

高エネルギー重イオン衝突における クォーコンニウム測定の実状

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- ▶ 浅川さんと私
- ▶ クォーコニウム測定 of 歴史
 - ▶ (私のD論を中心に…個人的なバイアスあり)
- ▶ 最近の結果
- ▶ まとめ

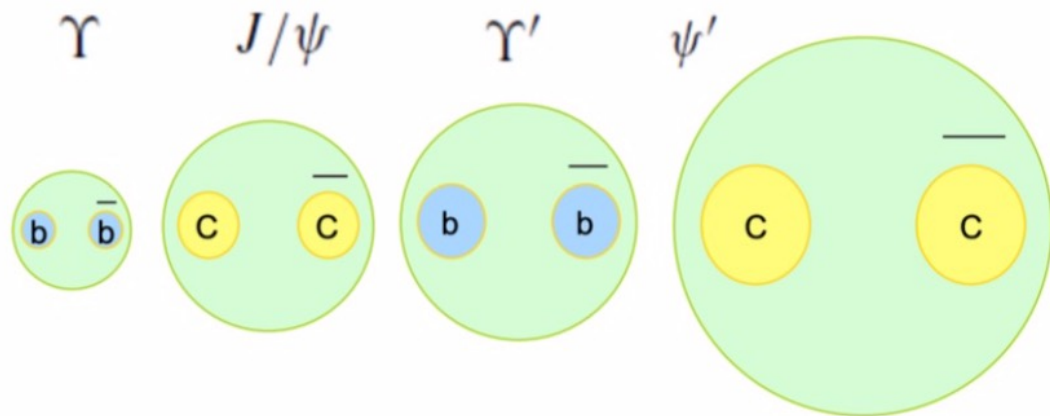
浅川さんと私

- ▶ 浅川さんとの出会い
 - ▶ 浅川さん：京都大学に赴任したばかり
 - ▶ 郡司：大学4回生（当時は宇宙線研究室に所属）
- ▶ 大学院進学に関する相談
 - ▶ 東大CNS（浜垣さん） or 京大・原子核ハドロン（今井さん）

クォーコニウム

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▶ チャームとボトムクォークのペアからなる束縛状態

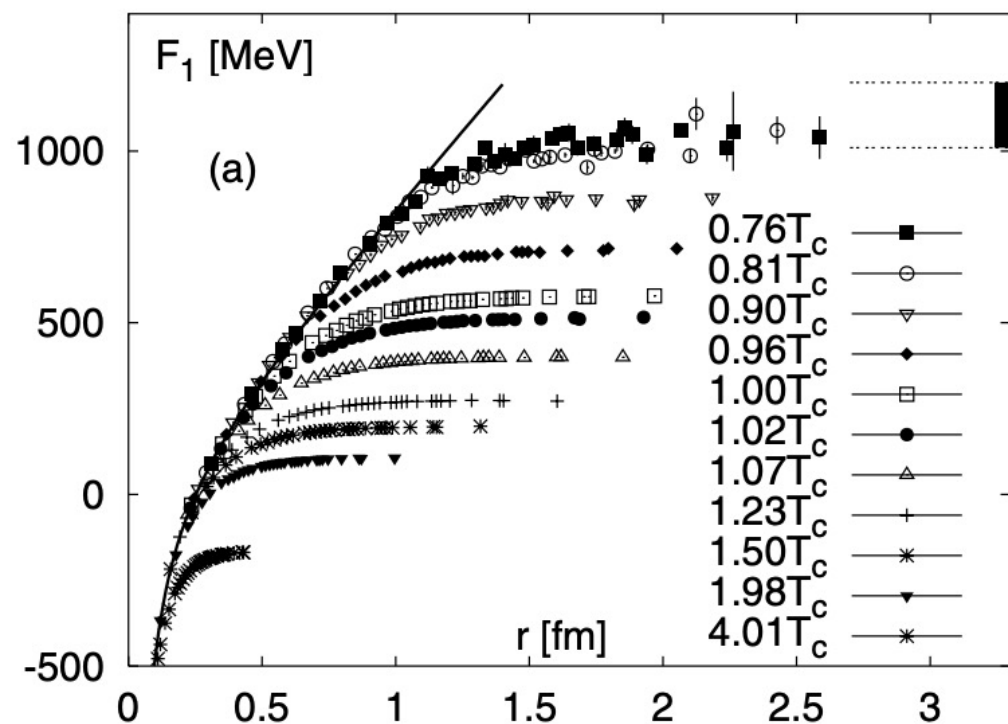


state	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ'_b	Υ''
mass [GeV]	3.10	3.53	3.68	9.46	9.99	10.02	10.26	10.36
ΔE [GeV]	0.64	0.20	0.05	1.10	0.67	0.54	0.31	0.20
ΔM [GeV]	0.02	-0.03	0.03	0.06	-0.06	-0.06	-0.08	-0.07
r_0 [fm]	0.50	0.72	0.90	0.28	0.44	0.56	0.68	0.78

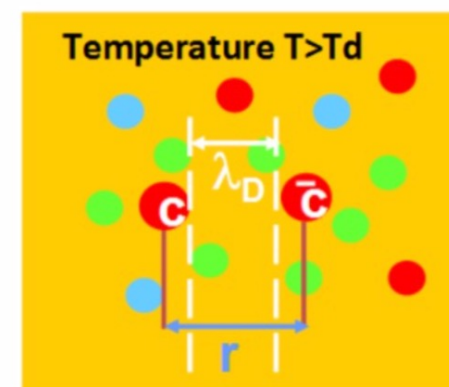
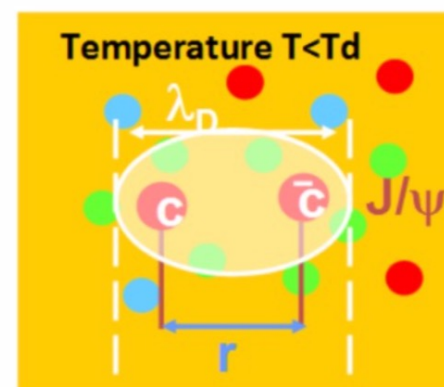
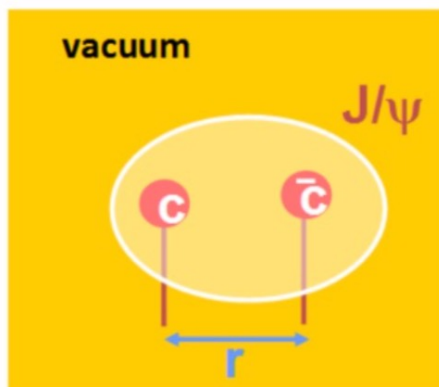
QGP中のクォーコンニウム

5

▶ QGP中でカラー遮蔽効果を受ける



Change of QQbar potential in the QGP
Color screening in QGP (T. Matsu and Satz, 1986)



溶解温度の推定

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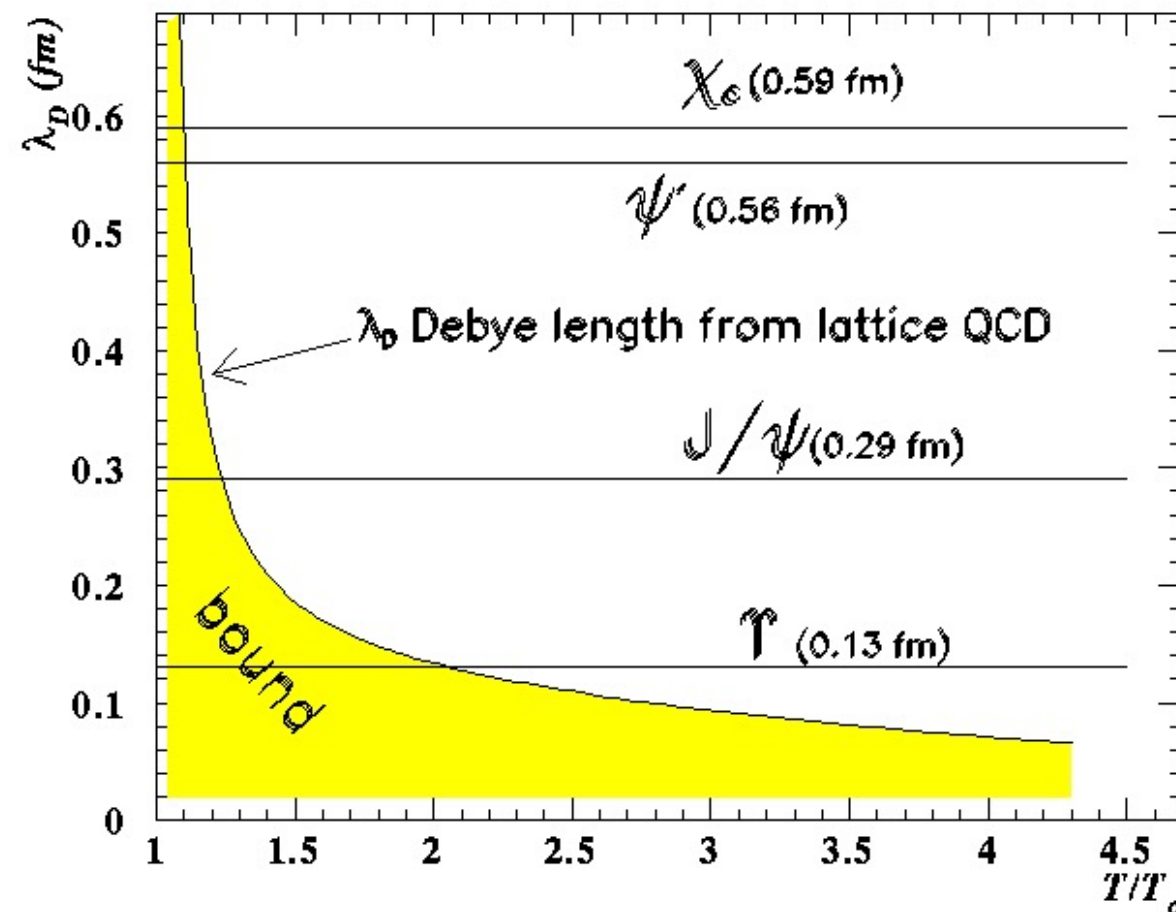
$$H = \frac{p^2}{2\mu} - \frac{\alpha_{eff} e^{-r/\lambda_d}}{r}$$

$$E(r) = \frac{1}{2\mu r^2} - \frac{\alpha_{eff} e^{-r/\lambda_d}}{r}$$

変分法($dE/dr = 0$) を解くと、次の条件を満たしたとき、束縛条件が形成されないことがわかる

$$\frac{dE(r)}{dr} = 0 \rightarrow \frac{r}{\lambda_D} \left(1 + \frac{r}{\lambda_D} \right) e^{-r/\lambda_D} = \frac{1}{\alpha_{eff} \mu \lambda_D}$$

$$f(x) = x(1+x)e^{-x} : f(x)|_{max} = 0.840 \text{ at } x = 1.62$$



$$\frac{1}{0.84\alpha_{eff}\mu} \geq \lambda_d \quad \lambda_d = \sqrt{\frac{2}{3}} \frac{1}{gT}, \quad g^2 = 4\pi\alpha_{eff}. \quad T_X = \frac{\mu}{0.840} \sqrt{\frac{2\alpha_{eff}}{9\pi}}$$

