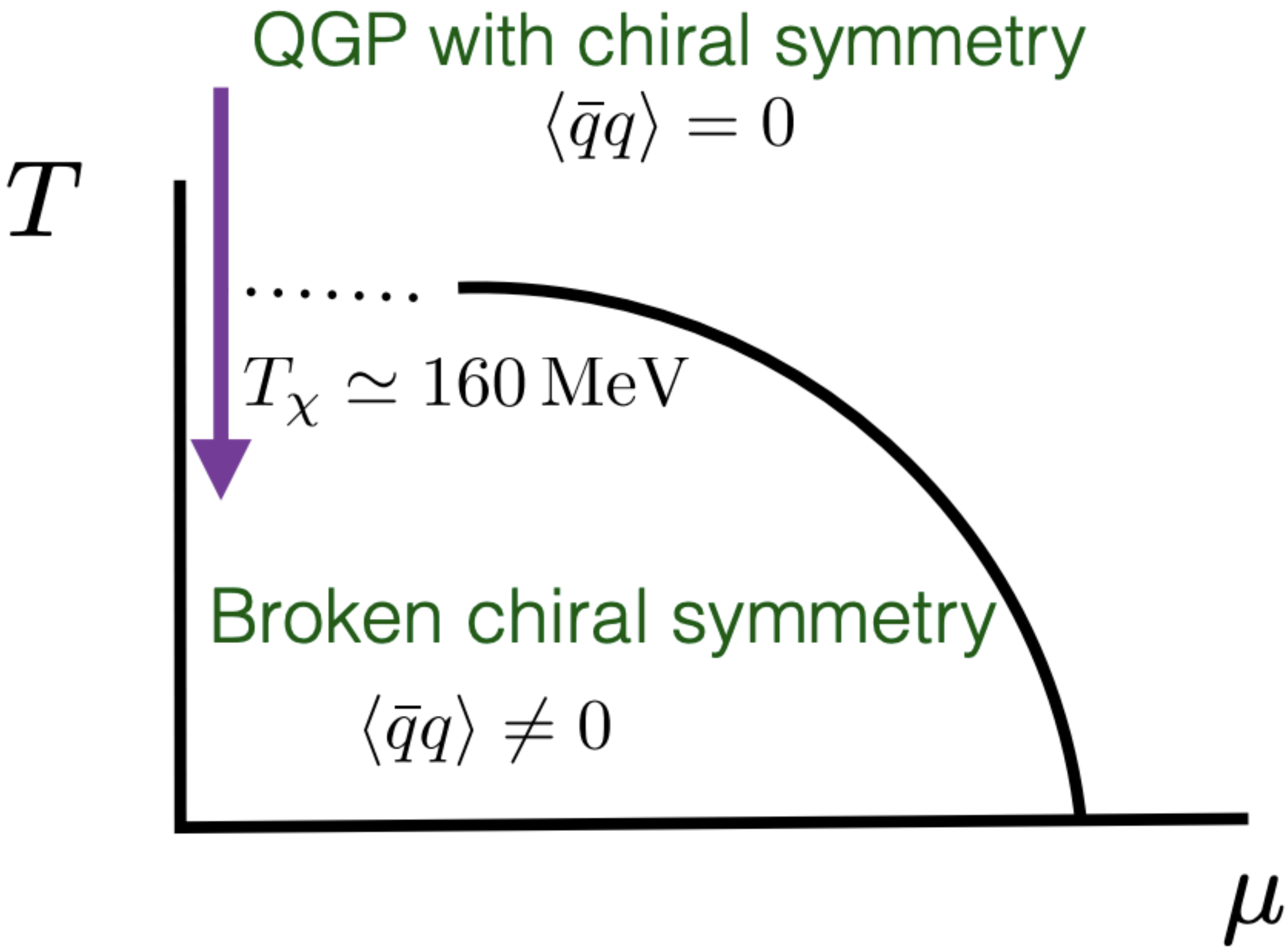
Soft pions near the QCD chiral critical point: transport and dynamics

Eduardo Grossi
IPhT Saclay, Ecole Polytechnique

E.G., A.Soloviev, D. Teaney, F. Yan PRD (2020)
E.G., A. Soloviev, D. Teaney, F. Yan PRD (2021)
A. Florio, E.G., A. Soloviev, D, Teaney PRD (2022)

QM 2022 Krakow 5-4-2022

Motivation



We are neglecting any hydro-dynamics of the chiral condensate !

Setup: the O(4) phase transition

The (approximated) conserved quantities of 2 flavour QCD are

$T^{\mu\nu}$	J_V^μ	J_A^μ
Stress	Iso-vector (isospin)	Iso-axial
(T, u^μ)	μ_V	μ_A
	$\bar{q}\gamma^0 t_I q$	$\bar{q}\gamma^0\gamma_5 t_I q$

The approximate flavour symmetry $SU(2)_L \times SU(2)_R \sim O(4)$

The order parameter is the chiral condensate

$$\langle \bar{q}q \rangle \sim \phi_\alpha = (\sigma, \varphi_\alpha) = (\text{sigma}, \text{pions})$$

We need the hydrodynamic theory of the charge and the order parameter

Rajagopal Wilczek (93)
Son and Stephanov (02)

Equation of motion (Model G)

Rajagopal Wilczek (93)

Chiral condensate ϕ_a + Axial and Vector charge $n_{ab} = \chi_0 \mu_{ab}$

$$\partial_t \phi_a + g_0\, \mu_{ab}\phi_b = \Gamma_0 \nabla^2 \phi_a - \Gamma_0 (m_0^2 + \lambda\phi^2)\phi_a + \Gamma_0 H_a + \theta_a\,,$$

$$\partial_t n_{ab} + g_0\, \nabla\cdot(\nabla\phi_{[a}\phi_{b]}) + H_{[a}\phi_{b]} = D_0 \nabla^2 n_{ab} + \partial_i \Xi^i_{ab}\,.$$

Ideal part — Dissipative part — Gaussian Noise

- The ideal part is charge conservation and Josephson constraint
- Two dissipative coefficient Γ_0 and D_0 and noise
- The simulation of the stochastic process is done with an ideal step and metropolis update.

Diffusion at high temperature, pion propagation at low temperature as the vev develops

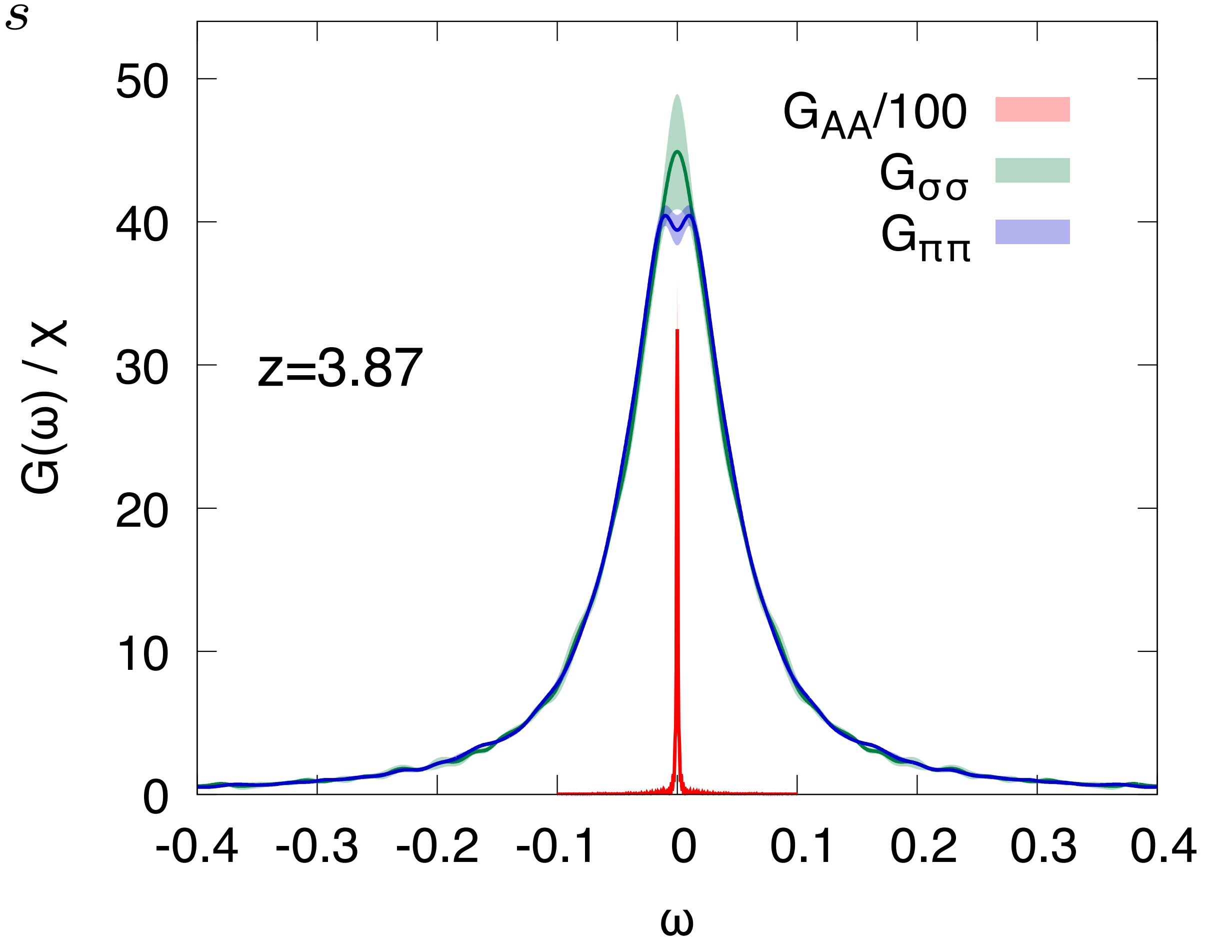
High temperature

$$G_{\sigma\sigma}(t,k) \equiv \frac{1}{V}\langle \sigma(t,\mathbf{k})\sigma(0,-\mathbf{k})\rangle_c\,,$$

$$G_{\pi\pi}(t,k) \equiv \frac{1}{3V}\sum_s \langle \pi_s(t,\mathbf{k})\pi_s(0,-\mathbf{k})\rangle_c\,,$$

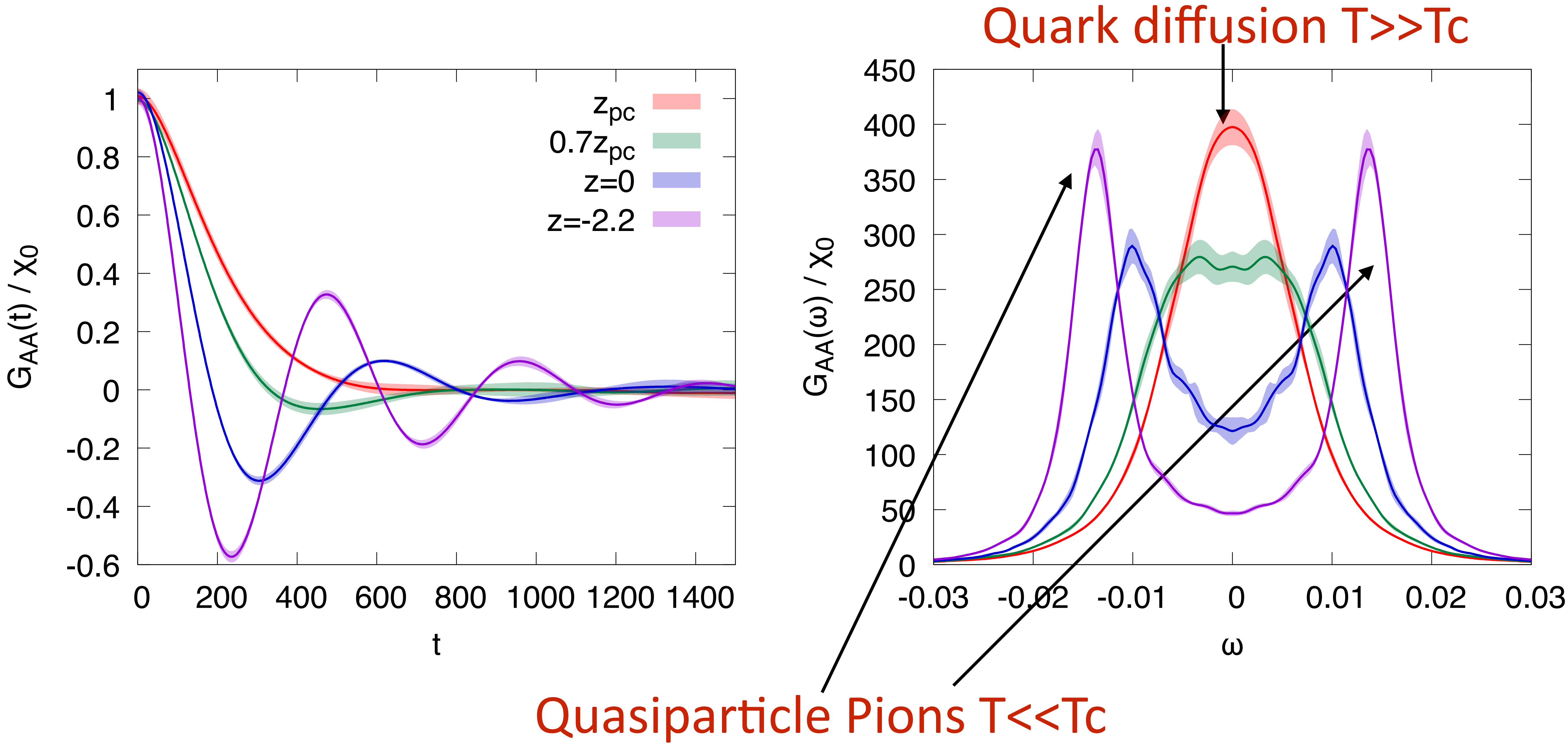
$$G_{AA}(t,k) \equiv \frac{1}{3V}\sum_s \langle n_A^s(t,\mathbf{k})n_A^s(0,-\mathbf{k})\rangle_c\,,$$

We focus on the statistical correlator at $k=0$



The axial charge is almost conserved the O(4) field are simply dissipate with a broad width

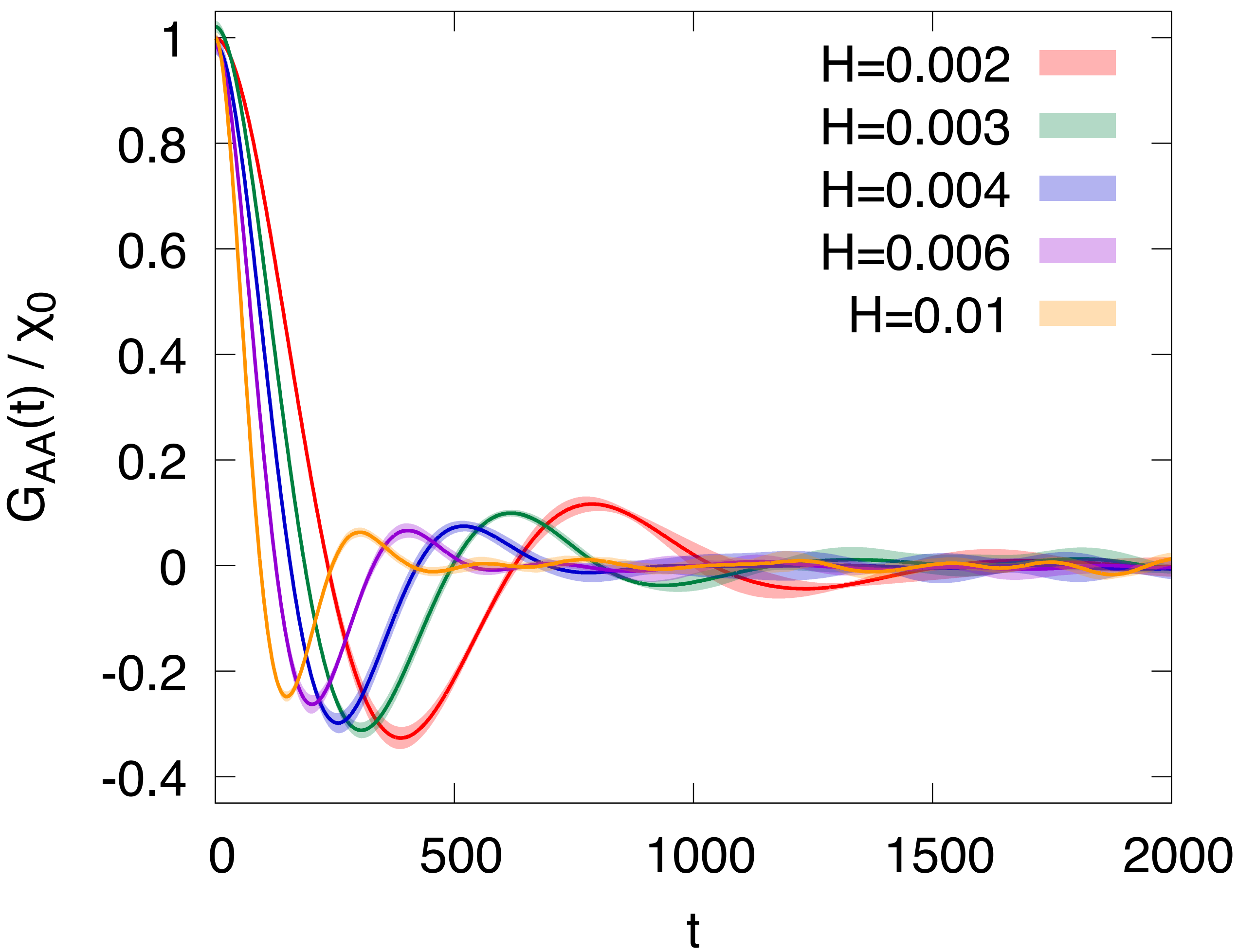
Propagation of axial charge across the transition