浅川-初田：J/ψは2T_cまで溶けない

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VOLUME 92, NUMBER 1

PHYSICAL REVIEW LETTERS

week ending
9 JANUARY 2004

J/ψ and η_c in the Deconfined Plasma from Lattice QCD

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(Received 28 August 2003; published 5 January 2004)

Analyzing correlation functions of charmonia at finite temperature (T) on $32^3 \times (32 - 96)$ anisotropic lattices by the maximum entropy method (MEM), we find that J/ψ and η_c survive as distinct resonances in the plasma even up to $T \approx 1.6T_c$ and that they eventually dissociate between $1.6T_c$ and $1.9T_c$ (T_c is the critical temperature of deconfinement). This suggests that the deconfined plasma is nonperturbative enough to hold heavy-quark bound states. The importance of having a sufficient number of temporal data points in MEM analyses is also emphasized.

DOI: 10.1103/PhysRevLett.92.012001

PACS numbers: 12.38.Gc, 12.38.Mh, 14.40.Gx, 25.75.Nq

Whether hadrons survive even in the deconfined quark-gluon plasma is one of the key questions in quantum chromodynamics (QCD). This problem was first examined in [1] and [2] in different contexts. In the former, it was shown that collective $q\bar{q}$ excitations with a low mass and a narrow width in the $\pi\text{-}\sigma$ channels exist even above T_c (the critical temperature) from analyses of the spectral functions in the Nambu–Jona-Lasinio model. The fate of heavy mesons such as J/ψ in the deconfined plasma was also investigated in a phenomenological potential picture taking into account the Debye screening [3]. In general, there is no *a priori* reason to believe that the dissociation of bound states should take place exactly at the phase transition point [4].

Experimentally, measurements of dileptons (diphotons) in heavy ion collisions may provide a clue to the properties of vector (pseudoscalar) mesons in hot/dense matter. Indeed, data from CERN Super Proton Synchrotron indicate anomalies in the dilepton spectra relevant to ρ and J/ψ . Also, BNL Relativistic Heavy Ion Collider is going to produce ample data of dileptons in a few years [5].

From the theoretical point of view, the spectral function (SPF) at finite temperature T , which has all the information of in-medium hadron properties, is a key quantity to be studied. Recently, the present authors have shown [6] that the first-principle lattice QCD simu-

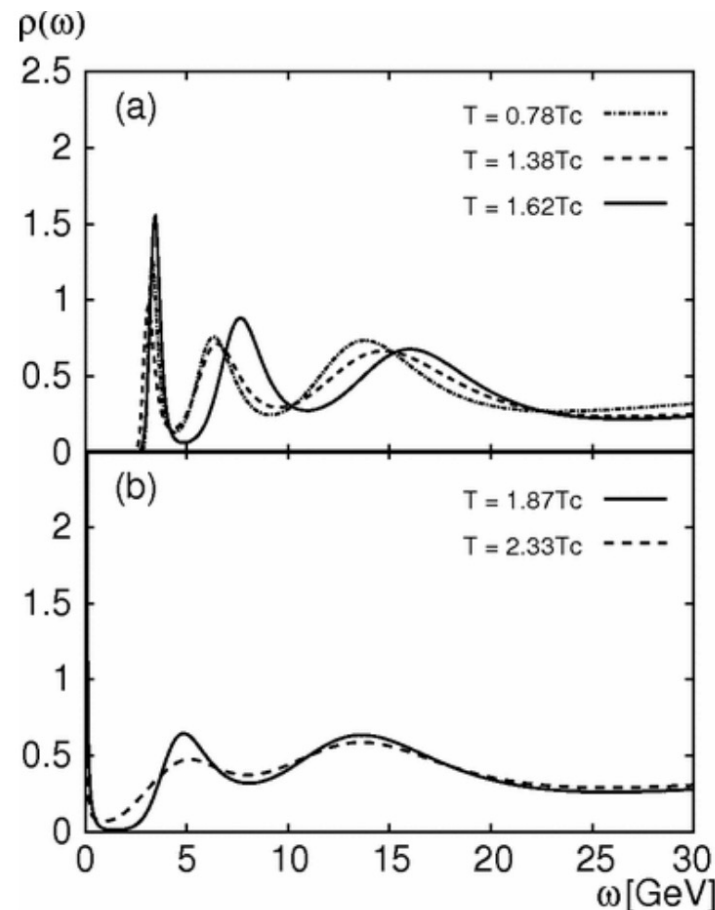
To draw the above conclusion with a firm ground, we put special emphasis on (i) the MEM error analysis of the resultant SPFs and (ii) the sensitivity of the SPFs to N_{data} (the number of the temporal data points adopted in MEM). These tests are crucial to prevent fake generation and/or smearing of the peaks and must be always carried out as emphasized in [7,8].

Let us first summarize the basic formulation of MEM applied to lattice data at finite T [7]. We consider the Euclidean correlation function of the local interpolating operator $J_l(\tau, \mathbf{x}) = \bar{c}i\gamma_l c$ with $l = 1, 2, 3$ for J/ψ and $l = 5$ for η_c . The spectral decomposition of the correlator in the imaginary time $0 < \tau < 1/T$ reads

$$D(\tau) = \int \langle J_l(\tau, \mathbf{x}) J_l^\dagger(0) \rangle d^3x = \int_0^\infty K(\tau, \omega) A(\omega) d\omega, \quad (1)$$

where ω is a real frequency and $A(\omega)$ is the spectral function. The sum over $l = 1, 2, 3$ is taken for J/ψ . $K(\tau, \omega)$ is the integral kernel, $K(\tau, \omega) = (e^{-\tau\omega} + e^{-(1/T-\tau)\omega})/(1 - e^{-\omega/T})$. For simplicity, we take the three momentum of the correlation function to be zero.

Monte Carlo simulations provide $D(\tau_i)$ with statistical error at a discrete set of temporal points τ_i . Although there exist infinitely many $A(\omega)$ which give the same $D(\tau_i)$ through the integral Eq. (1), MEM provides a way to select a unique $A(\omega)$ without introducing any



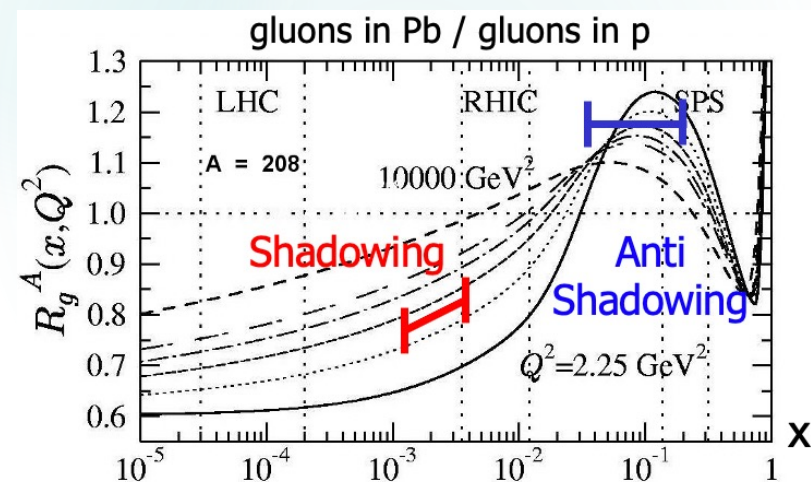
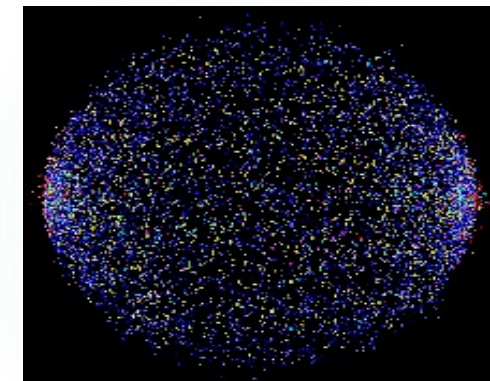
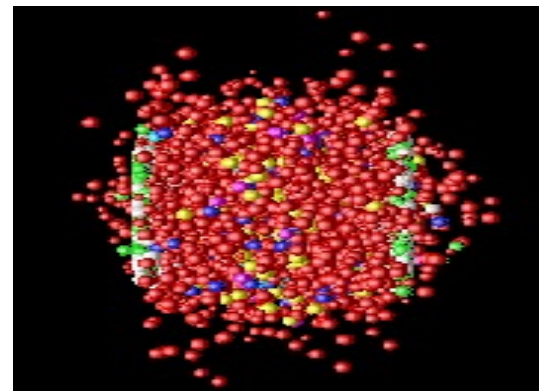
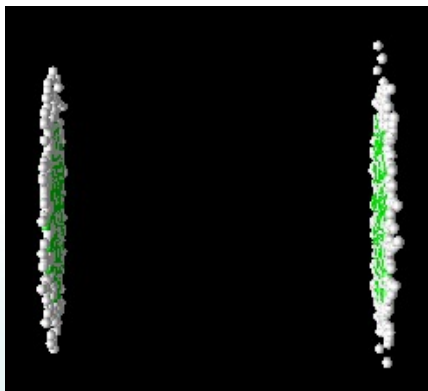
S. Digal, F. Karsch and
H. Satz

state	J/ψ(1S)	χ _c (1P)	ψ'(2S)	Υ(1S)	χ _b (1P)	Υ(2S)	χ _b (2P)	Υ(3S)
T _d /T _c	2.10	1.16	1.12	> 4.0	1.76	1.60	1.19	1.17

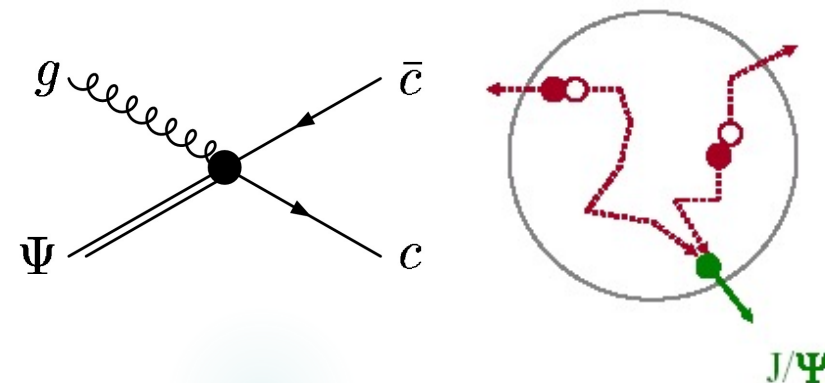
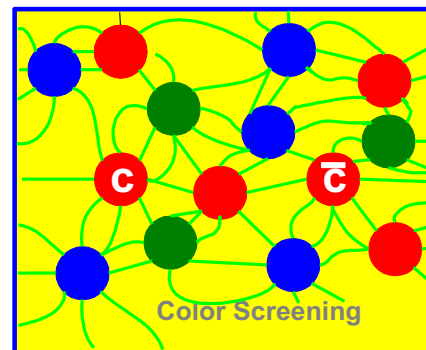
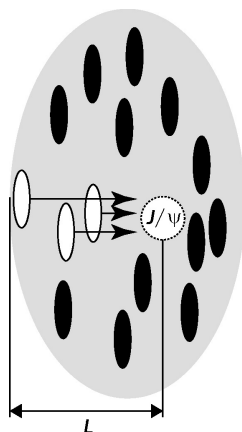
重イオン衝突での測定

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▶ 様々な過程からの寄与：原子核効果+QGP効果



$$e^{-L\rho\sigma_{abs}}$$



SPSでの測定：NA50

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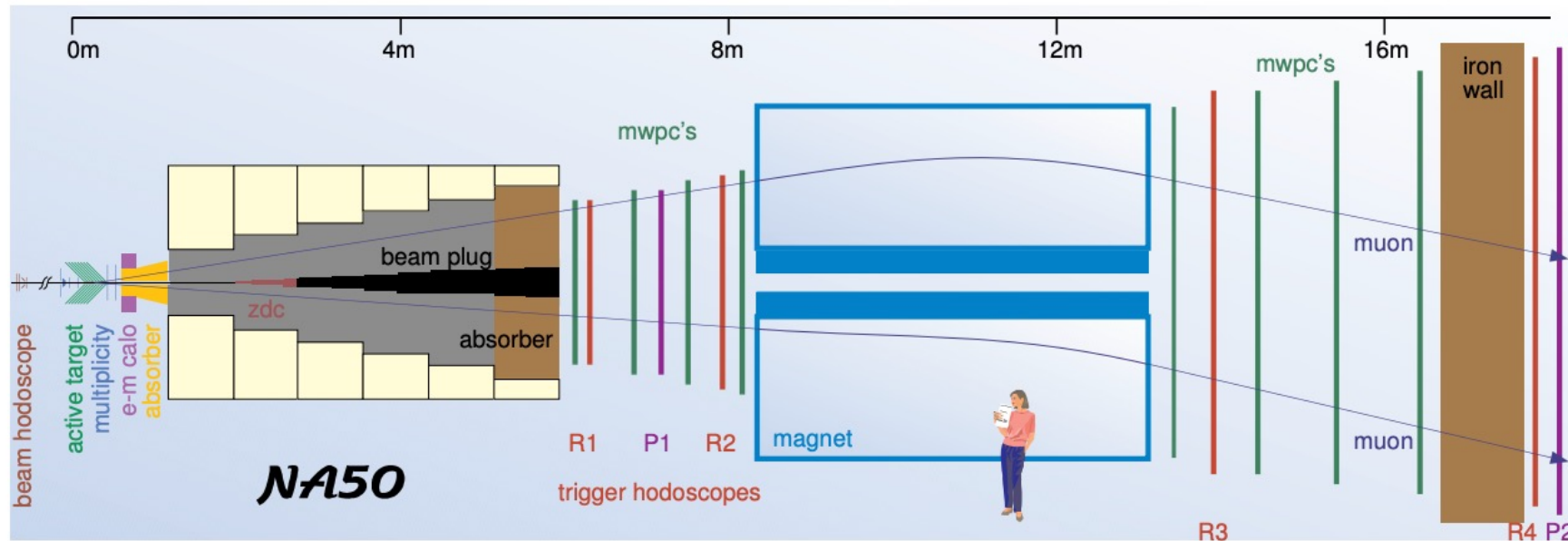
NA50 experimental apparatus

Same spectrometer as for NA38 and NA51

EM calorimeter (E_T)

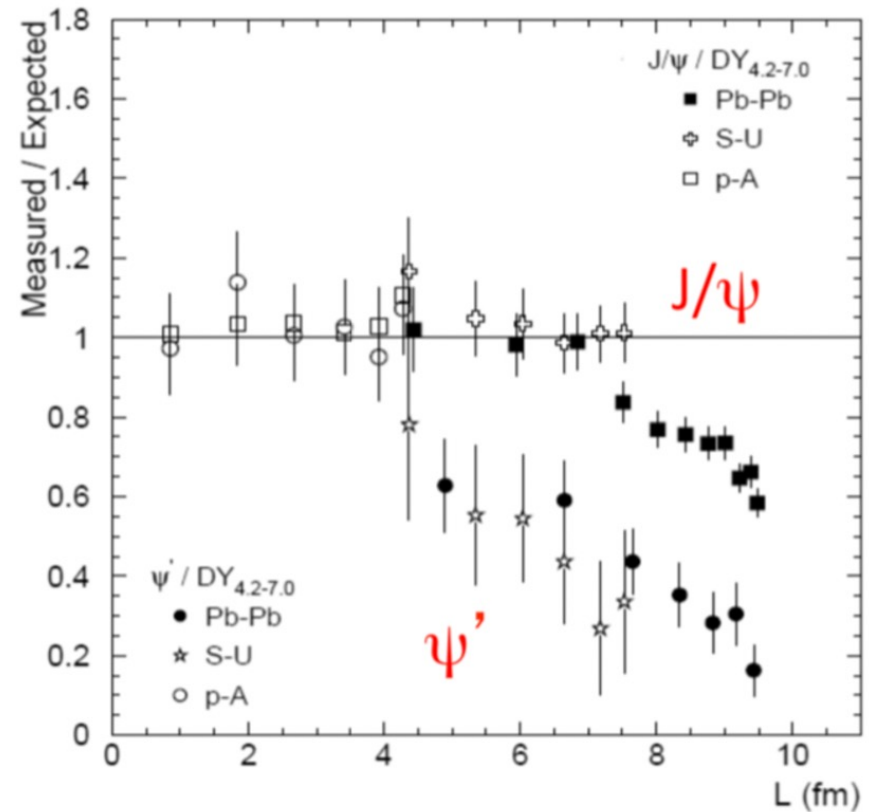
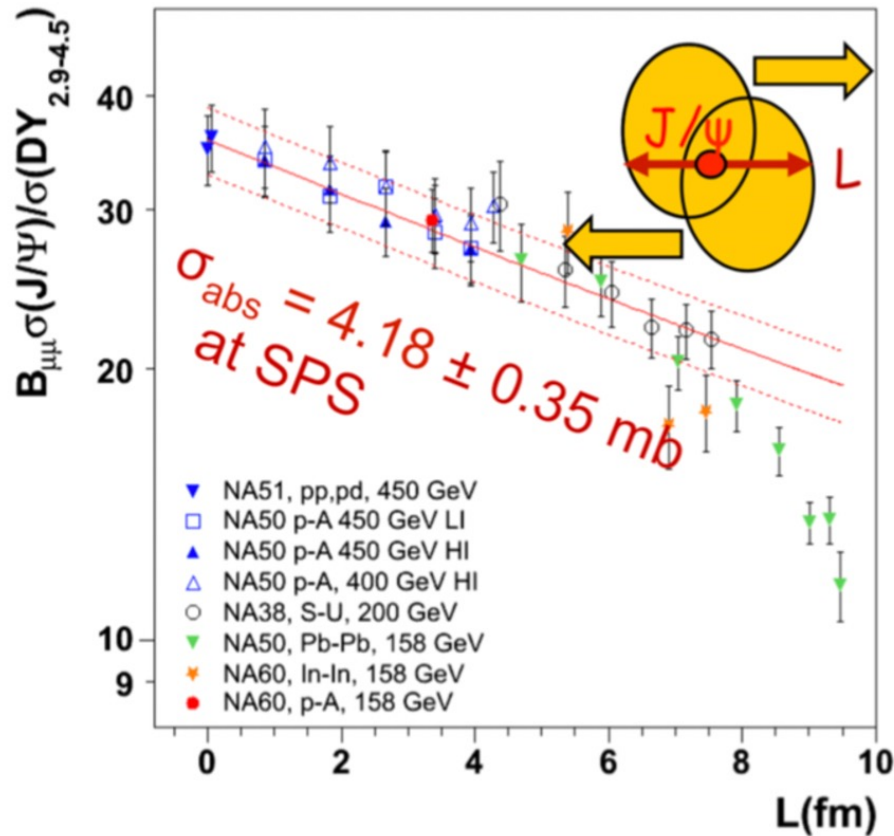
Zero Degree Calorimeter (E_{ZDC})

Multiplicity Detector (N_{ch})



J/ψ と ψ' の収量抑制

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Stronger suppression than the normal nuclear matter effects in central collisions

QGPの証拠!