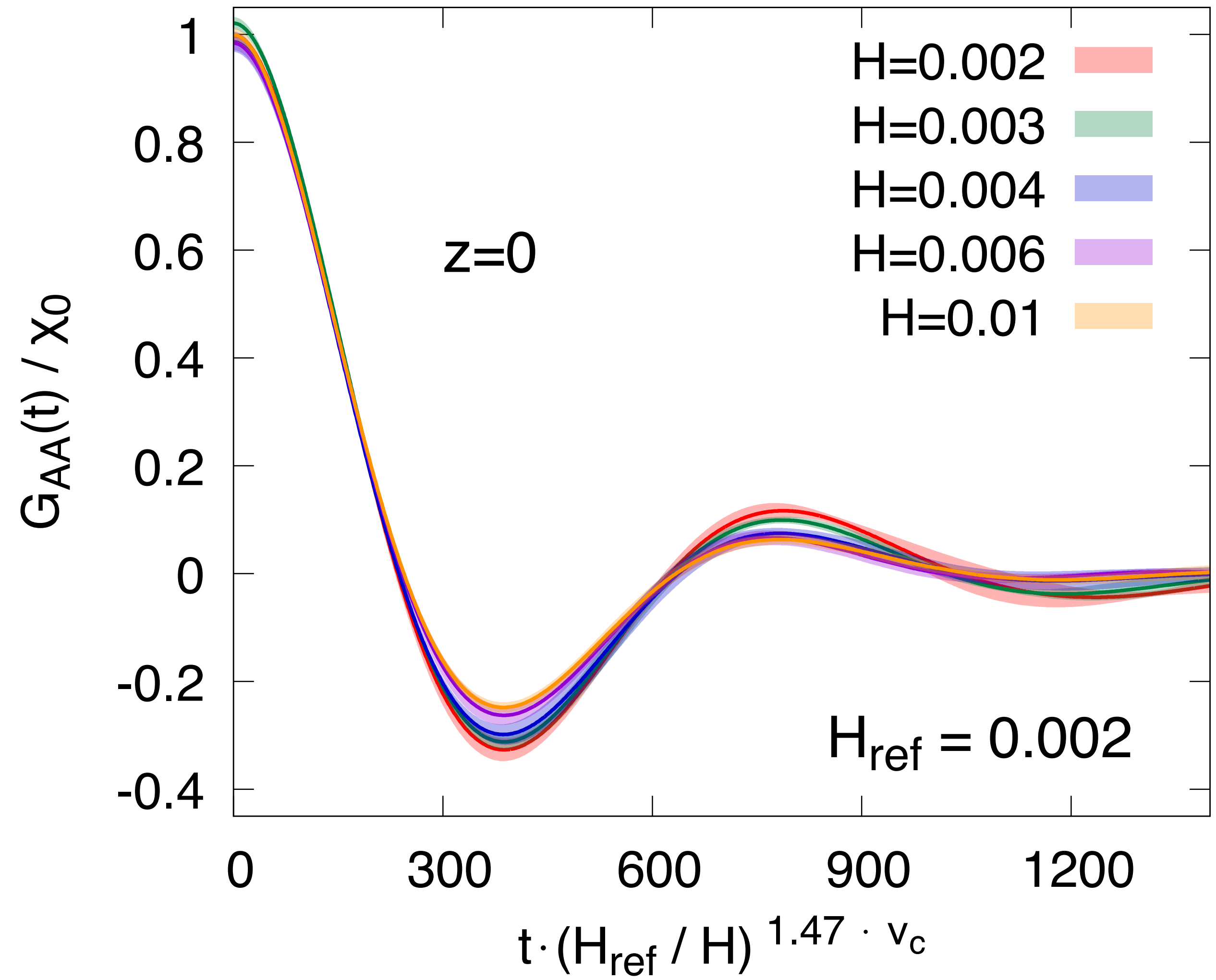


Around T_{pc} the axial charge start changing form a diffusive form to a quasiparticle one

Dynamical scaling on the critical line

On the Critical line z=0 we should have scaling with a dynamical critical exponent ζ





$$\frac{G_{AA}(t,H)}{\chi_0} = Y_A^c\left(H^{\zeta\nu_c} t\right) ,$$

Mesure: $\zeta = 1.47 \pm 0.01$

Expected: $\zeta = d/2 = 1.5$

Rajagopal Wilczek (93)

Effective Boltzmann equation

E.G., A.Soloviev, D. Teaney, F. Yan PRD (2020)

From the linear propagator we can define (using the Wigner transform) an effective kinetic description of the soft pions distribution function

$$\partial_t f_\pi + \frac{\partial E_{\mathbf{p}}}{\partial \mathbf{q}} \frac{\partial f_\pi}{\partial \mathbf{x}} - \frac{\partial E_{\mathbf{p}}}{\partial \mathbf{x}} \frac{\partial f_\pi}{\partial \mathbf{q}} = \text{interaction terms}$$

Well below the phase transition the pions propagate like quasipartilce with a modified energy dispersion from the medium

$$E_{\mathbf{p}} = v^2(p^2 + m^2)$$

Depends on $\bar{\sigma}$

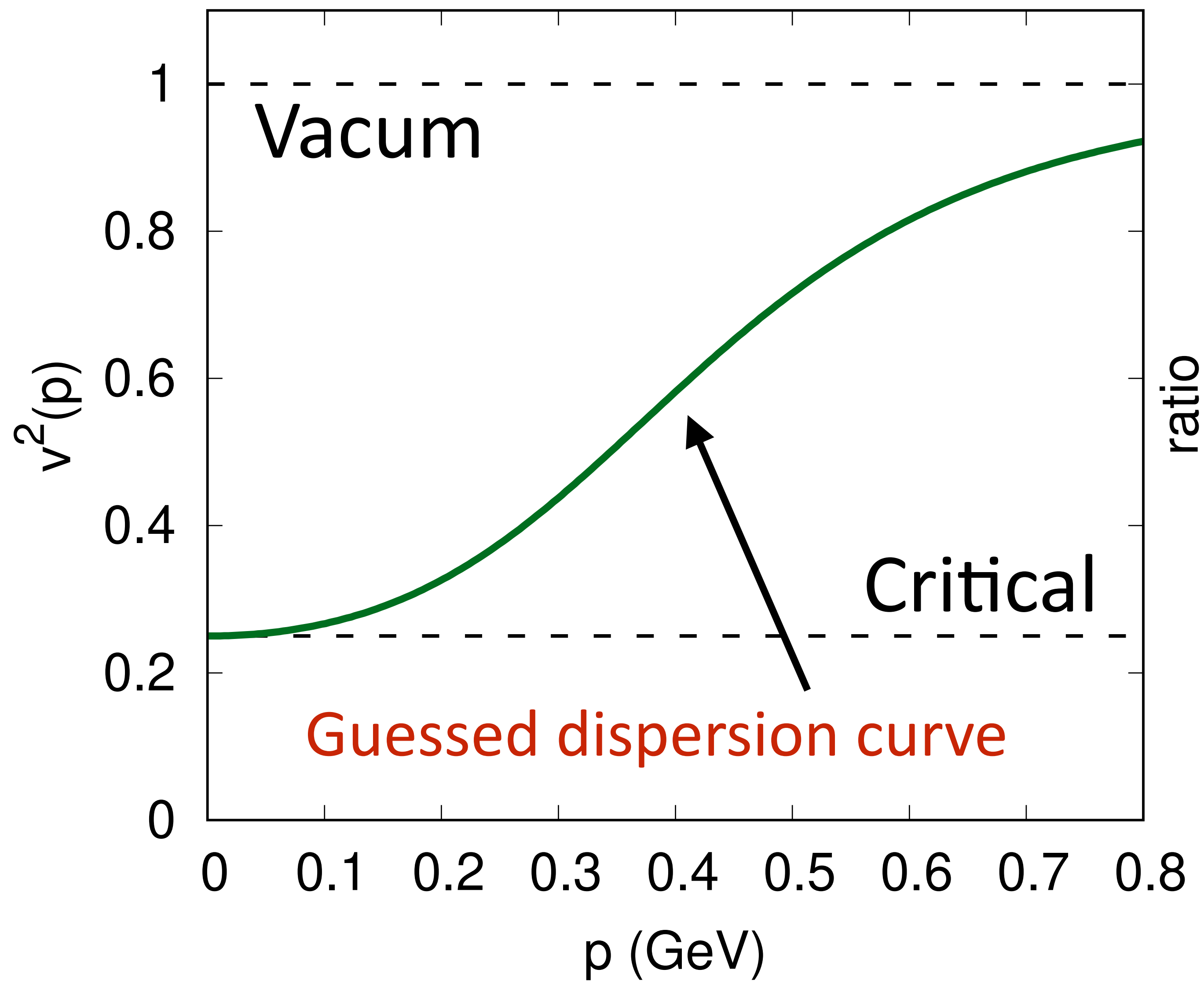
Soft pion enhancement

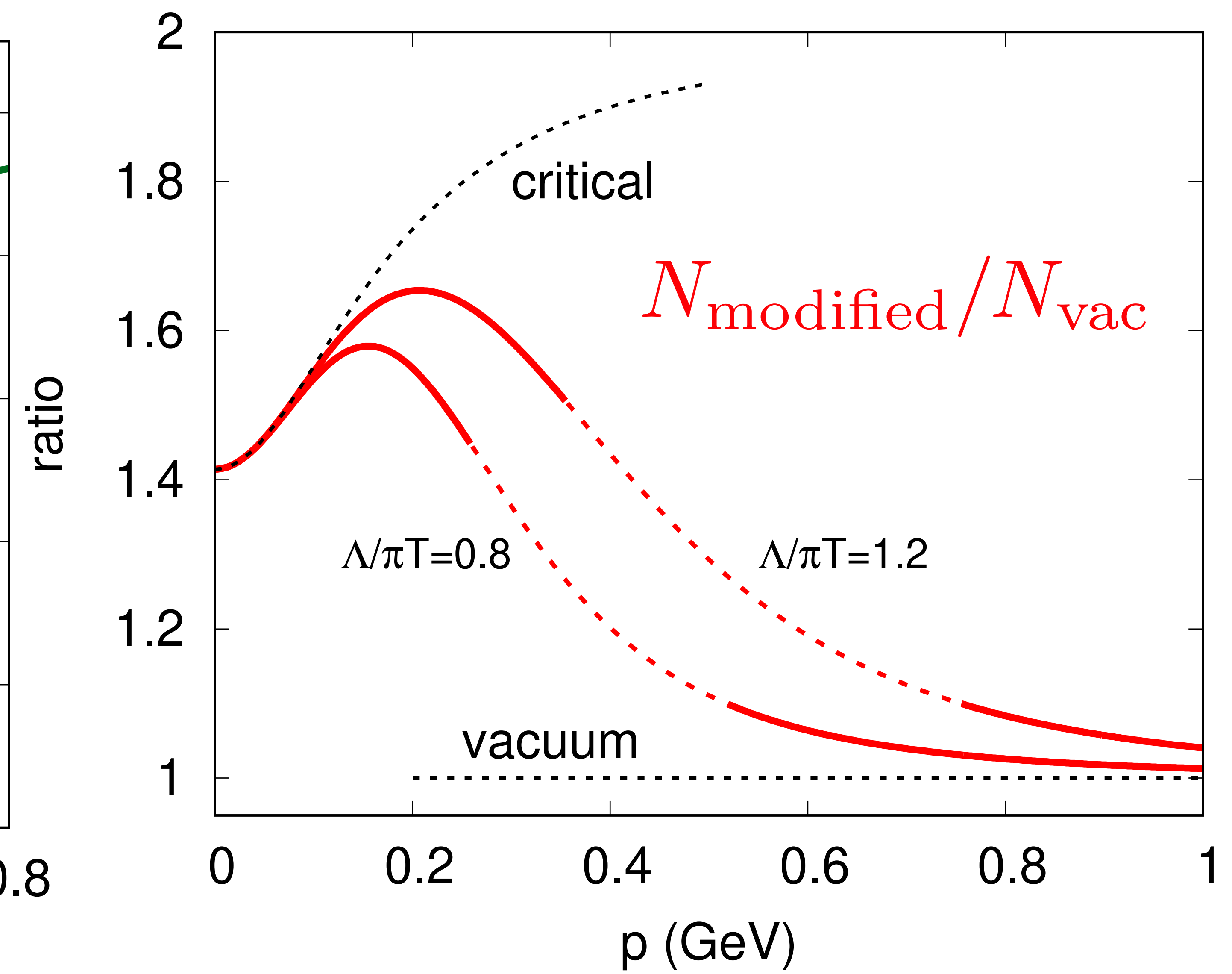
E.G., A. Soloviev, D. Teaney, F. Yan PRD (2021)

The dispersion curve get modified form the phase transition

$$E_{\mathbf{p}} = v^2(p)(p^2 + m^2)$$

$$n(E_p) = \frac{1}{e^{E_p/T} - 1}$$





Pion enhanced $p < 0.5$ GeV

臨界ダイナミクス その2

Talk by Xin An

Evolution of Non-Gaussian Hydrodynamic Fluctuations

Xin An

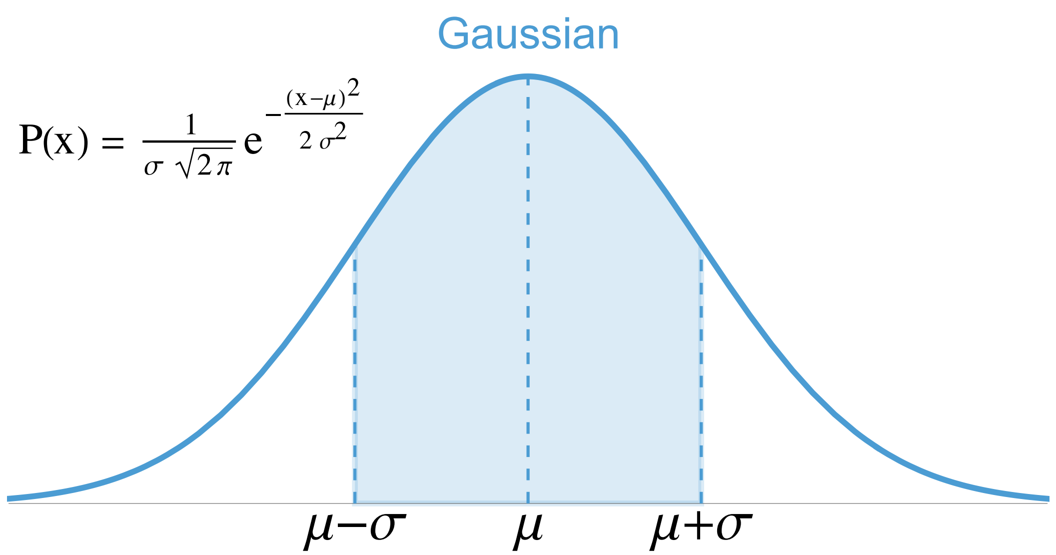
based on *PRL* **127**, 072301 and work in progress
with G. Başar, M. Stephanov and H.-U. Yee

XXIXth Quark Matter, Kraków, Poland
Apr 5, 2022

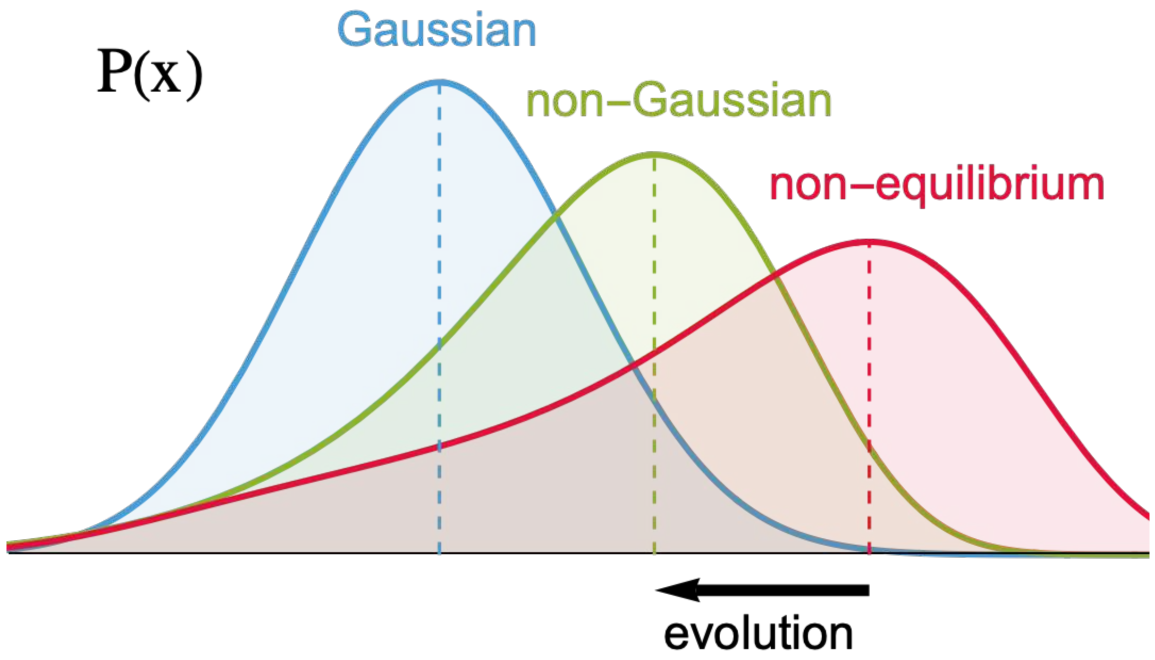
UNC

Non-Gaussian & non-equilibrium fluctuations

- **Gaussian** distribution : normality manifested by central limit theorem .



- Fluctuations in general deviate from normal distribution (i.e., **non-Gaussian**) and/or evolve towards equilibration (i.e., **non-equilibrium**) .



Stochastic description

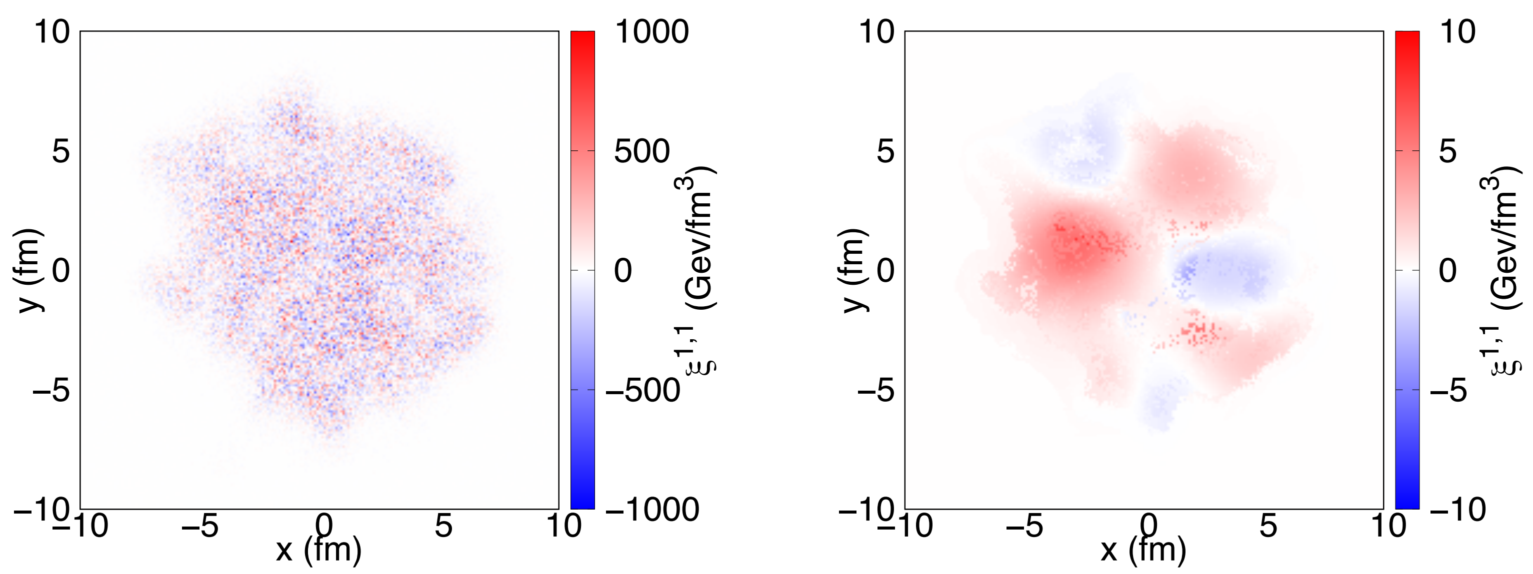
- **Langevin equation** for a set of variables ψ_i :

$$\partial_t \psi_i = F_i + \xi_i \,, \quad \text{(Newton's 2nd law)}$$

$$\langle \xi_i(x_1)\xi_j(x_2)\rangle = 2Q_{ij}\delta^{(4)}(x_1 - x_2)\,, \quad \text{(FDT)}$$

where $F_i \equiv$ drift force ; $\xi_i \equiv$ random force (noise) ; $Q_{ij} \equiv$ Onsager matrix .

- It suffers from the problem of **infinite noise** and **multiplicative noise** .



regularize the infinite noise in stochastic sampling Singh *et al.*, 2019

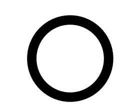
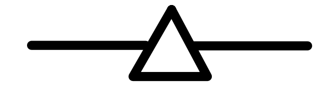
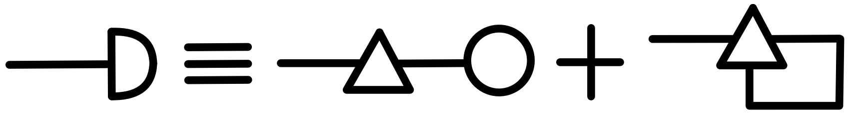
Landau *et al.*, 1959; Kapusta *et al.*, 2012; Young *et al.*, 2015; Sakaida *et al.*, 2017; Nahrgang *et al.*, 2018; etc. See also Posters by De and Pihan

Deterministic description

- The Langevin equation is complementary to **Fokker-Planck equation** for probability distribution $P[t;\psi_i]$:

$$\partial_t P = \overbrace{\left(-F_i P + (Q_{ij}P)_{,j}\right)}^{\text{probability flux}}{}_{,i} \,, \qquad \text{(Ito's prescription)}$$

where $(\ldots)_{,i} = \delta(\ldots)/\delta\psi_i$ and

entropy	$S = \log P_{\text{eq}}$	
Poisson/Onsager matrix	$M_{ij} = \Omega_{ij} + Q_{ij}$	
drift force	$F_i = M_{ij}S_{,j} + M_{ij,j}$	

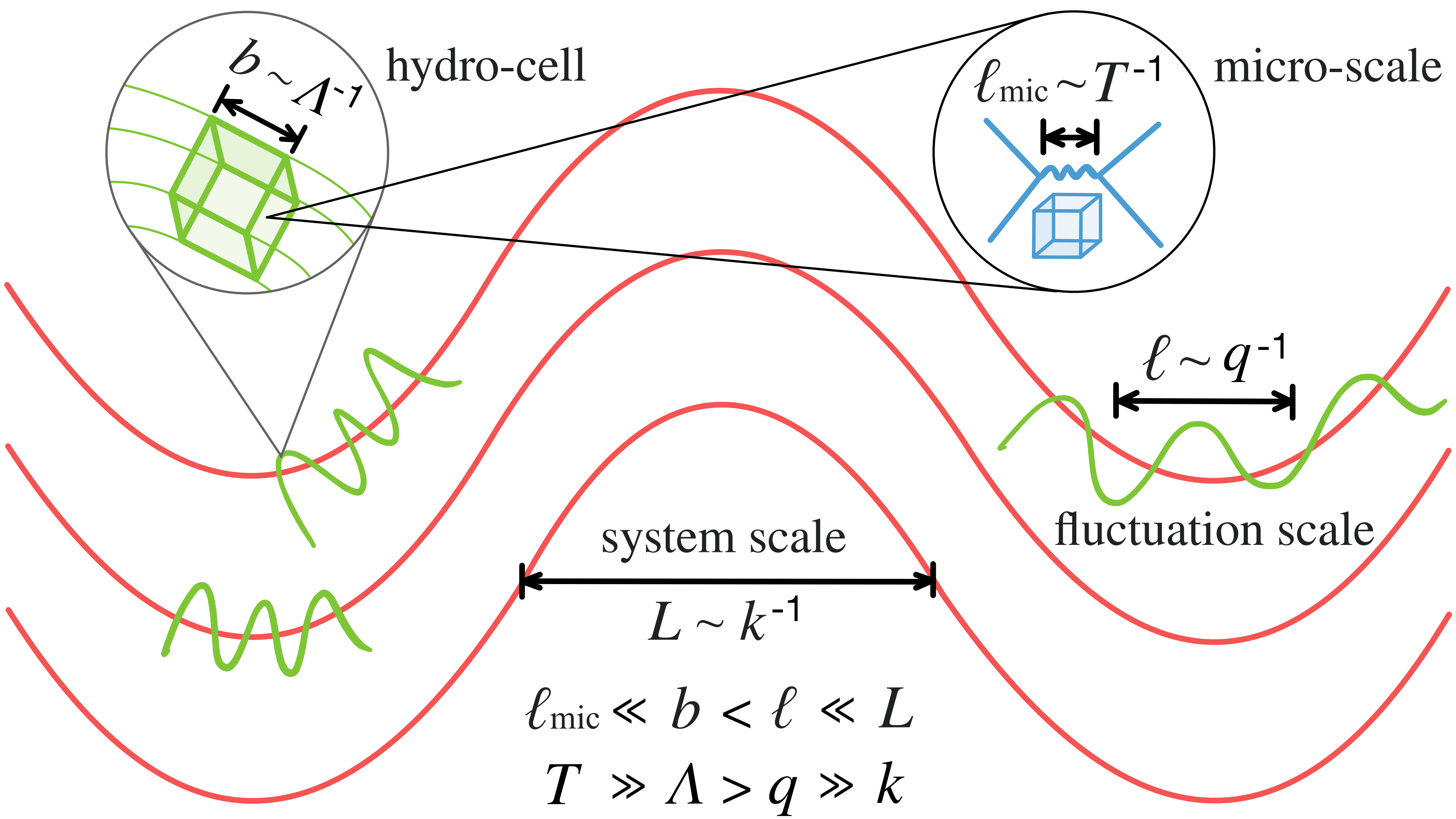
- Evolution equation for n-th **cumulant** (n-point connected function) $G_n^{\text{c}} \equiv \langle \psi_1 \ldots \psi_n \rangle_{\text{c}}$: see An *et al.*, PRL, 2021 for details

$$\partial_t P \quad \Longrightarrow \quad \partial_t G_n^{\text{c}} = \mathscr{F}[\{G_n^{\text{c}}\}]\,.$$

Small parameters

- Small parameters in, e.g., hydrodynamics : Akamatsu *et al.*, 2017; An *et al.*, 2019

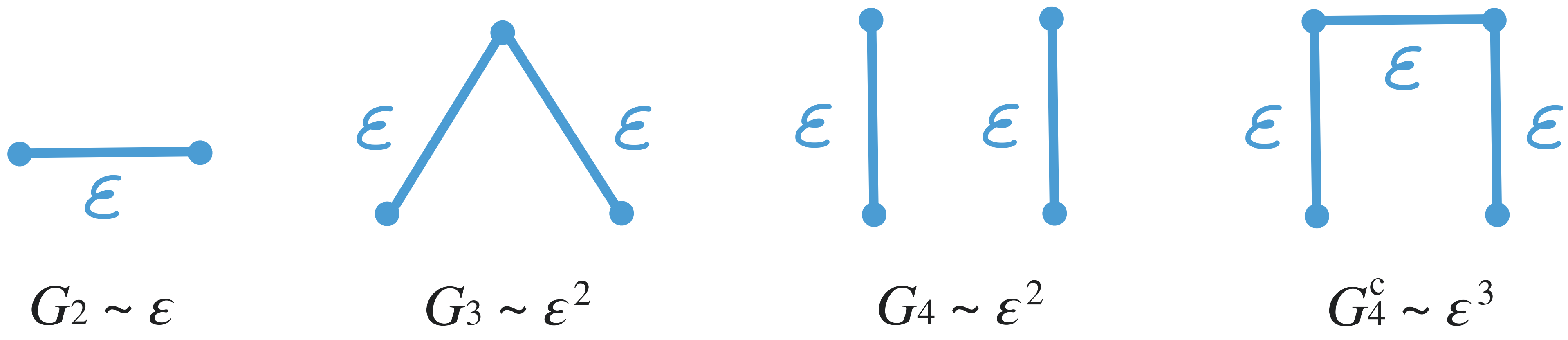
parameter	expression	meaning	role
δ	$\ell_{\rm mic}/\ell$	Knudsen number	controls gradient expansion
ε	$(\ell_{\rm mic}/\ell)^3$	inverse of uncorrelated DOF	controls loop expansion



Truncated equations

- Effective power counting:

$$S \sim \varepsilon^{-1}, \qquad M_{ij} \sim \varepsilon\delta^2, \qquad G_n^{\mathrm{c}} \sim \varepsilon^{n-1}.$$



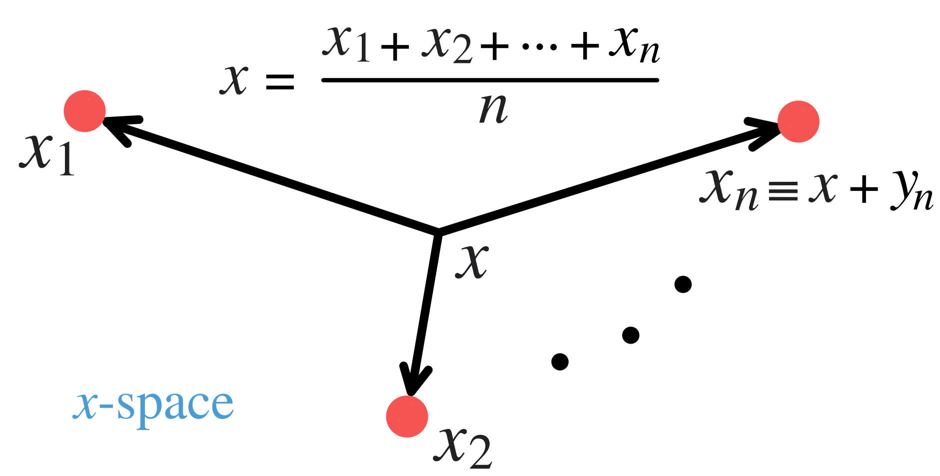
- Perturbed in small parameters, the evolution equations for cumulants can be systematically truncated and iteratively solved (e.g., in hydrodynamics):

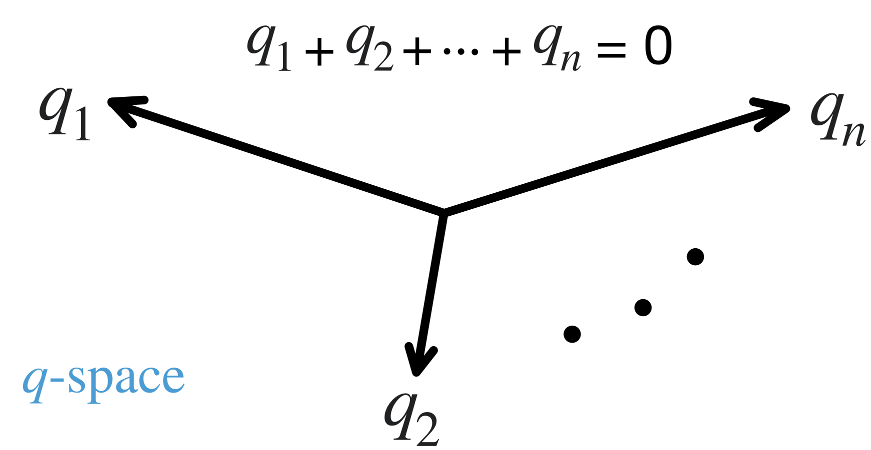
$$\partial_t G_n^{\mathrm{c}} = \underbrace{\mathscr{F}_{\mathrm{tree}}[\{G_n^{\mathrm{c}}\}]}_{\mathcal{O}(\varepsilon^{n-1}\delta^2)} + \text{higher orders}.$$

Multi-point Wigner function

- We introduced the novel **n-point Wigner function** (cumulant in q-space)

$$W_n^{\mathrm{c}}(x, q_1, \ldots, q_n) = \int \left[\prod_{i=1}^{n} d^3 y_i e^{-i q_i \cdot y_i}\right] \delta^{(3)}\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right) G_n^{\mathrm{c}}(x + y_1, \ldots, x + y_n),$$





being traditional Wigner function when $n = 2$:

$$W_2(x, q, -q) = \int d^3 y \, e^{-i q \cdot y} G_2\left(x + \frac{y}{2}, x - \frac{y}{2}\right).$$

- Evolution equation for W_n^{c}:

$$\partial_t W_n^{\mathrm{c}} = \mathscr{F}[\{W_n^{\mathrm{c}}\}].$$

Cumulant evolution equations for diffusive charge

- Evolution equations for $W_{n=2,3,\ldots}$ of diffusive charge:

$$\partial_t W_2(q_1,q_2) = -\left[\gamma q_1^2 W_2(q_1,q_2) + \lambda q_1\cdot q_2\right]_{\text{Perm.}}\,, \quad \text{(Hydro+ equation)}$$

$$\partial_t W_3(q_1,q_2,q_3) = -\Bigg[\frac{1}{2}\gamma q_1^2 W_3(q_1,q_2,q_3) + \frac{1}{2}\gamma' q_1^2 W_2(-q_2,q_2)W_2(-q_3,q_3) + \lambda' q_1\cdot q_2 W_2(-q_3,q_3)\Bigg]_{\text{Perm.}}\,, \quad \text{(non-Gaussian Hydro+)}$$

$\cdots$ where $\gamma = \lambda\alpha' \equiv$ diffusion coefficient.