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NA50 Publications

Evidence for deconfinement of quarks and gluons

from the J/psi suppression pattern
measured in Pb-Pb collisions at the CERN-SPS

Physics Letters B, in print; CERN-EP-2000-013 [PS file](#)

Dimuon and charm production in nucleus-nucleus collisions at the CERN-SPS

Euro. Phys. J. C, in print; CERN-EP-2000-012 [PS file](#)

Low mass dimuon production in proton and ion induced interactions at the SPS

European Physics Journal C13 (2000) 69; CERN-EP/99-112-Rev [PS file](#)

Observation of a threshold effect in the anomalous J/psi suppression

Physics Letters B450 (1999) 456; CERN-EP/99-13 [PS file](#)

Observation of Fission in Pb-Pb Interactions at 158 A GeV

Physical Review C59 (1999) 876 [Text \(PS file\)](#) and [Figures \(PS file\)](#)

J/psi, psi' and Drell-Yan production in pp and pd interactions at 450 GeV/c

(NA51 Collaboration)

Physics Letters B438 (1998) 35 [PS file](#)

The quartz-fiber Zero-Degree Calorimeter for the NA50 experiment at CERN SPS

Nuclear Instruments and Methods in Physics Research A411 (1998) 1

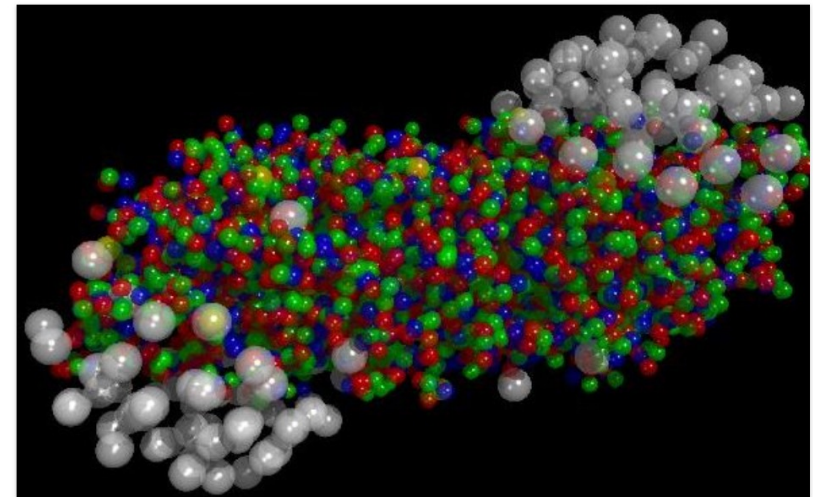
Anomalous J/Psi suppression in Pb-Pb interactions at 158 GeV/c per nucleon

Physics Letters B410 (1997) 337 [PS file](#)

Voir en [français](#)

New State of Matter created at CERN

10 FEBRUARY, 2000



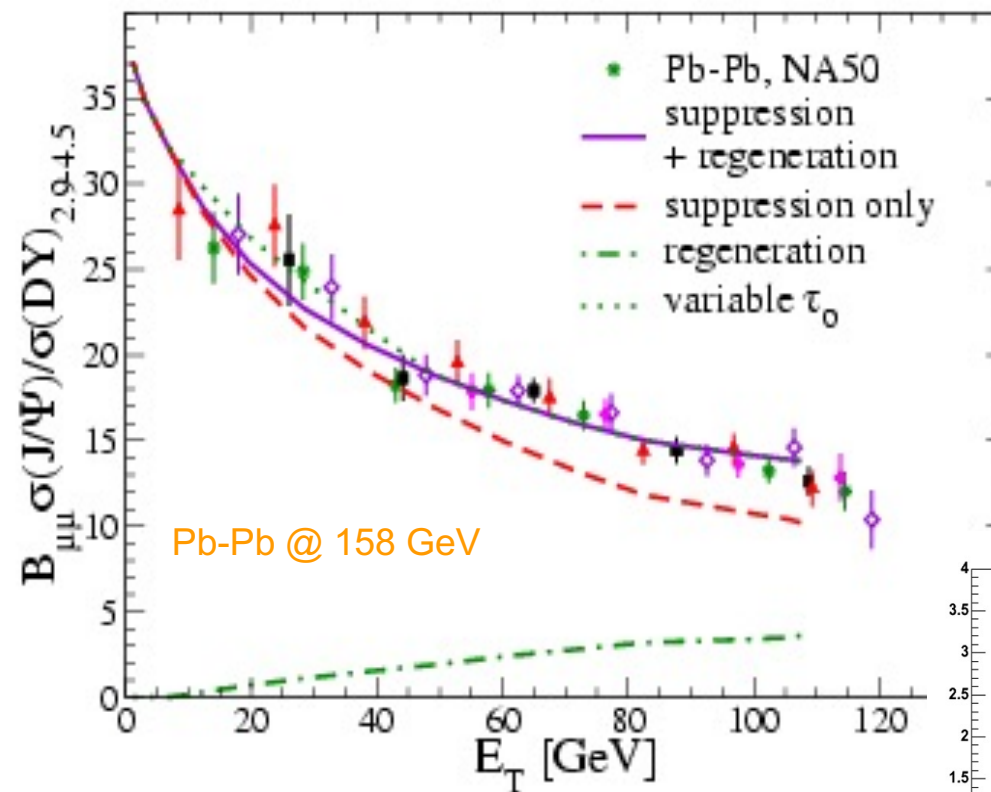
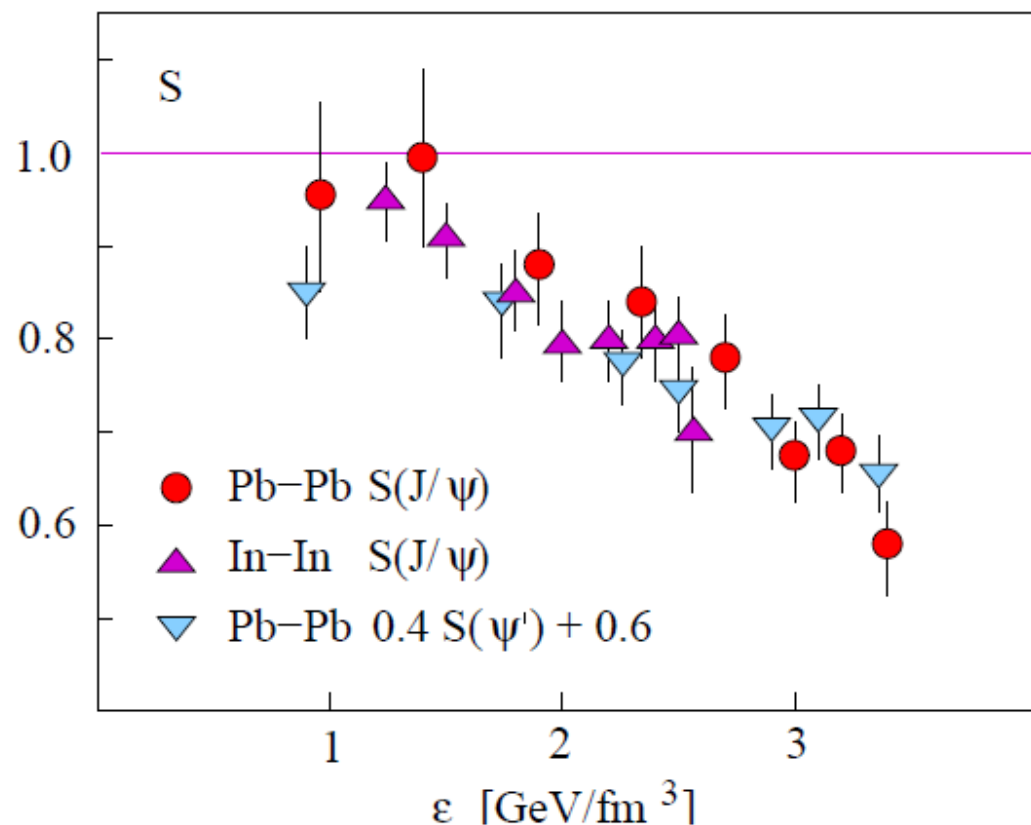
Geneva, 10 February 2000. At a special seminar on 10 February, spokespersons from the experiments on CERN's Heavy Ion programme presented compelling evidence for the existence of a new state of matter in which quarks, instead of being bound up into more complex particles such as protons and neutrons, are liberated to roam freely.