- Equilibrium solutions match thermodynamics:

$$W_2^{\text{eq}} = \frac{1}{\alpha'}\,, \qquad W_3^{\text{eq}} = -\frac{\alpha''}{\alpha'^3}\,, \qquad W_4^{\text{c,eq}} = -\frac{\alpha'''}{\alpha'^4} + \frac{3\alpha''^2}{\alpha'^5}\,, \quad \cdots$$

- In critical regime $(T^{-1} \ll \xi \ll q^{-1})$: Hohenberg *et al.*, 1977; Stephanov, 2009

$$\lambda \sim \xi\,, \quad \lambda' \sim \xi^{\frac{3}{2}}, \quad \gamma \sim \xi^{-1}, \quad \gamma' \sim \xi^{-\frac{1}{2}}, \quad W_n^{\text{c}} \sim \xi^{\frac{5n-6}{2}}, \quad \partial_t W_n^{\text{c}} \sim q^2 \xi^{\frac{5n-8}{2}}.$$

For confluentized equations see An *et al.*, to appear; for the EFT approach, see Poster by Sogabe

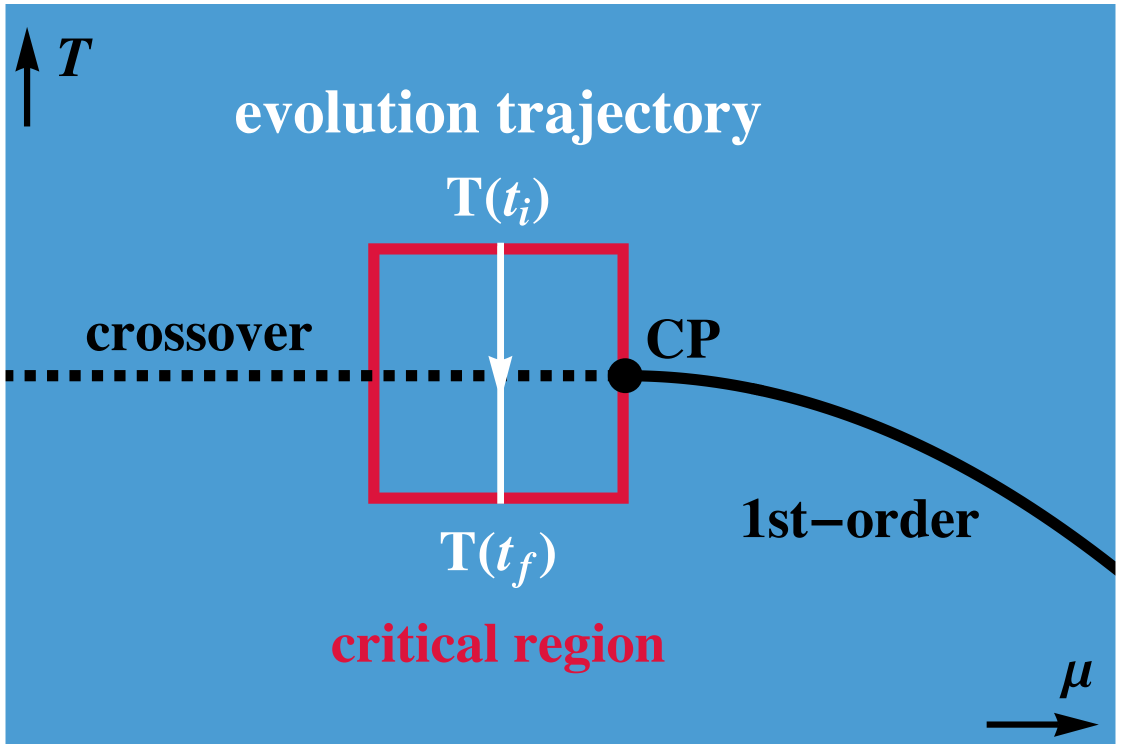
Evolution near critical point

- System evolution in the crossover region near CP :

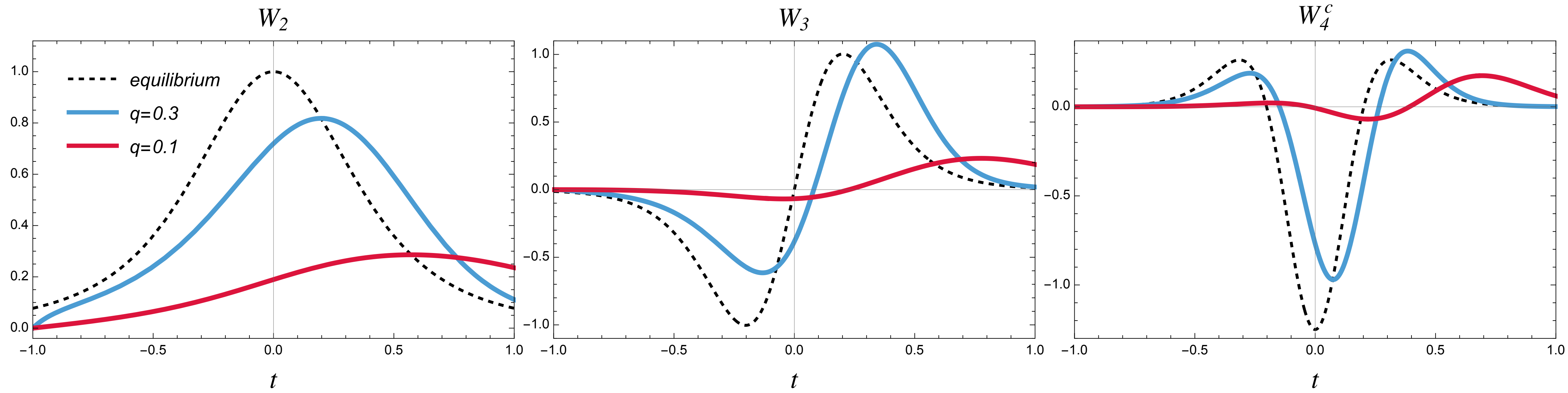
mapping Ising critical region

$$\xi_{\text{QCD}} = \xi_{\text{Ising}}\left(r(\mu,T), h(\mu,T)\right)$$

For more realistic EoS,
see e.g., Contributions by Kapusta and Ratti

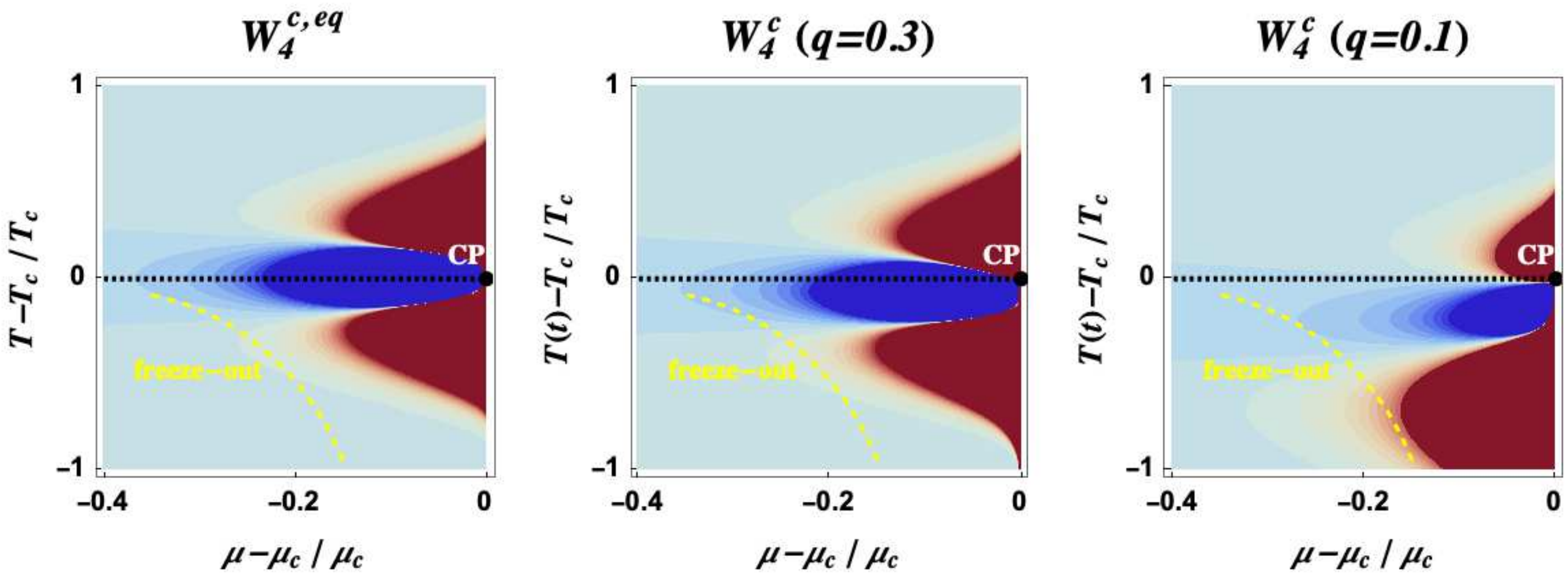


- Fluctuations are closer to equilibrium on smaller scale (larger q) , and retain longer memory on larger scale (smaller q) :

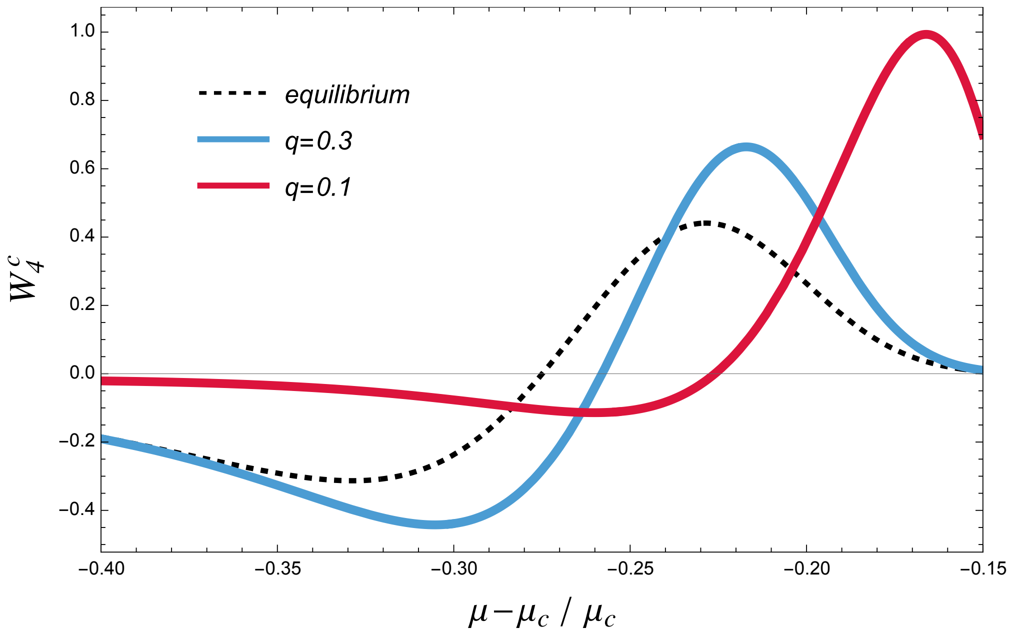


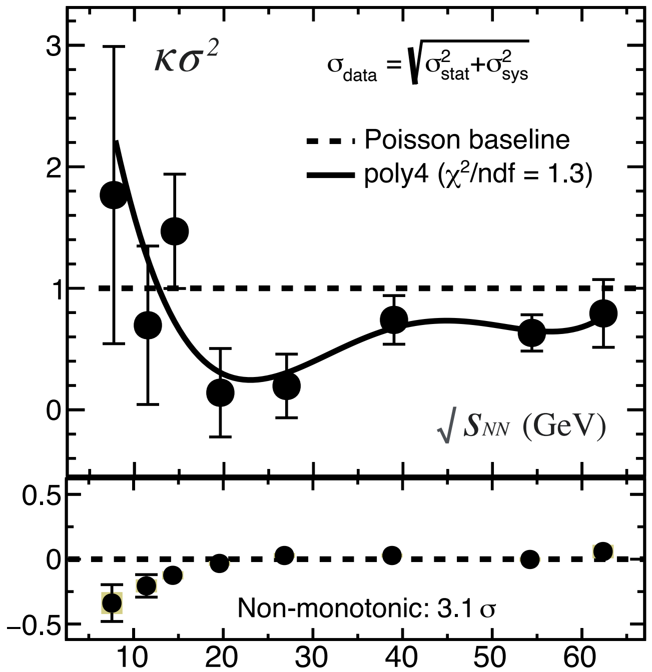
Freeze-out

- W_4^c ($\sim$ kurtosis) in and out of equilibrium along the **freeze-out** curve :



- Expected **non-monotonic** behavior : Stephanov, 2011; STAR, 2021; Pradeep *et al.*, 2022 (also Poster)





流体モデルを拡張する

1. 新しい自由度を導入する

臨界モード，パイオン，スピン

2. 新しい項を見つける

Thermal shear

(3. 上の2つを現象に適用する)

スピン偏極

シアーからのスピン偏極1

Talk by Baochi Fu

Shear-Induced Polarization & Spin Hall Effects in heavy-ion collisions

Baochi Fu

Peking University

with S. Liu, L.-G. Pang, H. Song and Y. Yin

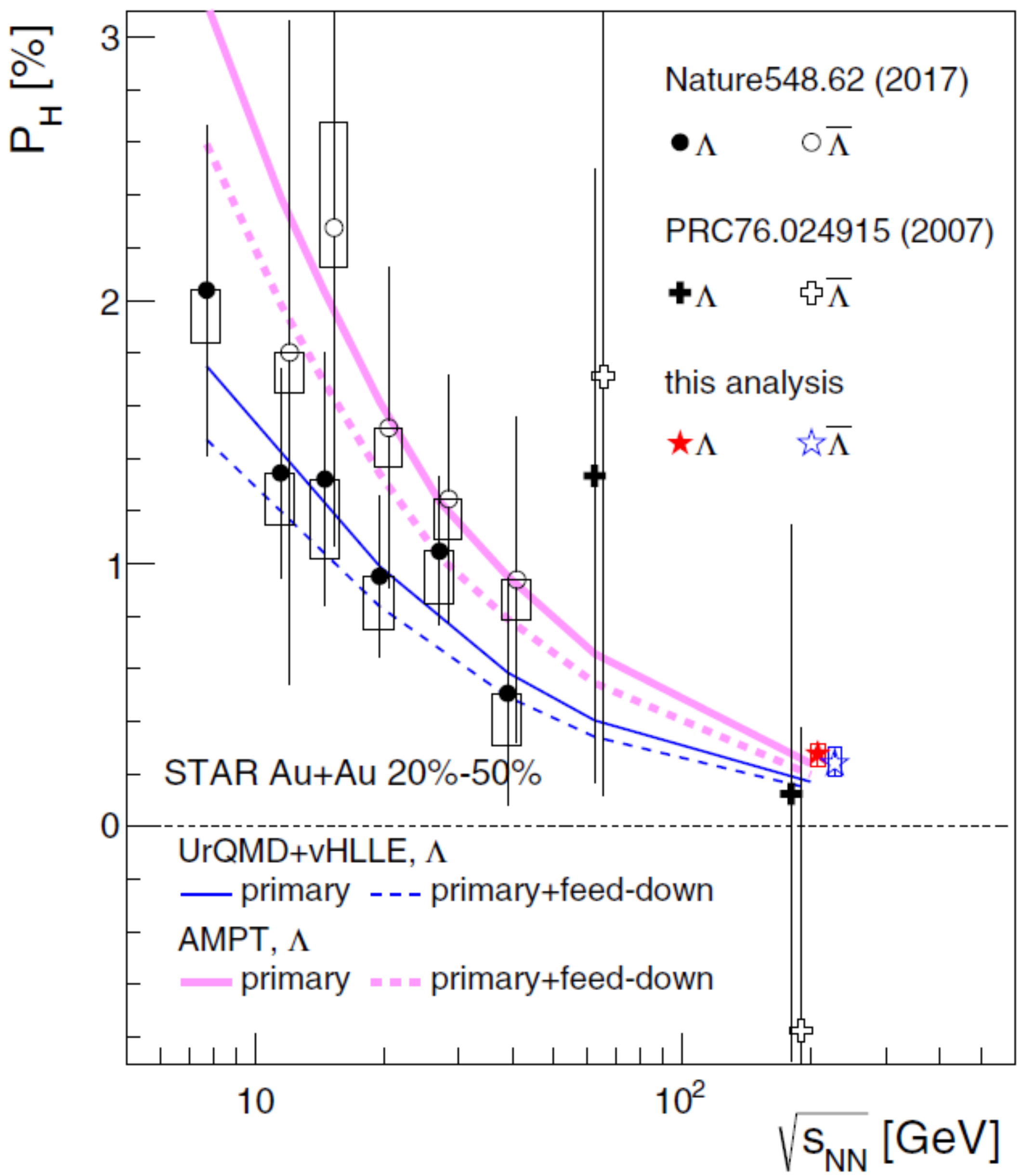
Shear-Induced Polarization: Phys.Rev.Lett. 127 14, 142301(2021)
Spin Hall Effects: arXiv: 2201.12970

PEKING UNIVERSITY

QUARK MATTER KRAKÓW 2022

Global polarization

STAR, Phys.Rev.C 98 (2018) 014910



- Spin-orbital coupling in non-central heavy ion collisions

- Signals observed at STAR BES energy:

  STAR Collaboration, Nature 548, 62 (2017)

- Data described be the statistic calculation

$$S^{\mu}(p) \leftarrow \varpi_{\nu\rho}(x)$$

Hydrodynamics:

I. Karpenko, F. Becattini, Eur.Phys.J.C 77 (2017) 4, 213
BF, K. Xu, X-G, Huang, H. Song, Phys.Rev.C 103 (2021) 2, 024903

Transport model:

H. Li, L. Pang, Q. Wang, X. Xia, Phys.Rev. C96 (2017) 054908
D. Wei, W. Deng, X. Huang, Phys.Rev. C99 (2019) 014905

local polarization: 'Sign puzzle'

- Different trend/sign in $P_y(\phi)$ and $P_z(\phi)$ results
- Long exist in hydrodynamic and transport calculations

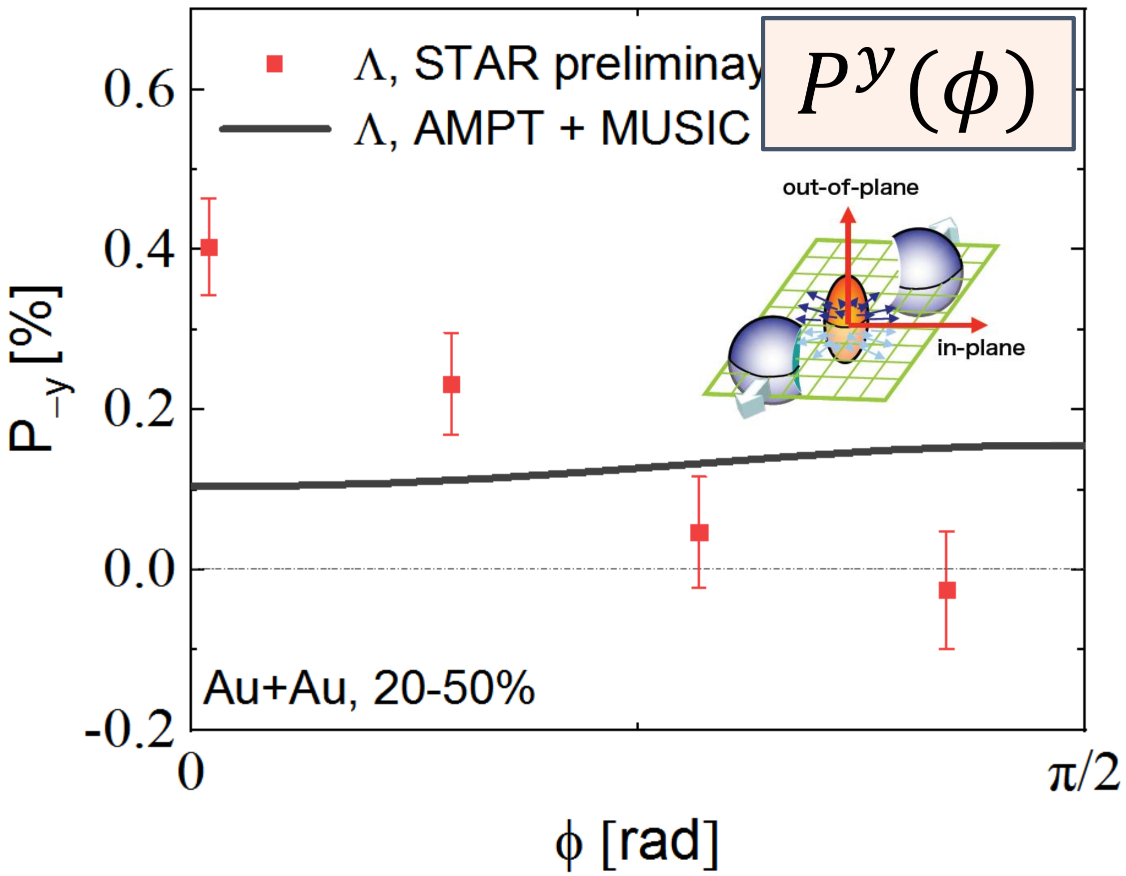
See also:

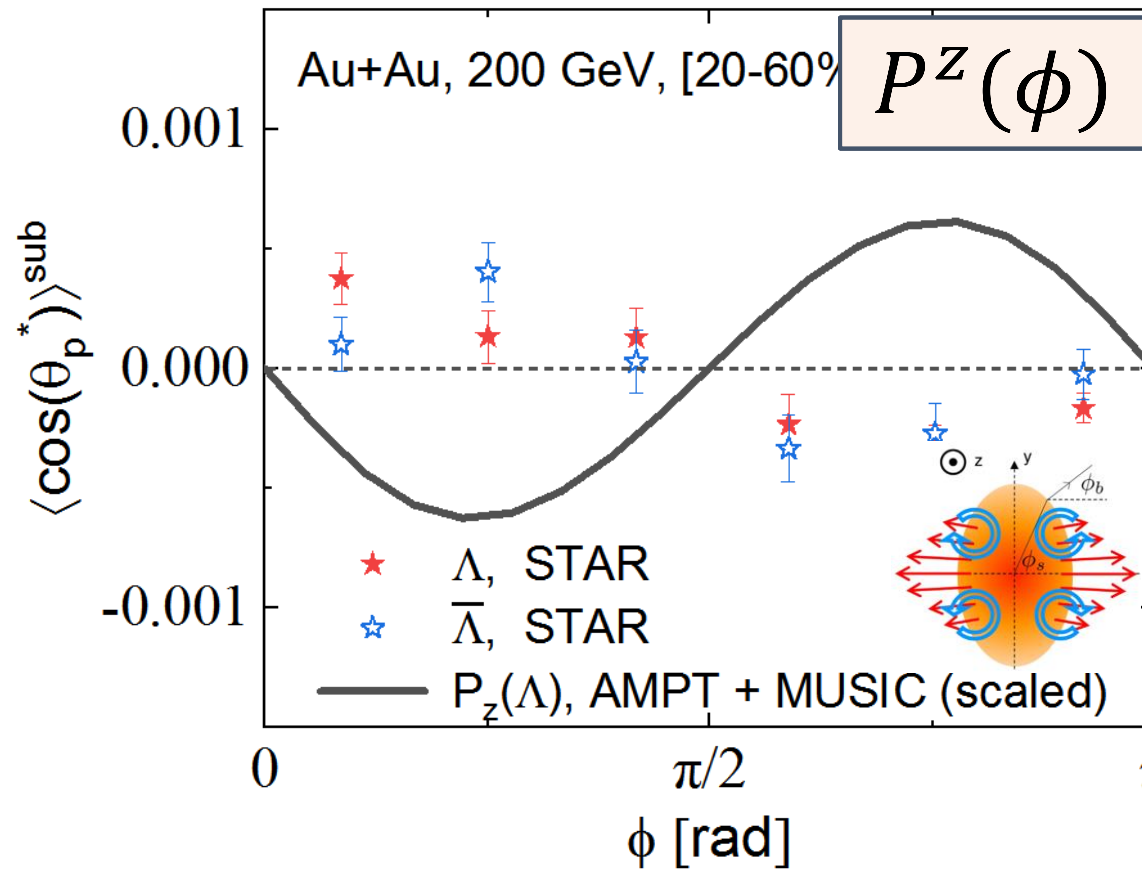
Karpenko, Becattini, EPJC 77 (2017) 4, 213

D. Wei, et al., PRC 99 (2019) 014905

X. Xia, et al., PRC 98 (2018) 024905

Becattini, Karpenko, PRL 120 (2018) 012302





BF, K. Xu, X-G, Huang, H. Song, Phys.Rev.C 103 (2021) 2, 024903

Shear Induced Polarization (SIP)

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin, Phys.Rev.Lett. 127 14, 142301(2021)

Axial Wigner function from CKT (Chen, Son, Stephanov, PRL 115 (2015) 2, 021601)

$$\mathcal{A}^\mu = \sum_\lambda \left(\lambda p^\mu f_\lambda + \frac{1}{2} \frac{\epsilon^{\mu\nu\alpha\rho} p_\nu u_\alpha \partial_\rho f_\lambda}{p \cdot u} \right)$$

Expand $\mathcal{A}^\mu$ to 1$^{\text{st}}$ order gradient of the fields:

$$\mathcal{A}^\mu = \frac{1}{2}\beta n_0(1-n_0)\left\{ \epsilon^{\mu\nu\alpha\lambda} p_\nu \partial_\alpha^\perp u_\lambda + 2\epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \left[\beta^{-1}(\partial_\lambda \beta)\right] - 2\frac{p_\perp^2}{\varepsilon_0}\epsilon^{\mu\nu\alpha\rho} u_\nu Q_\alpha^{\ \lambda} \sigma_{\rho\lambda} \right\}$$

Vorticity — T gradient (spin Nernst effect) — Shear-Induced Polarization

- $\sigma^{\mu\nu}$: shear stress tensor (symmetric)
- No free parameter
- Identical form by linear response theory with arbitrary mass (S. Liu and Y. Yin, JHEP 07 (2021) 188)

$$Q^{\mu\nu} = -p_\perp^\mu p_\perp^\nu / p_\perp^2 + \Delta^{\mu\nu}/3$$

$$\sigma^{\mu\nu} = \frac{1}{2}\left(\partial_\perp^\mu u^\nu + \partial_\perp^\nu u^\mu\right) - \frac{1}{3}\Delta^{\mu\nu}\partial_\perp \cdot u$$

Shear Induced Polarization (SIP)

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin, Phys.Rev.Lett. 127 14, 142301(2021)

Axial Wigner fu

To one-loop order (in charge neutral fluid)

$\epsilon^{\mu\nu\alpha\lambda} p_\nu \partial_\alpha (\beta u)_\lambda$

Thermal vorticity

$$\varpi_{\mu\nu} = \frac{1}{2}(\partial_\nu(\beta u_\mu) - \partial_\mu(\beta u_\nu))$$

Expand $\mathcal{A}^\mu$ to

$$\mathcal{A}^\mu = \frac{1}{2}\beta n_0(1-n_0)\left\{\epsilon^{\mu\nu\alpha\lambda} p_\nu \partial^\perp_\alpha u_\lambda + 2\epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \left[\beta^{-1}(\partial_\lambda \beta)\right] - 2\frac{p_\perp^2}{\varepsilon_0}\epsilon^{\mu\nu\alpha\rho} u_\nu Q_\alpha^{\ \lambda} \sigma_{\rho\lambda}\right\}$$

Vorticity

T gradient (spin Nernst effect)

Shear-Induced Polarization

Total P^μ = [Vorticity] + [T gradient] + [Shear]

Shear Induced Polarization (SIP)

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin,
Phys.Rev.Lett. 127 14, 142301(2021)

Axial Wigner fu

To one-loop order (in charge neutral fluid)

$$\epsilon^{\mu\nu\alpha\lambda} p_\nu \partial_\alpha (\beta u)_\lambda$$

Thermal vorticity

$$\varpi_{\mu\nu} = \frac{1}{2}(\partial_\nu(\beta u_\mu) - \partial_\mu(\beta u_\nu))$$

Expand $\mathcal{A}^\mu$ to

$$\mathcal{A}^\mu = \frac{1}{2}\beta n_0 (1 - n_0) \left\{ \epsilon^{\mu\nu\alpha\lambda} p_\nu \partial_\alpha^\perp u_\lambda + 2\epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \left[\beta^{-1}(\partial_\lambda \beta)\right] - 2\frac{p_\perp^2}{\varepsilon_0} \epsilon^{\mu\nu\alpha\rho} u_\nu Q_\alpha^{\ \lambda} \sigma_{\rho\lambda} \right\}$$

Vorticity

T gradient
(spin Nernst effect)

Shear-Induced Polarization

Total P^μ = [Vorticity] + [T gradient] + [Shear]

Total P^μ = [Thermal vorticity] + [Shear]

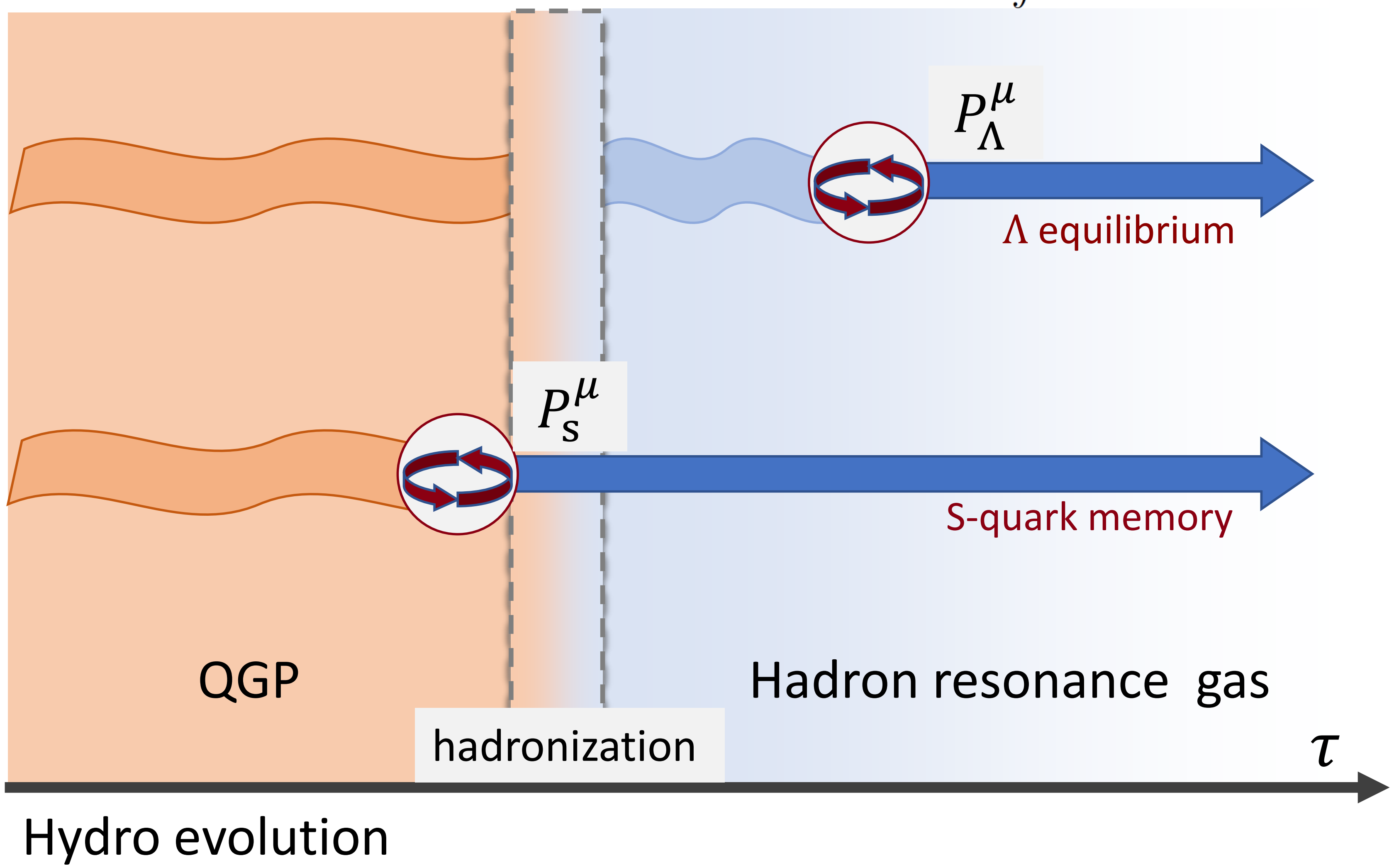
The only new effect

See also: Kumar@Tues. & Buzzegoli@Tues.

‘Λ equilibrium’ vs. ‘S-quark memory’

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin, Phys.Rev.Lett. 127 14, 142301(2021)

Spin Cooper-Frye: $$P^{\mu}(\boldsymbol{p}) = \frac{\int d\Sigma^{\alpha} p_{\alpha}\, \mathcal{A}^{\mu}(x, \boldsymbol{p}; m)}{2m \int d\Sigma^{\alpha} p_{\alpha} n(\beta\varepsilon_0)}$$



‘Λ equilibrium’

$\tau_{\text{spin},\Lambda} \to 0$

Polarization of Λ-hyperon

$P_{\Lambda}^{\mu}(p)$

F. Becattini (2013)
and later hydrodynamic(transport) calculations

‘S-quark memory’

$\tau_{\text{spin},\Lambda} \to \infty$

Polarization of S-quark

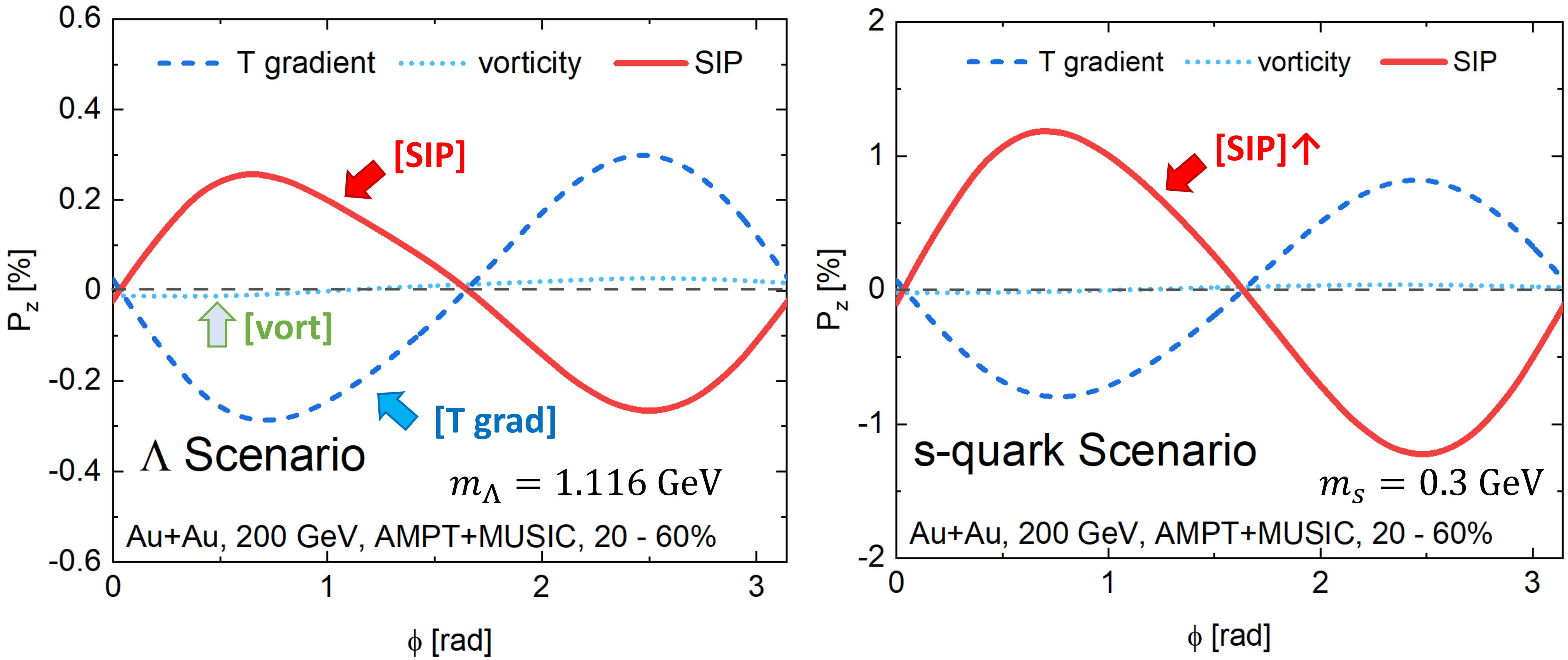
$P_{\Lambda}^{\mu}(p) = P_{s}^{\mu}(p)$

Z.-T. Liang, X.-N. Wang, PRL 94 (2005) 102301

Competition of P_z: Grad T vs. SIP

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin, PRL 127 14, 142301(2021)

Total P^{μ} = [vorticity] + [T grad] + [SIP]



- [SIP]: "$+\sin(2\phi)$" structure for P_z (same as exp.)
- Total polarization: a competition between [SIP] and [Grad T]

Competition between:

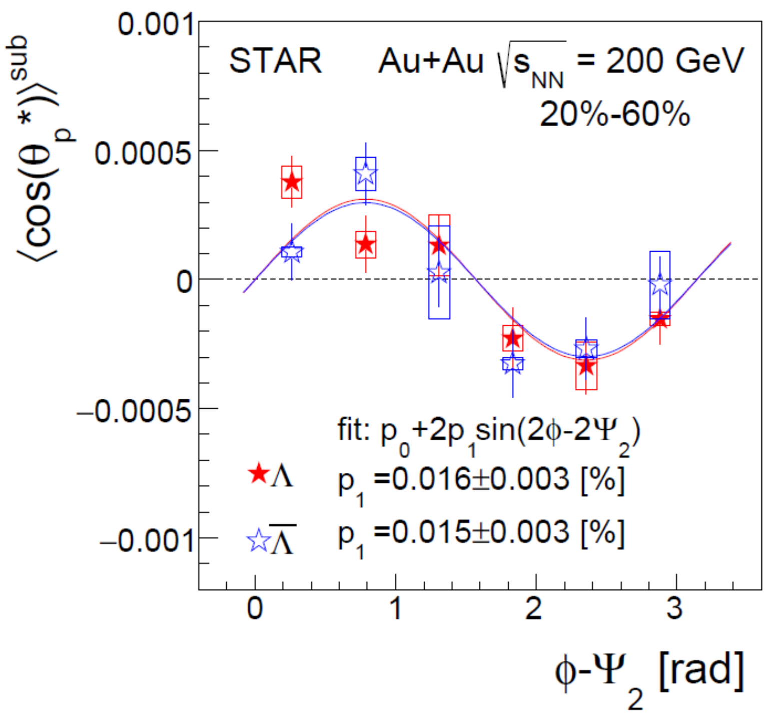
T-grad: $\epsilon^{\mu\nu\alpha\lambda}u_\nu p_\alpha[\beta^{-1}\partial_\lambda\beta]$

Shear: $\epsilon^{\mu\nu\alpha\rho}u_\nu p_\rho\left(\frac{p^\lambda}{\epsilon_0}\right)\partial_{(\alpha}u_{\lambda)}$

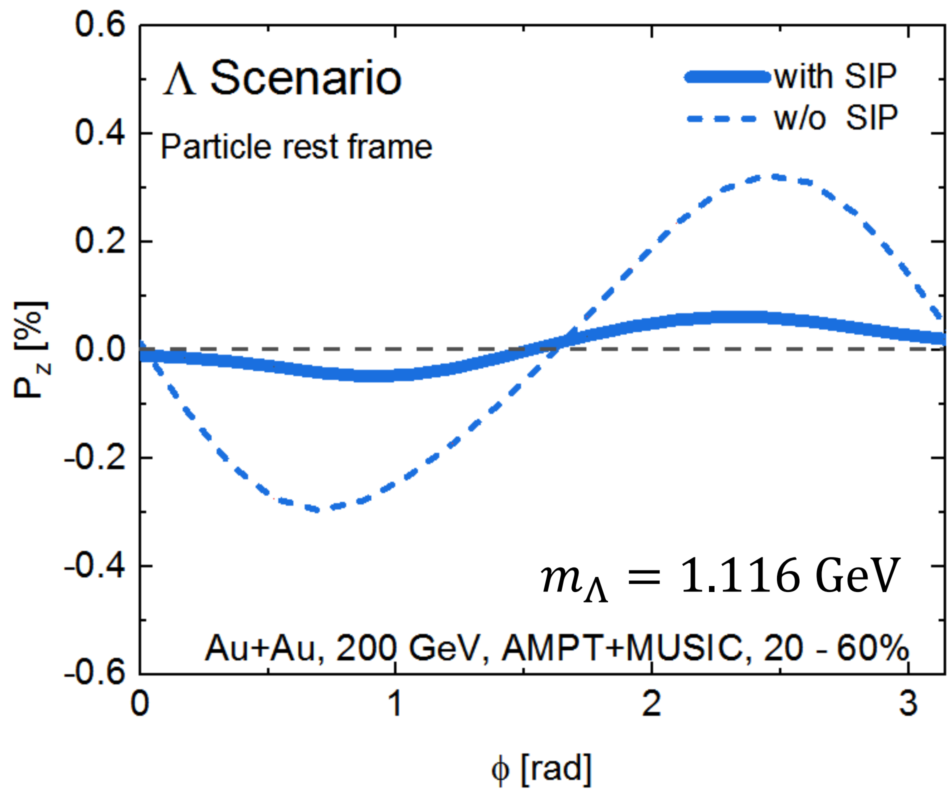
$P_z(\phi)$ with SIP

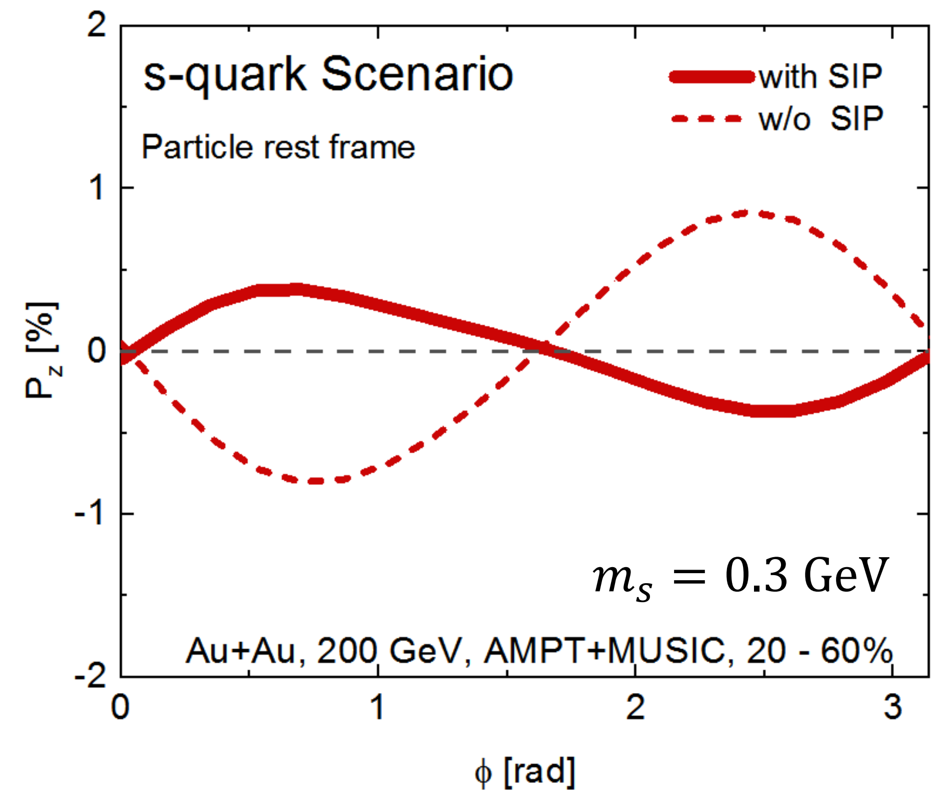
BF, S. Liu, L. -G. Pang, H. Song, Y. Yin, Phys.Rev.Lett. 127 14, 142301(2021)

Total P^μ = [vorticity] + [T grad] + [SIP]
= [thermal vorticity] + [SIP]



STAR, PRL 123 (2019) 132301





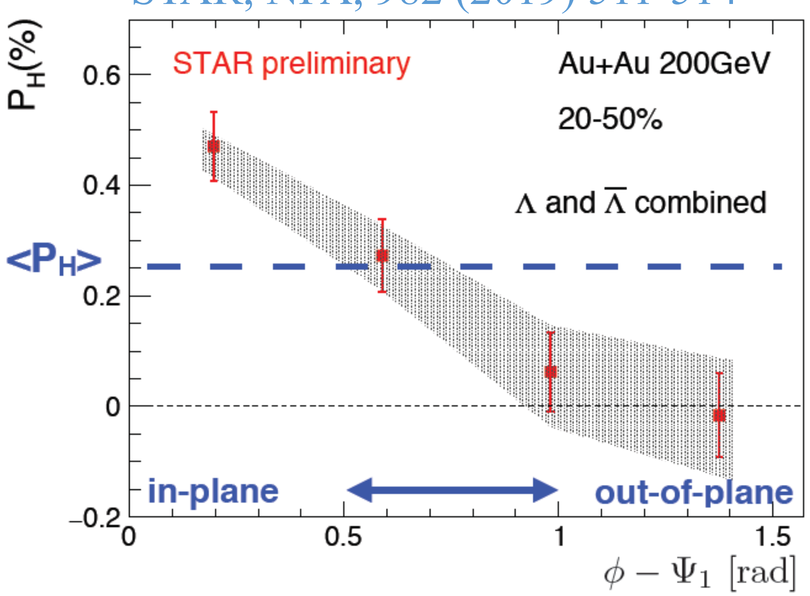
- In the scenario of ‘S-quark memory’, the total P^μ with SIP qualitatively agrees with data

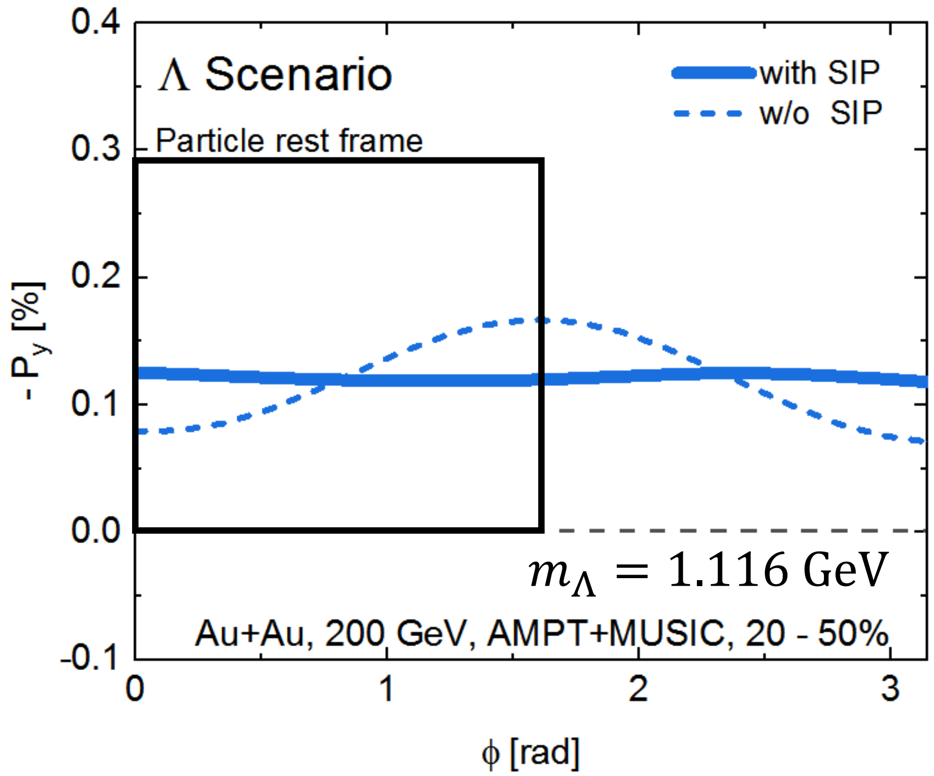
$P_y(\phi)$ with SIP

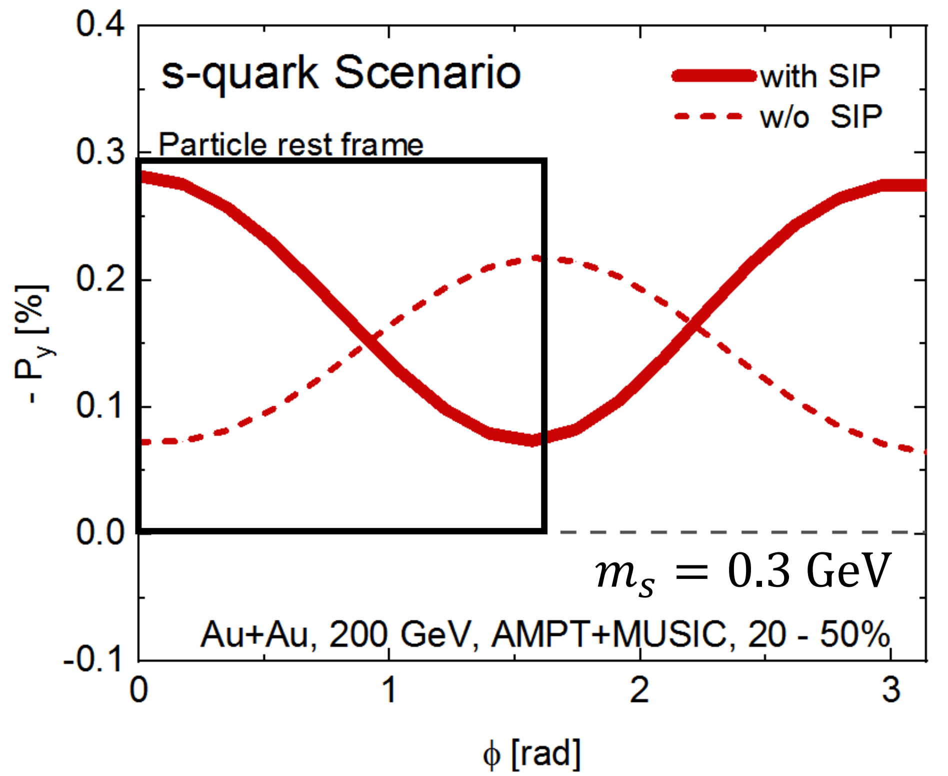
BF, S. Liu, L. -G. Pang, H. Song, Y. Yin, Phys.Rev.Lett. 127 14, 142301(2021)

Total P^μ = [vorticity] + [T grad] + [SIP]
= [thermal vorticity] + [SIP]

STAR, NPA, 982 (2019) 511-514



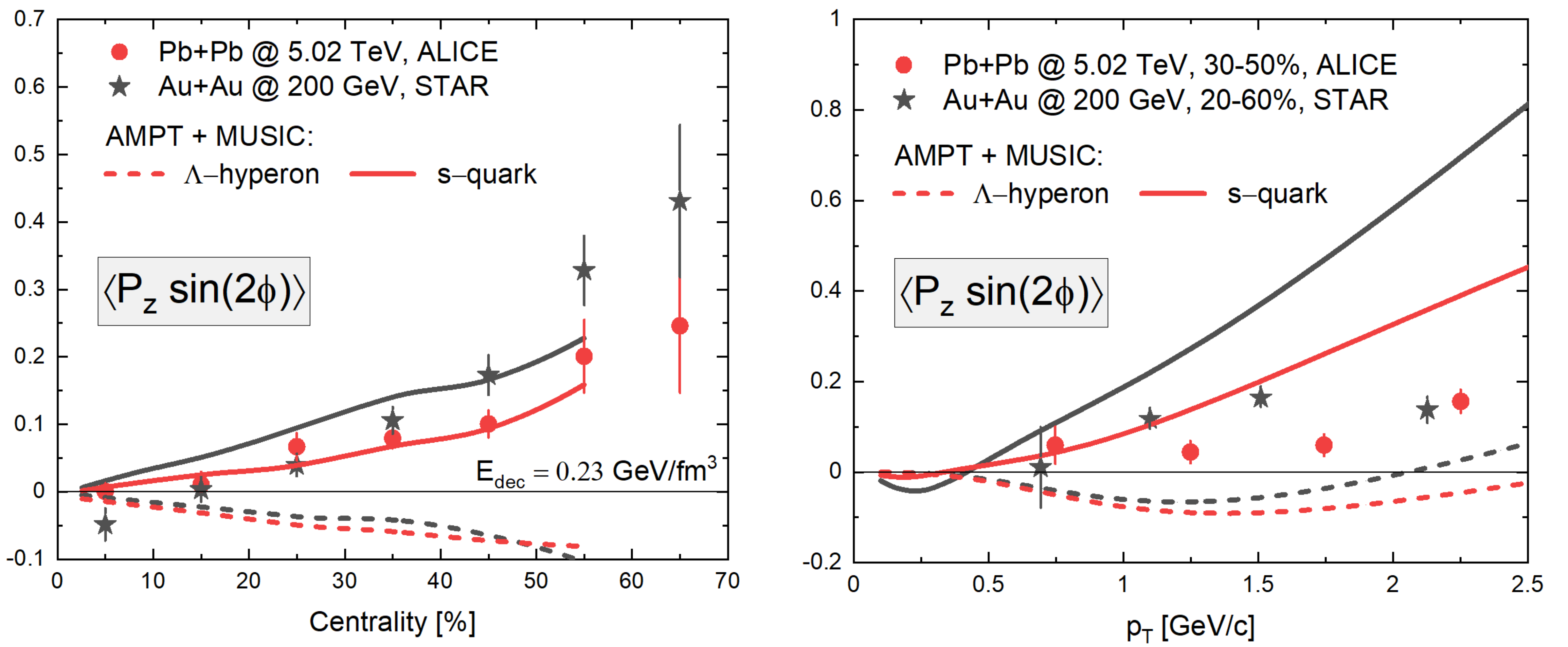




- In the scenario of ‘S-quark memory’, the total P^μ with SIP qualitatively agrees with data

From RHIC to LHC

Same hydrodynamic model: AMPT + MUSIC
(LHC parameter from EPJC 77 (2017) 9, 645)



- "Strange Memory" scenario qualitatively describes the centrality & p_T dependence
- More precise model needed to quantitative description

BF & H. Song, in preparation

Summary

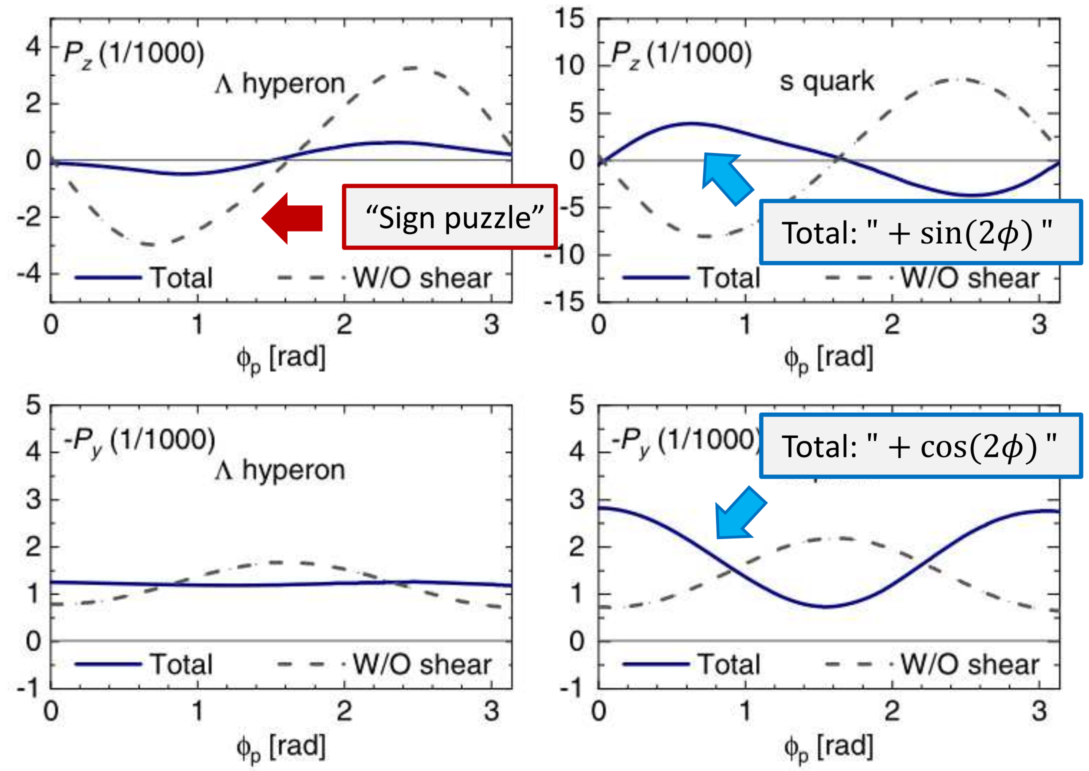
Shear-Induced Polarization: Phys.Rev.Lett. 127 14, 142301(2021)
Spin Hall Effects: arXiv: 2201.12970