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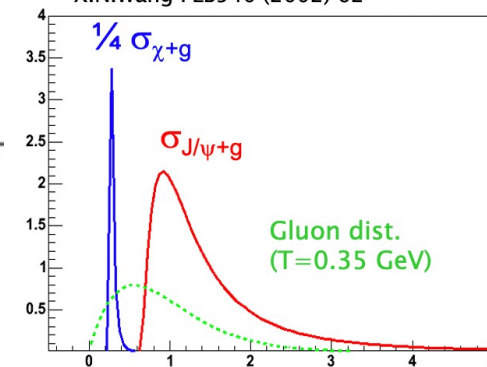
Theory predicts that this state must have existed at about 10 microseconds after the Big Bang, before the formation of matter as we know it today, but until now it had not been confirmed experimentally. Our understanding of how the universe was created, which was previously unverified theory for any point in time before the formation of ordinary atomic nuclei, about three minutes after the Big Bang, has with these results

SPSでのJ/ψ抑制の解釈

12



X.N.Wang PLB540 (2002) 62

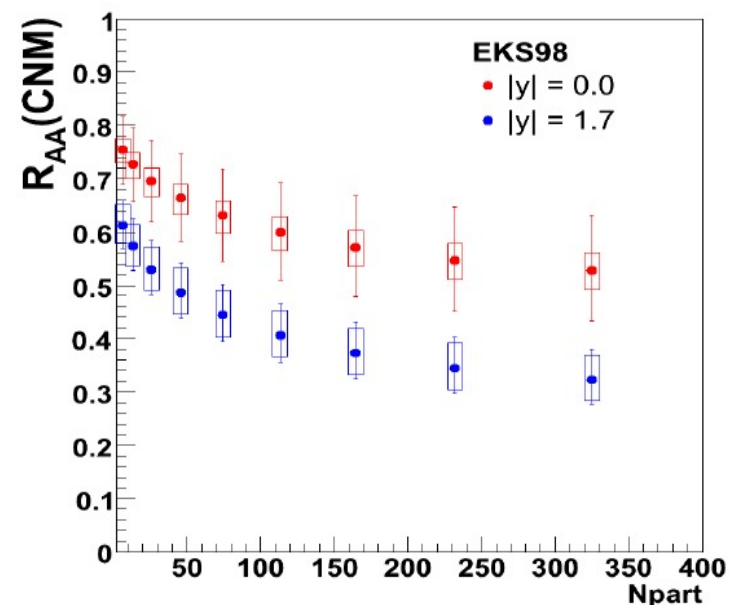
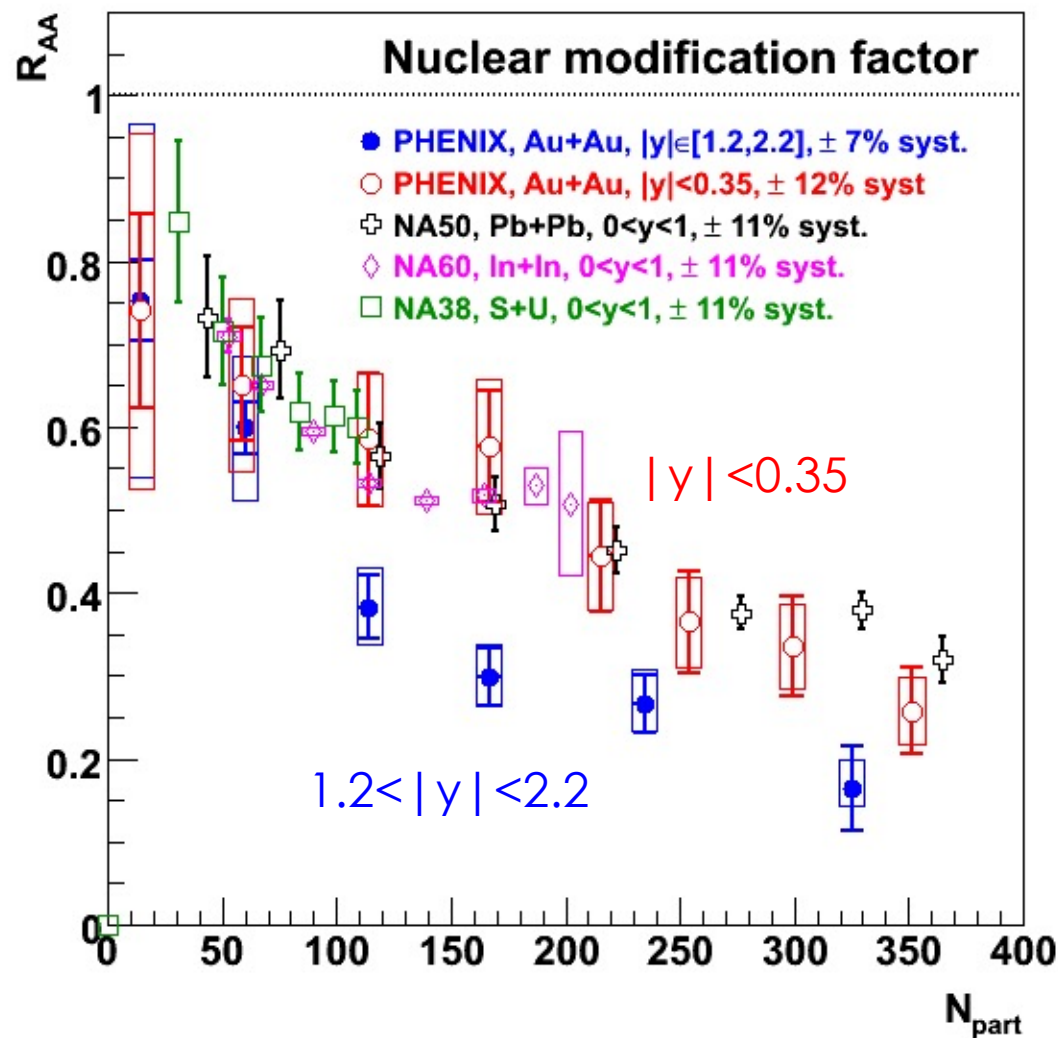


40% of J/ψ is from ψ' and χ_c , those are melt around T_c

J/ψ suppression at SPS can be the suppression of feed-down contributions from excited states.

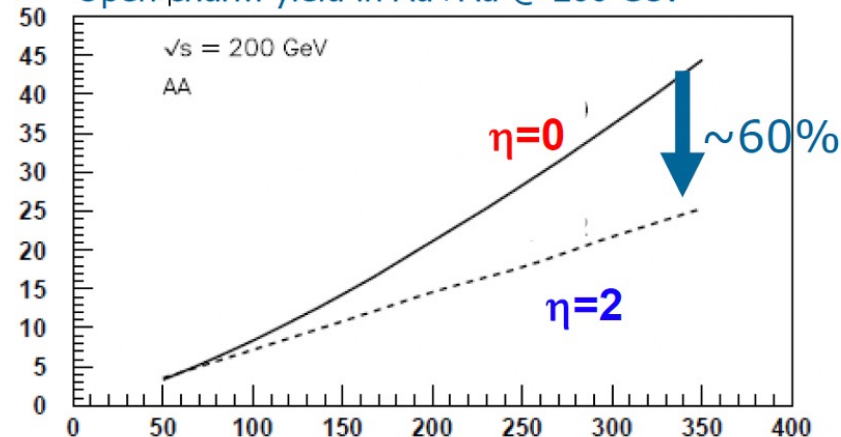
RHICでのJ/ψ R_{AA}

13



K. L. Tuchin hep-ph/0402298

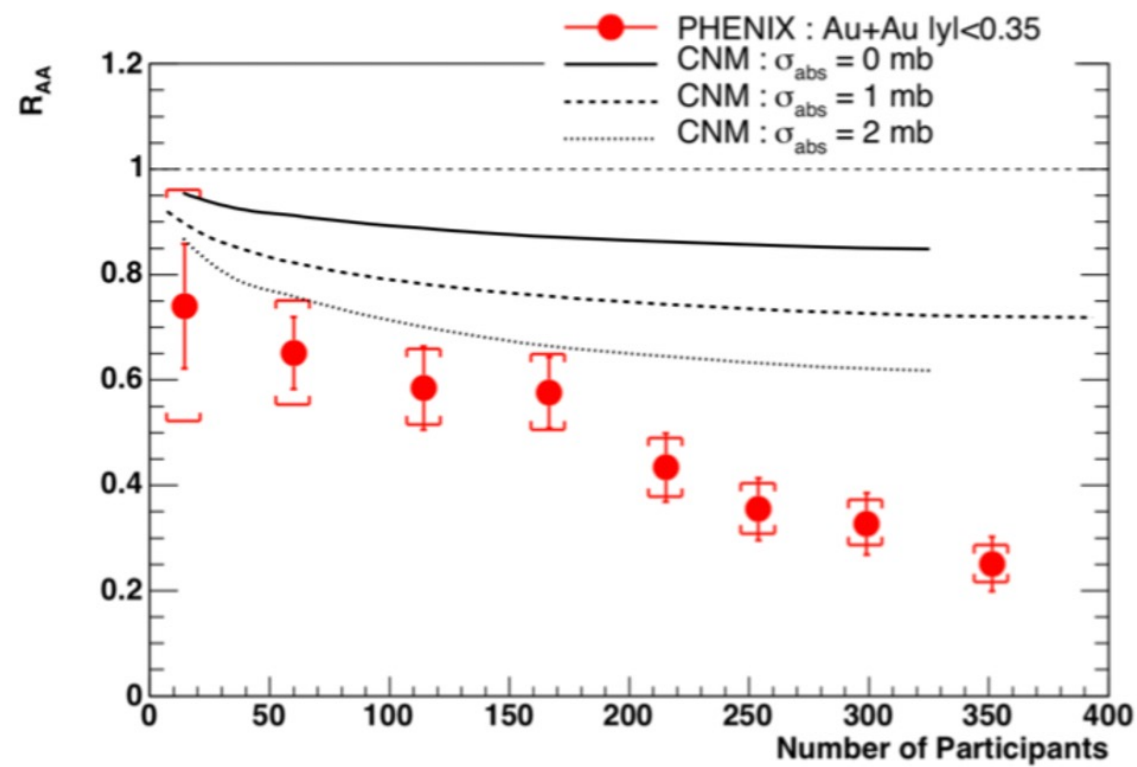
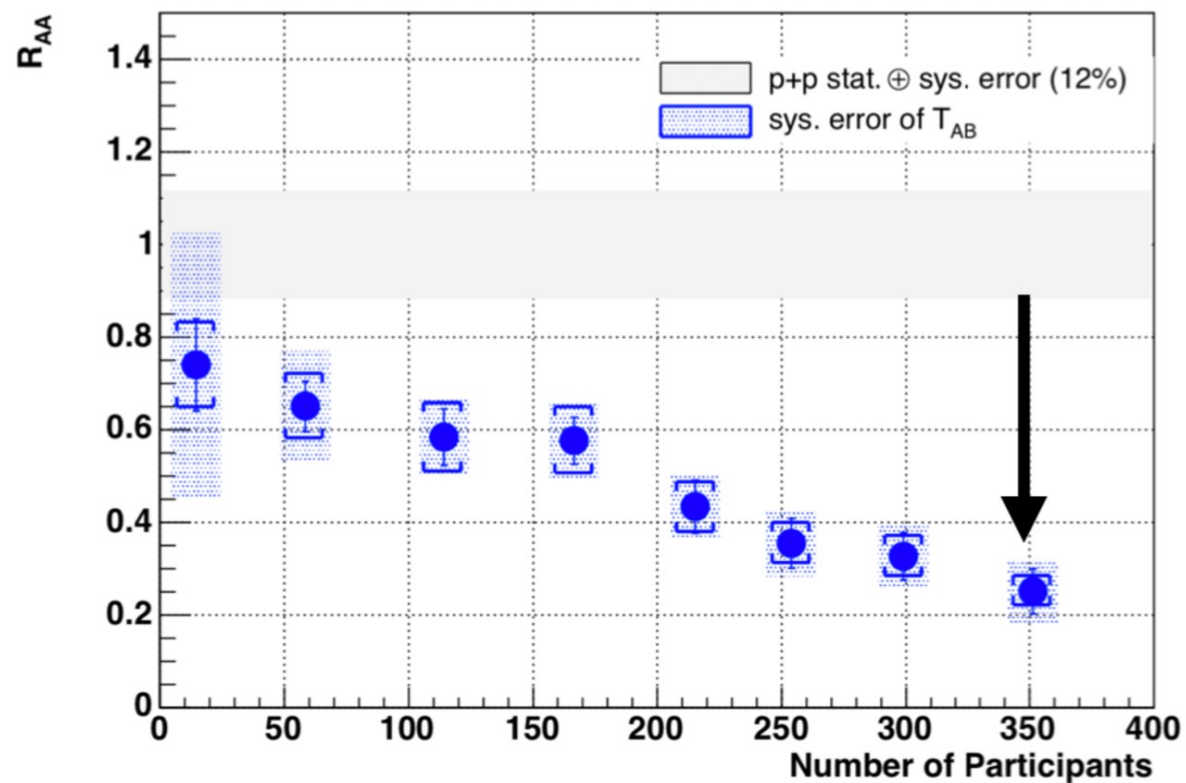
Open charm yield in Au+Au @ 200 GeV