Total P^{μ} = [vorticity] + [Grad T] + [SIP] + [SHE]

Shear-Induced Polarization

“**Strange memory**” + **Shear-Induced Polarization**

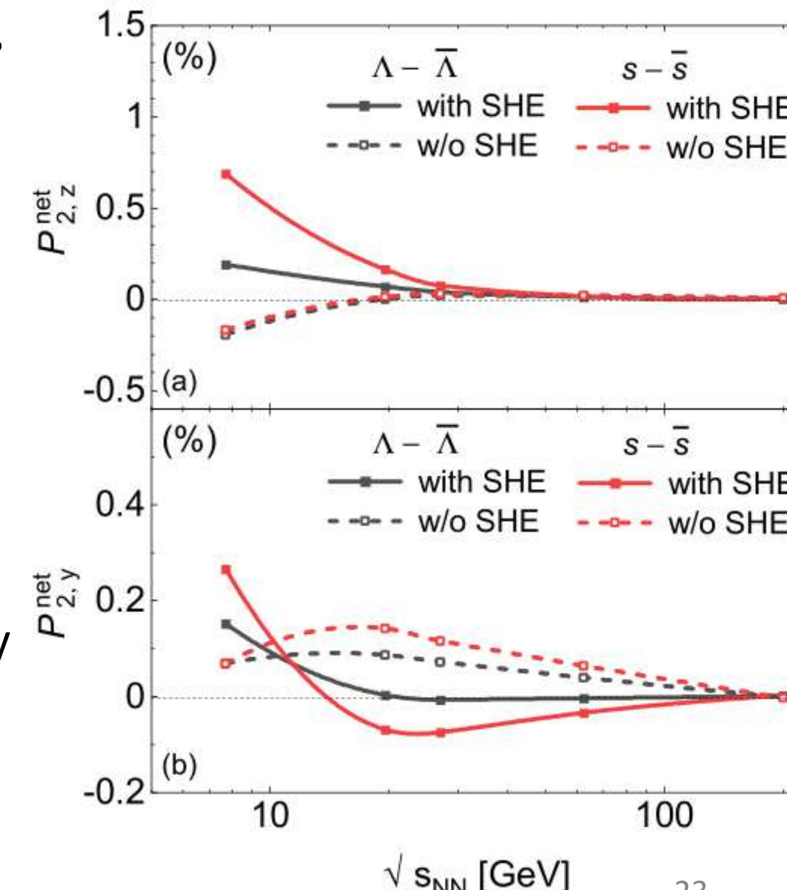
Describes $P_z(\phi)$ and $P_y(\phi)$ qualitatively at **top RHIC** and **LHC**



Spin Hall Effects

$$\vec{P}_{\pm} \propto \pm \vec{p} \times \vec{\nabla}\mu_B$$

- Particle – Anti-particle **separation**
- Relevant for RHIC-BES and RHIC/LHC forward rapidity
- Scenario independent



シアーからのスピン偏極2

Talk by Matteo Buzzegoli

Spin-thermal shear coupling in relativistic nuclear collisions

Quark Matter 2022 , April 5 2022

Matteo Buzzegoli
IOWA STATE UNIVERSITY

and F. Becattini, A. Palermo, G. Inghirami, I. Karpenko

Polarization from Wigner function

F. Becattini, Lect. Notes Phys. 987 (2021) 15-52.

The covariant Wigner function of the free Dirac field:

$$W(x,k)_{AB} = \frac{1}{(2\pi)^4} \int \mathrm{d}^4 y \, \mathrm{e}^{-\mathrm{i}k\cdot y} \langle : \bar{\Psi}_B(x+y/2)\Psi_A(x-y/2) : \rangle$$

where:

$$\langle \widehat{X} \rangle = \mathrm{tr}\left(\widehat{\rho}\, \widehat{X} \right)$$

It allows to calculate the mean spin vector:

$$S^\mu(p) = \frac{1}{2} \frac{\int \mathrm{d}\Sigma \cdot p \; \mathrm{tr}_4 \left(\gamma^\mu \gamma^5 W_+(x,p) \right)}{\int \mathrm{d}\Sigma \cdot p \; \mathrm{tr}_4 W_+(x,p)}$$

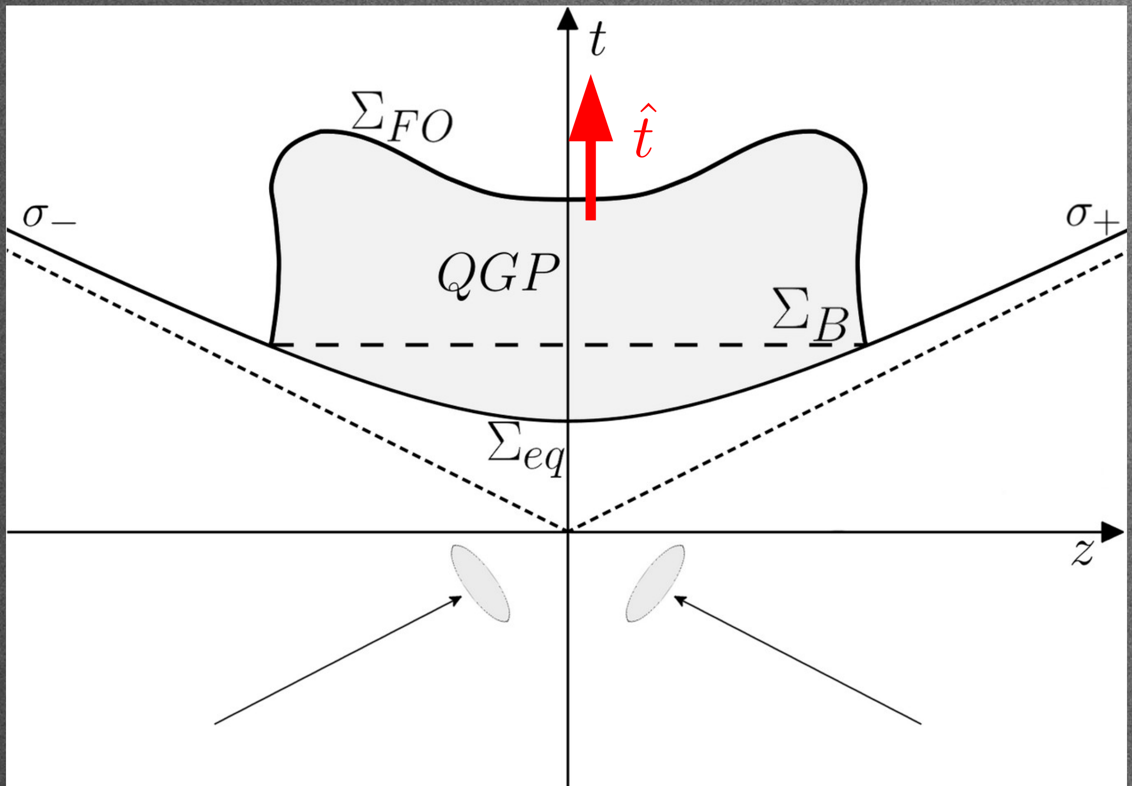
Spin polarization induced by thermal shear

Using **linear response theory** we eventually obtain:

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu \xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

F. Becattini, MB, A. Palermo, Phys. Lett. B 820 (2021) 136519

Same (not precisely the same) formula obtained by Liu and Yin with a different method:
S. Liu, Y. Yin, JHEP 07 (2021) 188



Dependence on a specific vector is not surprising as this term arises from the correlator

$$\langle \widehat{Q}^{\mu\nu}_x \widehat{W}(x,k)\rangle$$

But Q is not a tensor and, unlike J, it does depend on the hypersurface

Application to relativistic heavy ion collisions

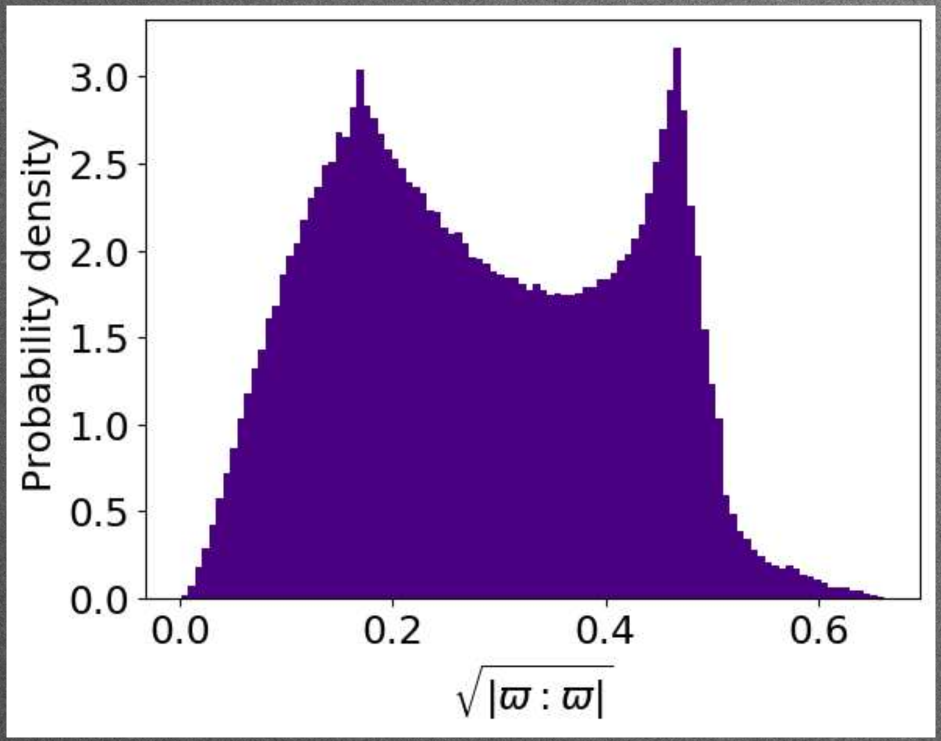
F. Becattini, MB, A. Palermo, G. Inghirami and I. Karpenko, Phys. Rev. Lett. 127 (2021) 27, 272302

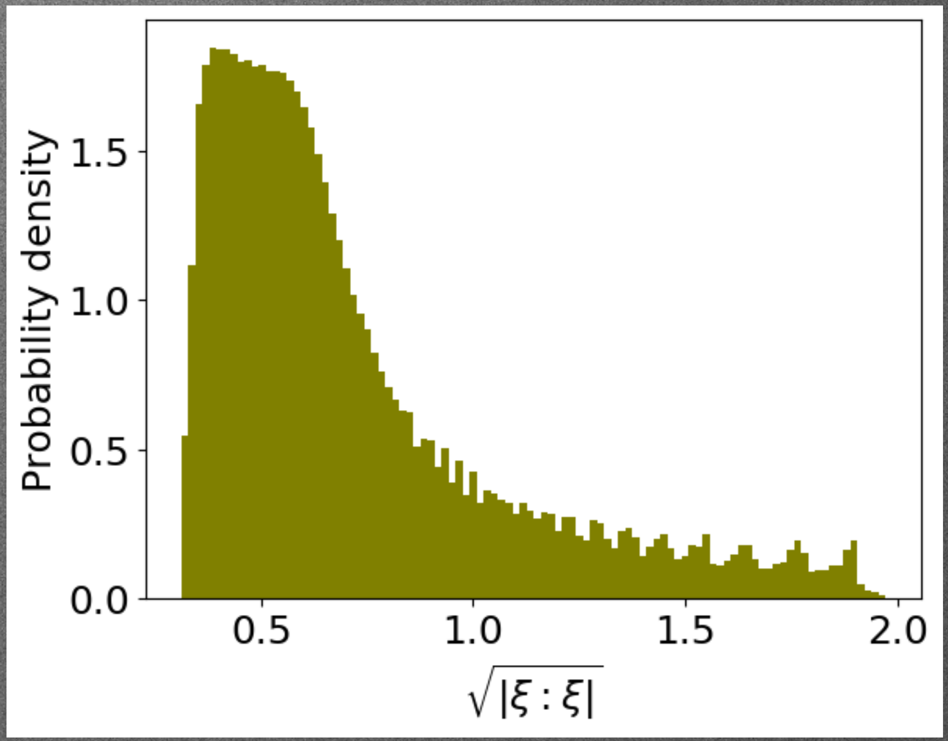
$$S^\mu = S^\mu_\varpi + S^\mu_\xi$$

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu\xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

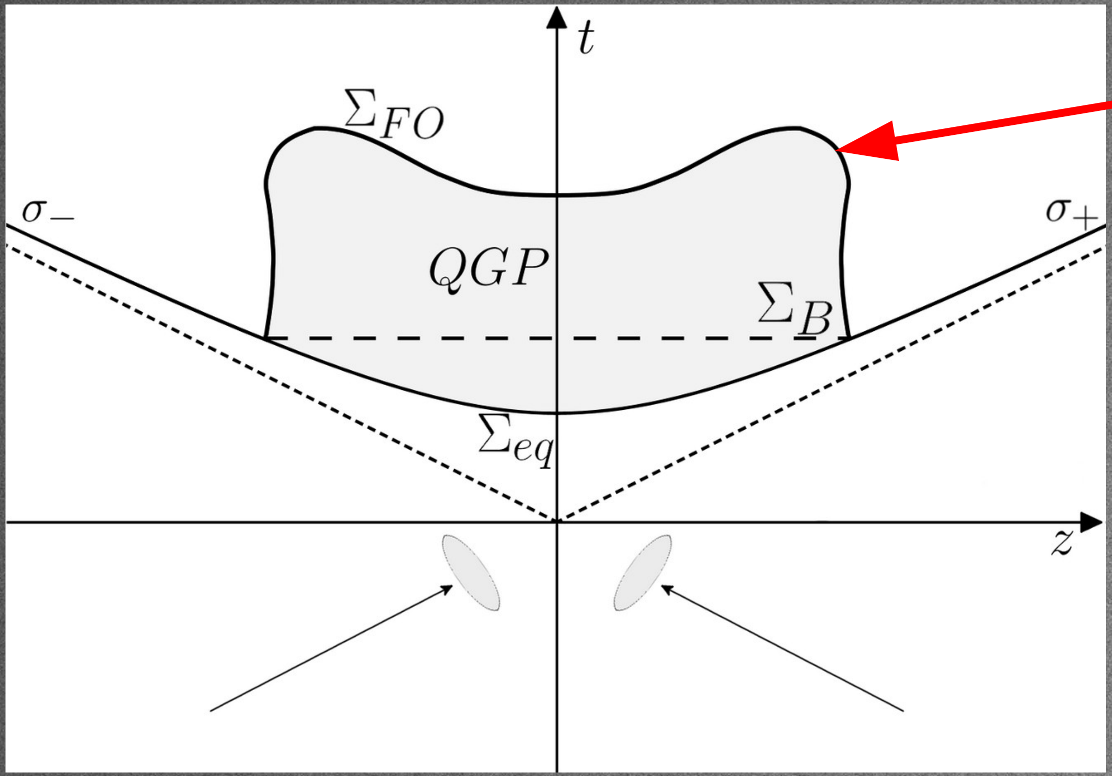
Modulus of thermal-vorticity and thermal-shear at the freeze-out hypersurface





Isothermal local equilibrium

The most appropriate setting for relativistic heavy ion collisions at very high energy!



At high energy, Σ_{FO} expected to be T= constant!

$$\beta^\mu = (1/T)u^\mu$$

$$\widehat{\rho}_{LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}\beta_\nu\right] = \frac{1}{Z}\exp\left[-\frac{1}{T}\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}u_\nu\right]$$

Only NOW u can be expanded!

$$u_\nu(y) \simeq u_\nu(x) + \partial_\lambda u_\nu(x)(y-x)^\lambda + \cdots$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu - \frac{1}{2T}(\partial_\mu u_\nu(x) - \partial_\nu u_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2T}(\partial_\mu u_\nu(x) + \partial_\nu u_\mu(x))\widehat{Q}^{\mu\nu}_x + \cdots\right]$$

Spin mean vector at leading order with isothermal local equilibrium (ILE)

Readily found by replacing the gradients of β with those of u

$$S^\mu_{\rm ILE}(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p \, n_F(1-n_F) \left[\omega_{\rho\sigma} + 2\, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma}\right]}{8 m T_{\rm FO} \int_\Sigma {\rm d}\Sigma \cdot p \, n_F}$$

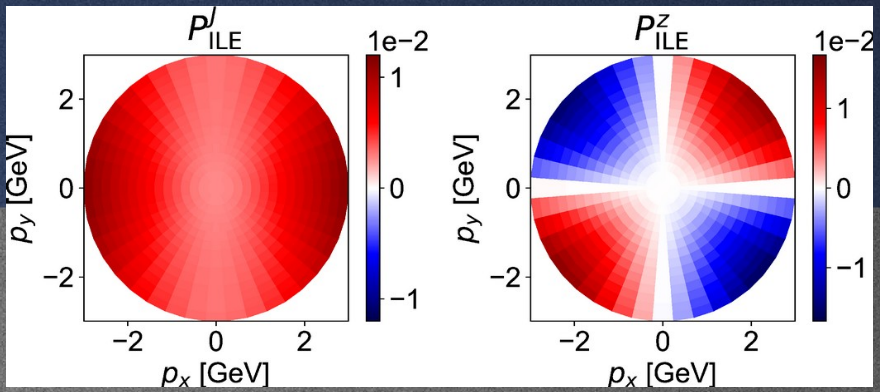
$$\omega_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho - \partial_\rho u_\sigma\right)$$ Kinematic vorticity

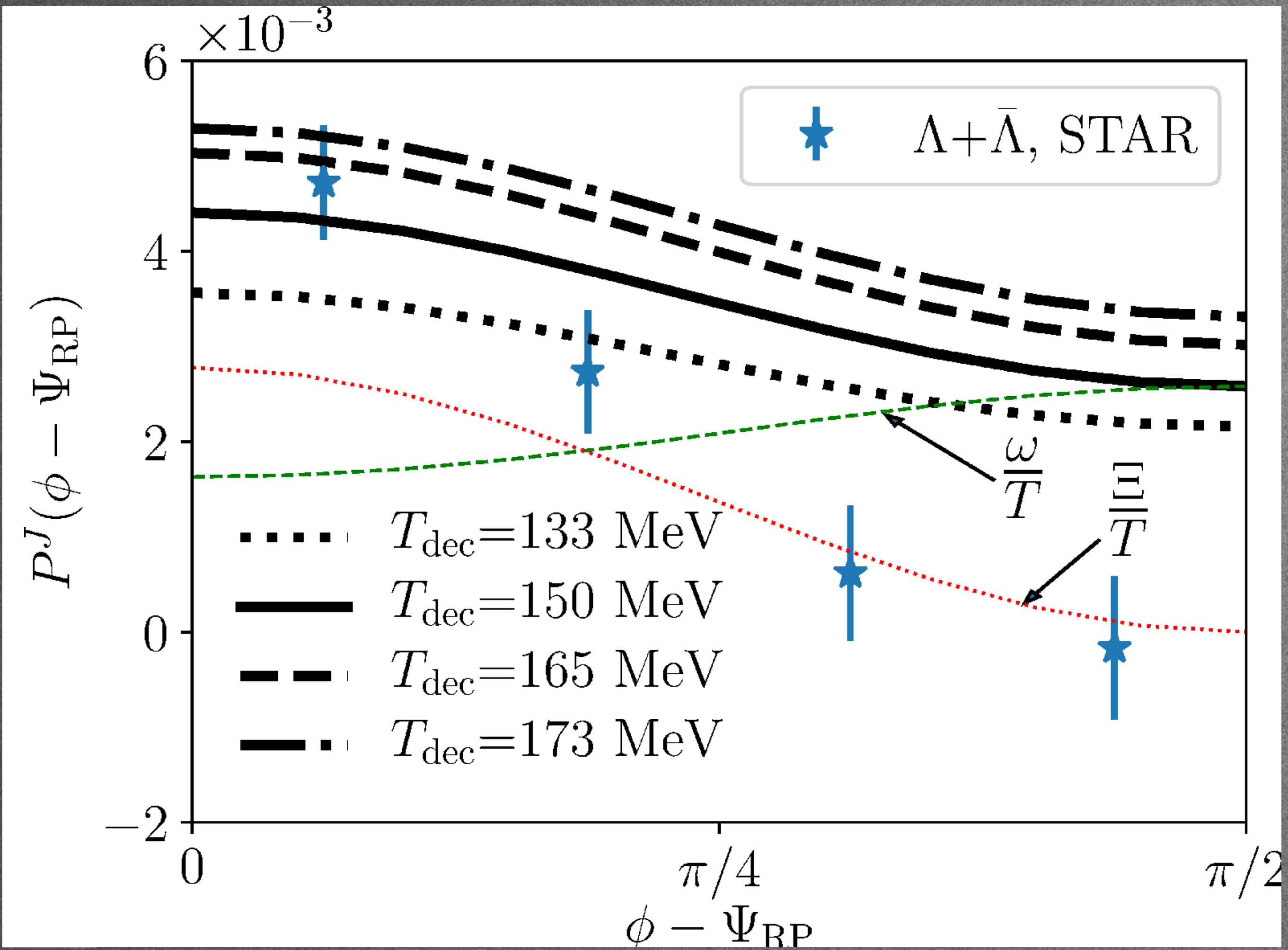
$$\Xi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho + \partial_\rho u_\sigma\right)$$ Kinematic shear

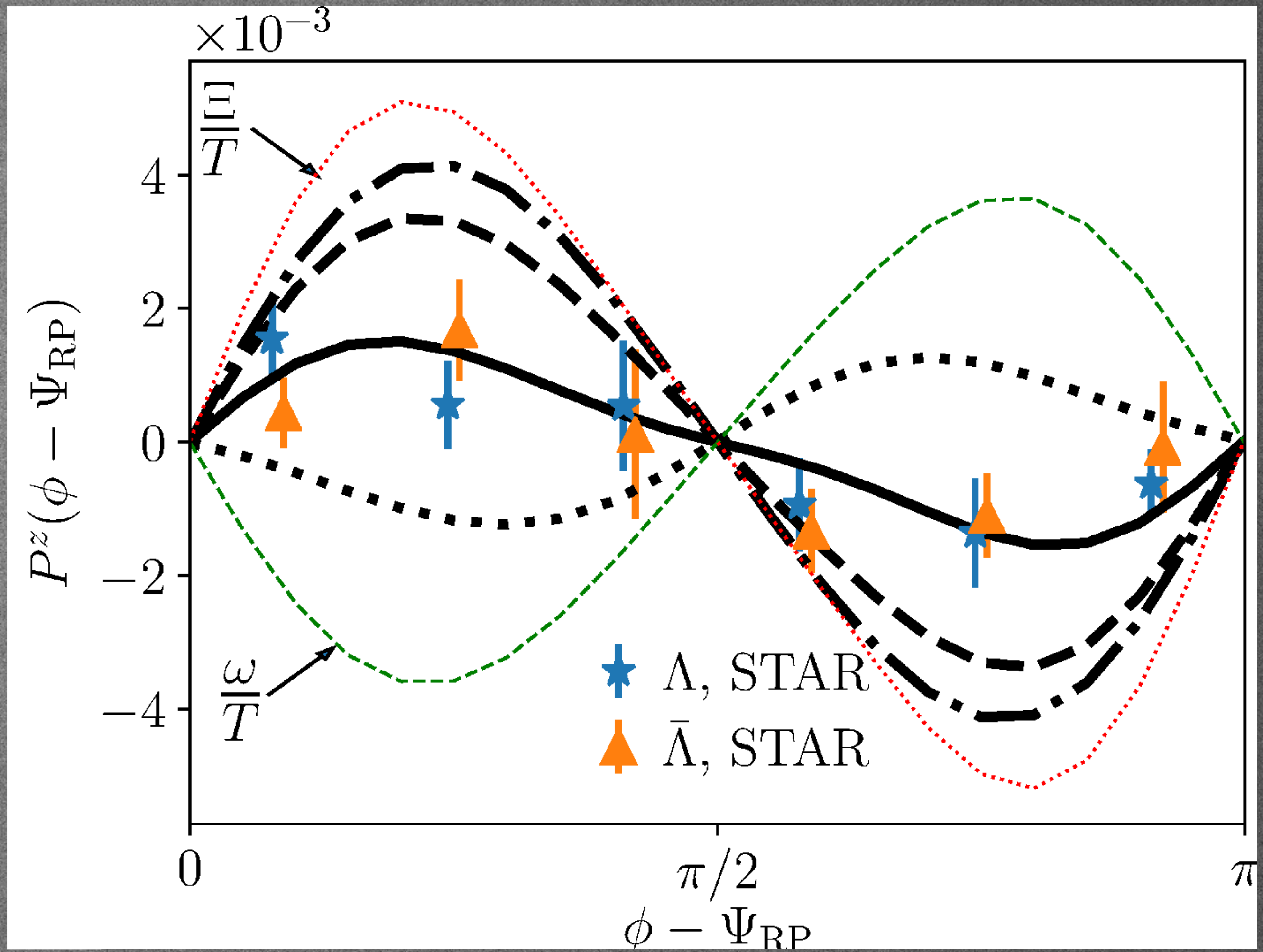
Isothermal local equilibrium: result

Apply the new formula (for primary hadrons):

$$S^{\mu}_{\rm ILE}(p) = -\epsilon^{\mu\rho\sigma\tau} p_{\tau} \frac{\int_{\Sigma} {\rm d}\Sigma \cdot p \, n_F (1-n_F) \left[\omega_{\rho\sigma} + 2\, \hat{t}_{\rho} \frac{p^{\lambda}}{\varepsilon} \Xi_{\lambda\sigma} \right]}{8 m T_{\rm dec} \int_{\Sigma} {\rm d}\Sigma \cdot p \, n_F}$$





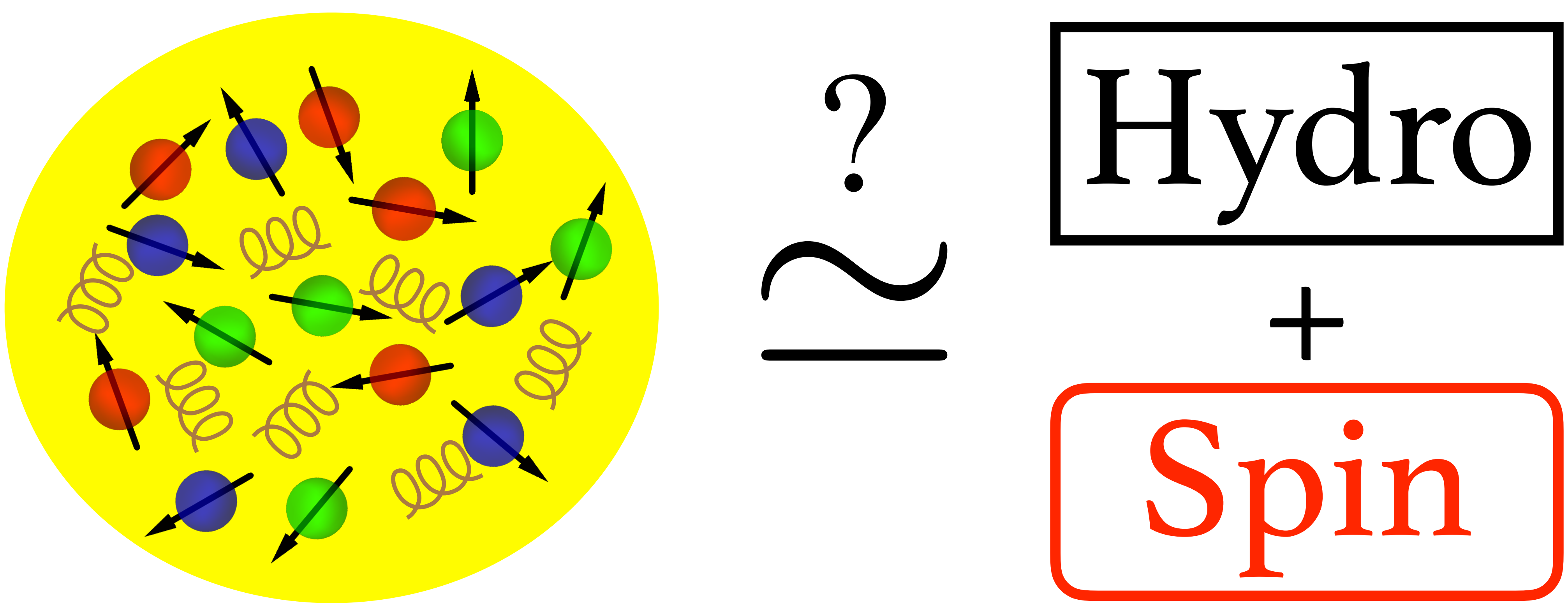


- Sensitive to the decoupling temperature $T_{\rm dec}$
- Quantitative agreement with the data for realistic $T_{\rm dec}$=150 MeV

スピンを含む流体

Talk by Masaru Hongo

Relativistic spin hydrodynamics **with torsion** and **linear response theory** for spin relaxation



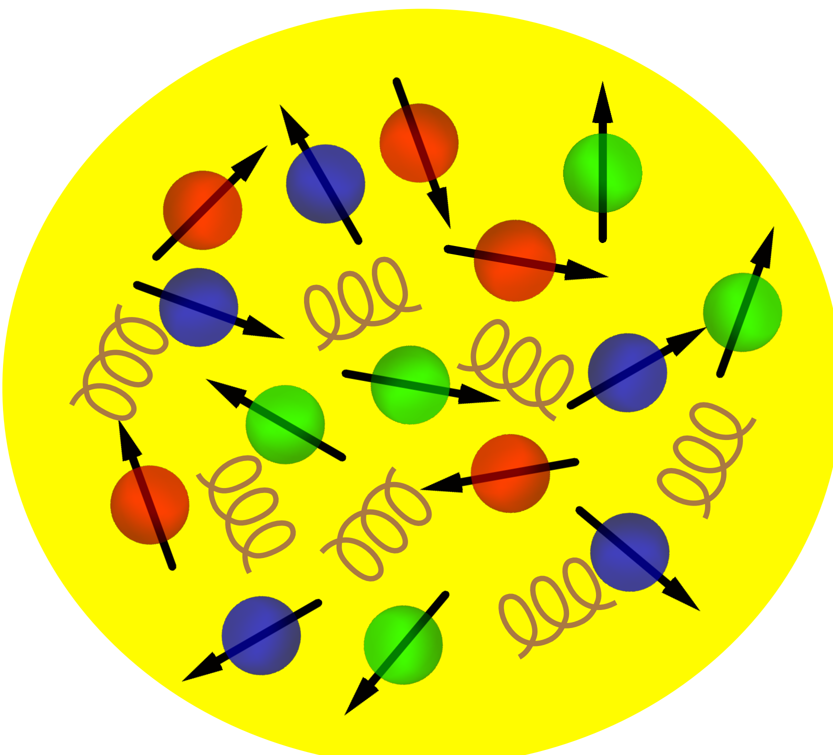
Masaru Hongo (Niigata University)

2022/04/04, Quark Matter 2022

MH-Huang-Kaminski-Stephanov-Yee, JHEP 2021, 150 (2021) [arXiv:2107.14231]

One-page Summary

Extending hydrodynamics to include spin



$\simeq$ Hydro + Spin

MH-Huang-Kaminski-Stephanov-Yee, JHEP 2021, 150 (2021) [arXiv:2107.14231]

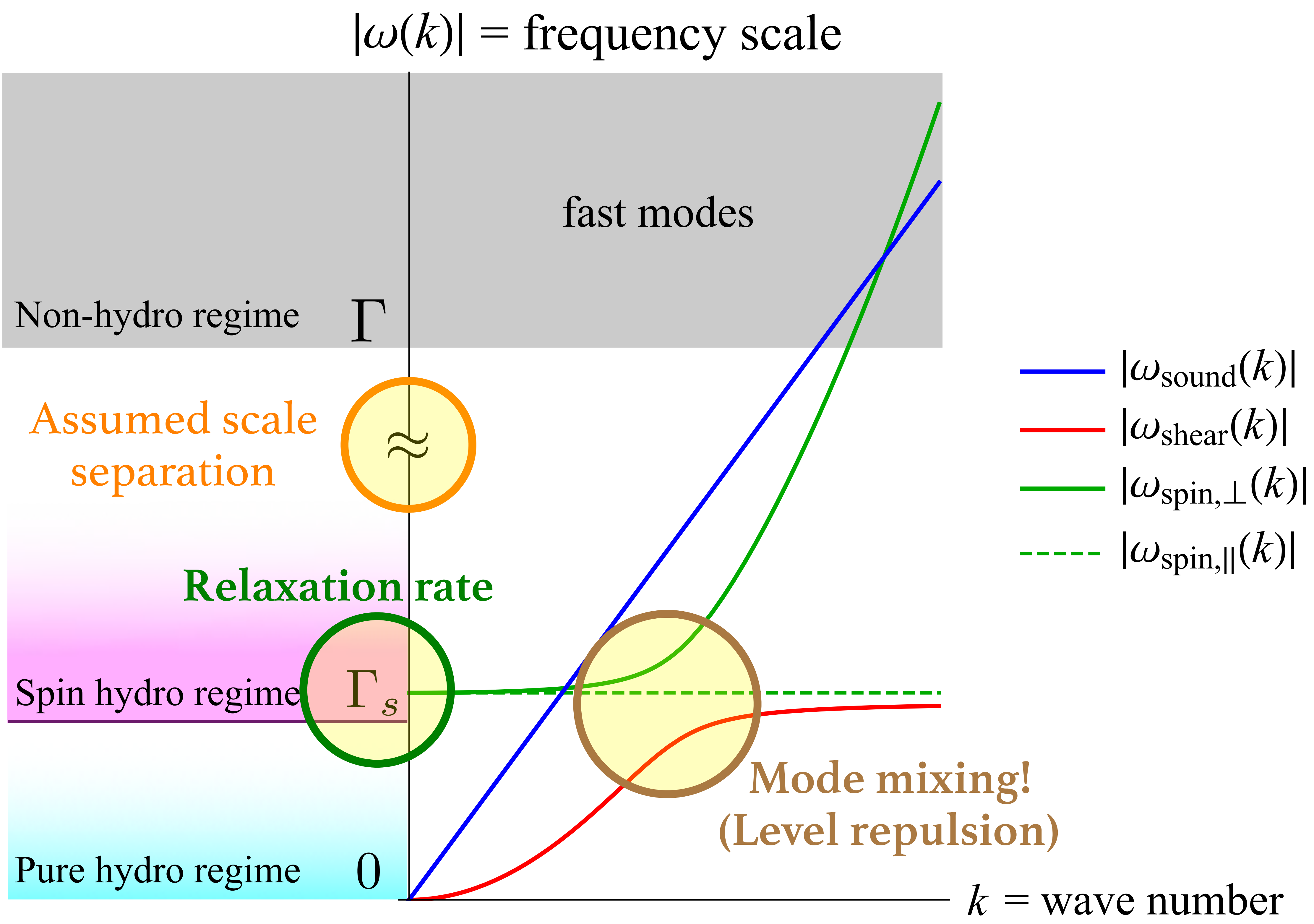
Three main messages from our paper:

(1) Spin hydrodynamic equations in a **torsionful** geometry

(2) **Mode mixing** between shear and spin modes

(3) **Green-Kubo formula** for a rotational viscosity

Sketch of our result



まとめ

1. 新しい自由度を導入する

臨界モード，パイオン，スピン

2. 新しい項を見つける

Thermal shear

(3. 上の2つを現象に適用する)

スピン偏極

個人的見解

堅実な結果は出ているが
“理論的におもしろい(?)”進展は
あまり見られなくなった印象
(よく言えば分野として成熟した?)

今後の大きな方向性を考える時期?
(中性子星も念頭に高密度QCD?
EICを考えてpQCD??)