RHICでのJ/ψ R_{AA}

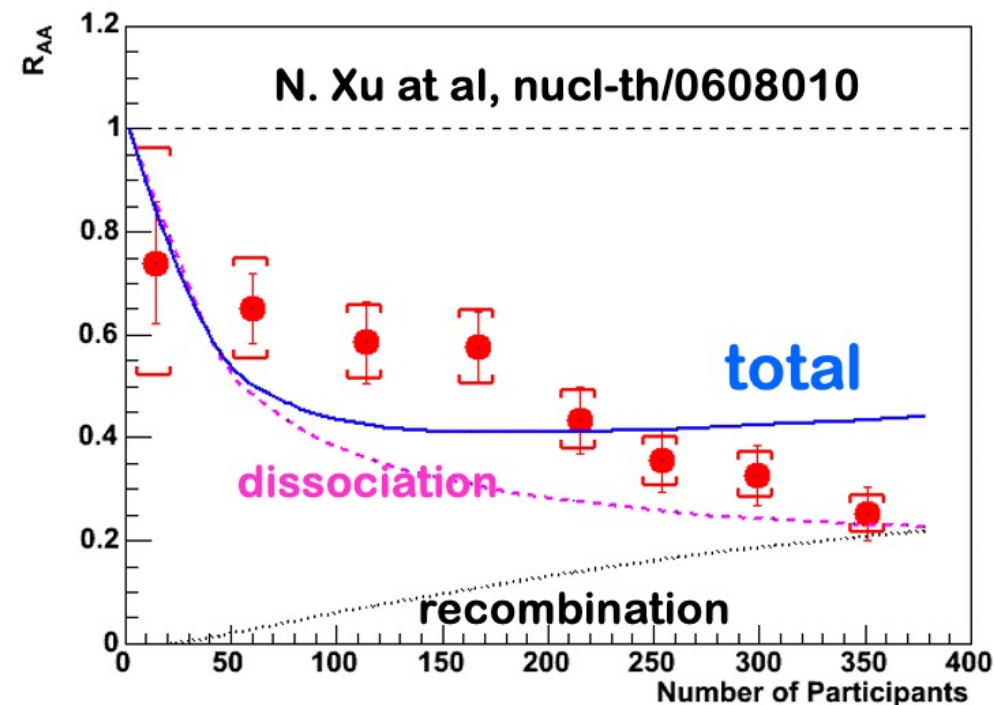
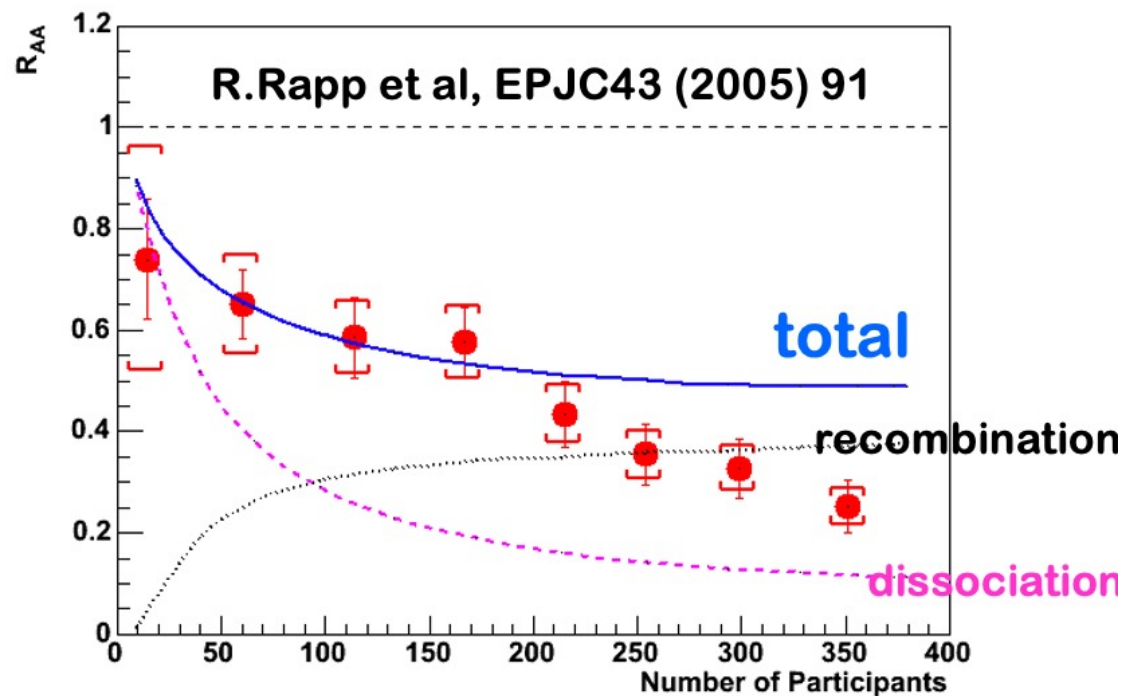
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Phys.Rev.Lett.98:232301,2007



モデルとの比較

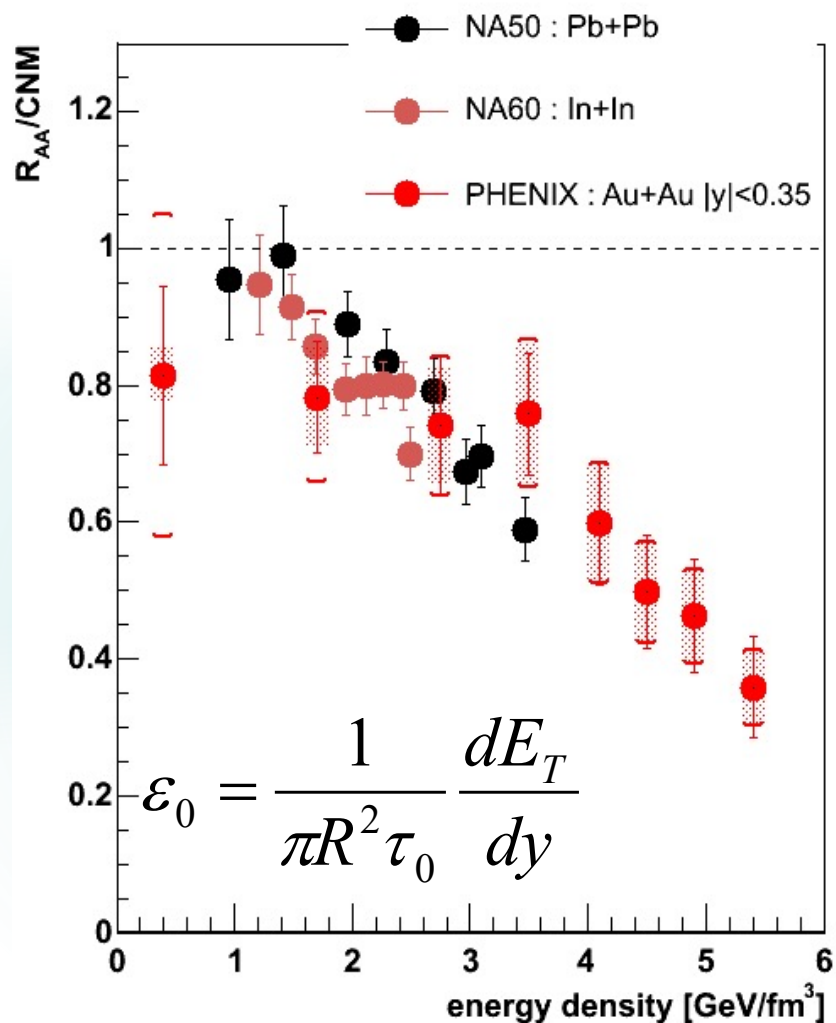
15



定性的には、再結合がないと収量はあわない。
でも、中心衝突度依存性はよく再現されていない

RHICでのJ/ψ R_{AA}

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▶ 横軸=Bjorken Energy density
($\tau_0 = 1 \text{ fm}/c$ と仮定。)

▶ 周辺衝突では、SPSの結果と良い一致。

▶ $N_{\text{part}} > 160 \rightarrow \varepsilon_{\text{bj}} > 3.5 [\text{GeV}/\text{fm}^3]$ で直接生成
J/ψの抑制？

再結合があるのか？

溶けているのは何なのか？

→一貫した結論になっていない

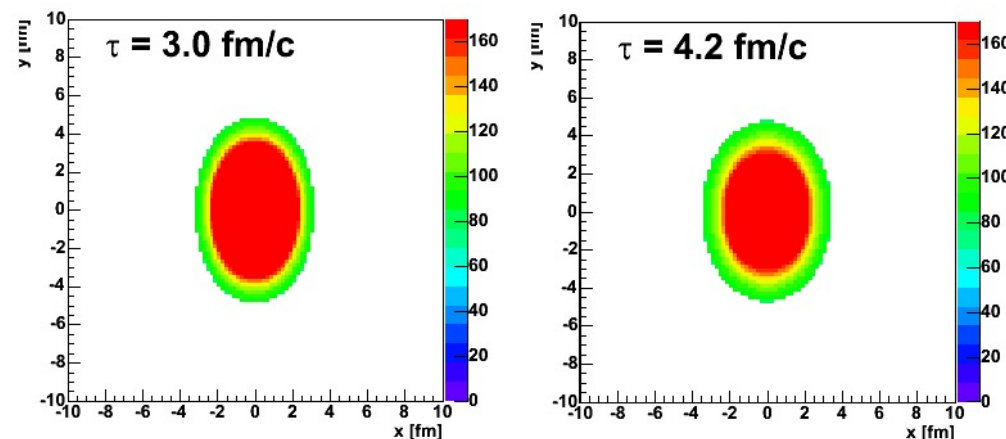
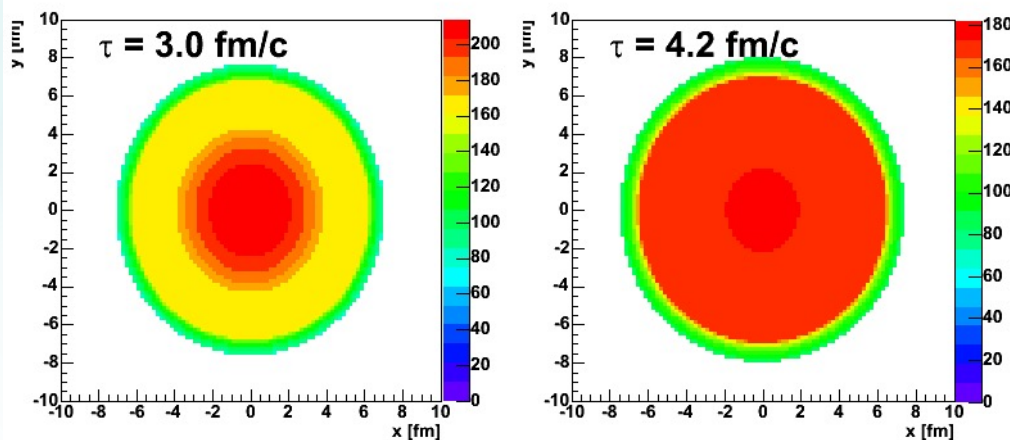
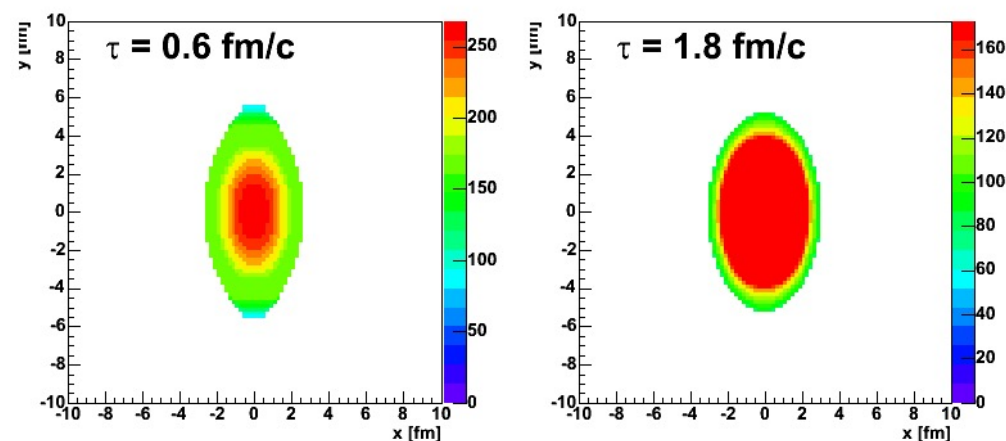
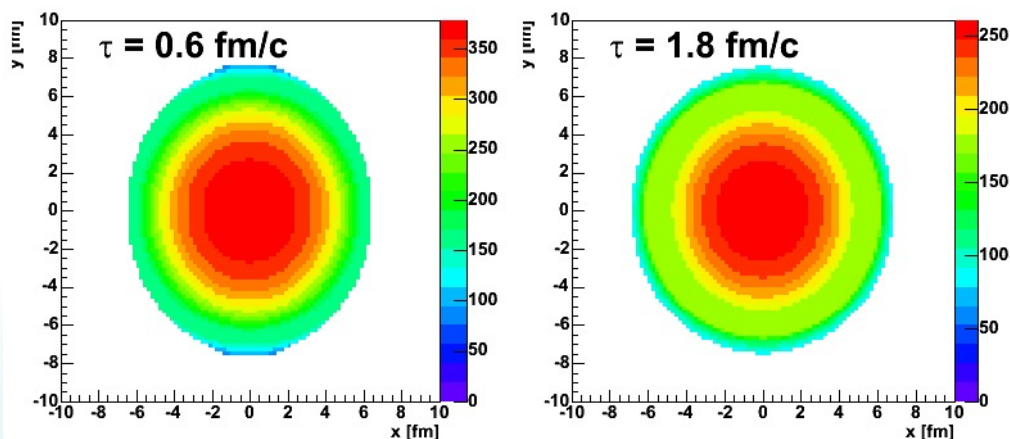
→自分の主張は？ ここままでは博士論文はダメかも…

平野さんの流体計算(2007)

17

$b=2.1$ fm (0-5% centrality)

$b=10.8$ fm (50-60%)



これを見て、「この中にJ/ψを入れて、溶解温度が推測できないか？」

Hydro+J/ψモデル

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- Survival Prob. In the medium:

$$S_{J/\psi}(\vec{x}_{J/\psi}(\tau)) = \exp\left[-\int_{\tau_0}^{\tau} \Gamma_{dis}(T(\vec{x}_{J/\psi}(\tau'))d\tau'\right]$$

- Decay Width:

– Currently, no (firm) theoretical information on Γ_{dis} for all charmonia in non-perturbative regime ($1-2T_c$).

$$\Gamma_{dis}(T) = \infty(T > T_{J/\psi}), \alpha(T/T_c - 1)^3 (T < T_{J/\psi})$$

T. Song, Y. Park and S. H. Lee
Phys.Lett.B659:621-627,2008.

- Motion of J/ψ: free streaming

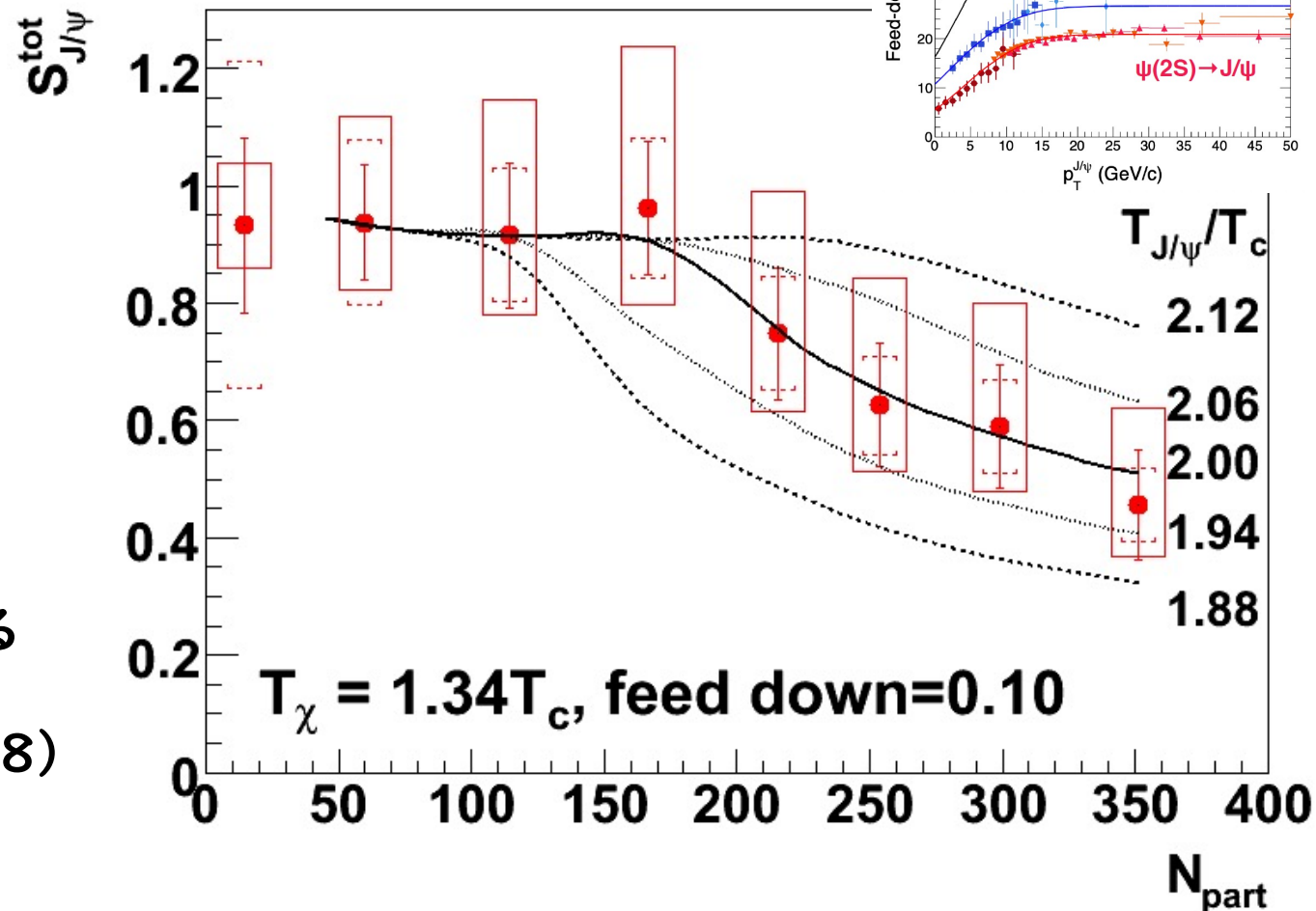
$$\vec{x}_{J/\psi}(\tau) = \vec{\beta}_{J/\psi}\tau + \vec{x}_0$$

信頼できる3+1Dの流体計算にのせて、
溶解温度が $2T_c$ のとき、結果が良く再現できる
これでD論は乗り切れるだろう、と確信
(このモデルは、所詮おもちゃレベル→QM08)
(再結合の有無はLHCを見てみよう…)

Hydro + J/ψ model

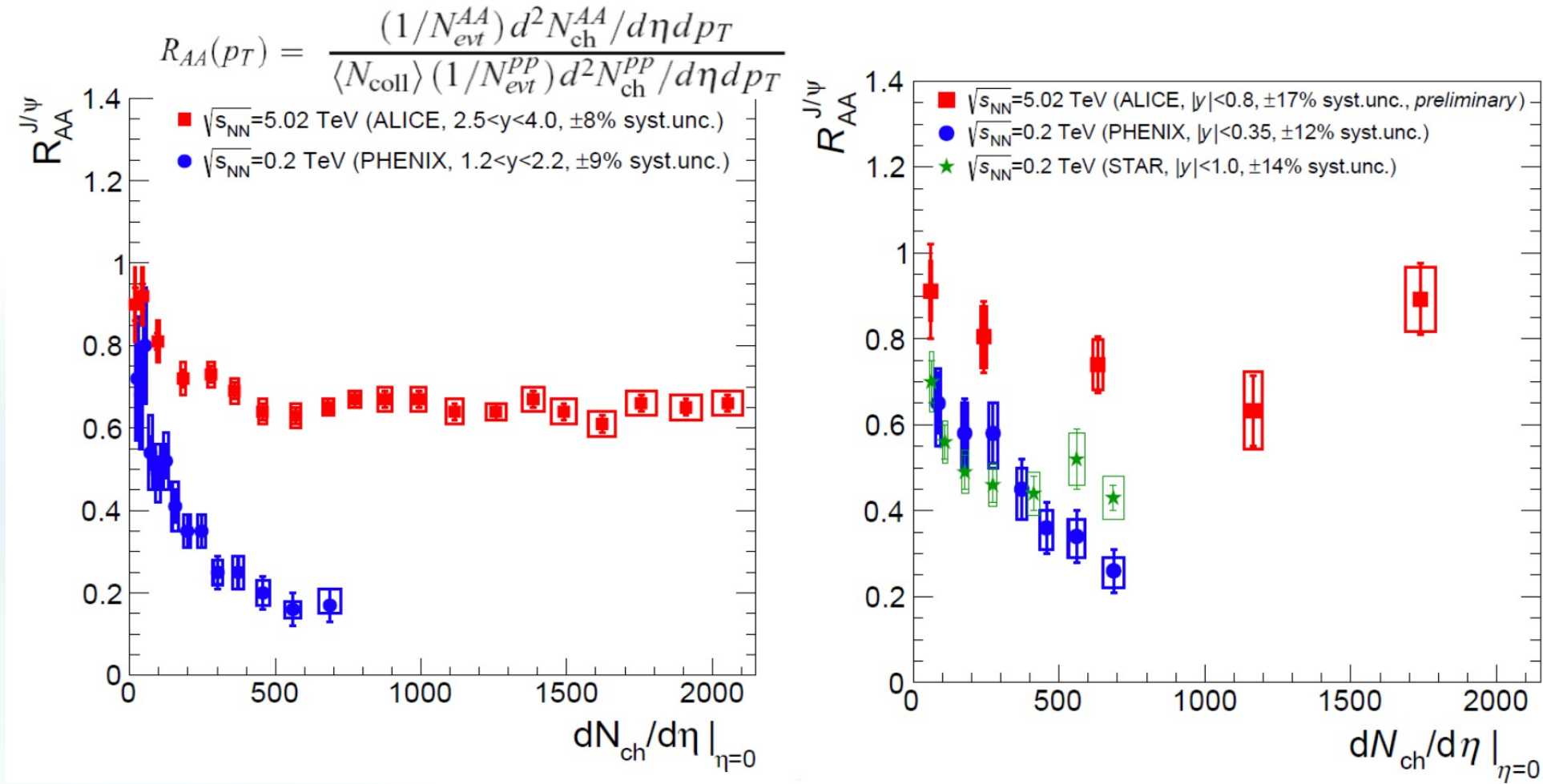
T. Gunji et al. PRC

76:051901, 2007



RHICからLHCへ：再結合の発見

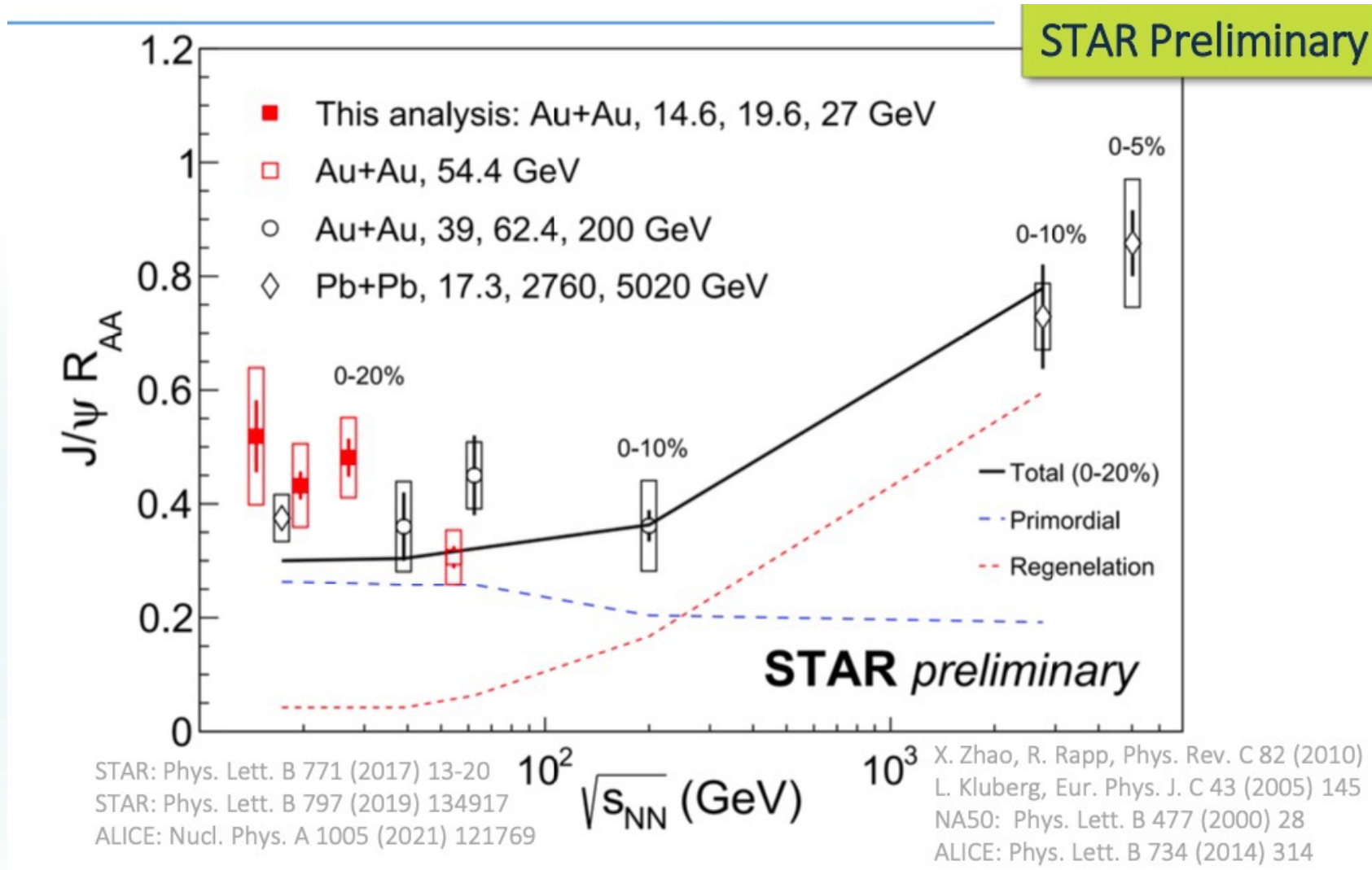
19



J/ ψ production is “less” suppressed at the LHC, even though higher temperature is reached at the LHC.

RHICからLHCへ：再結合の発見

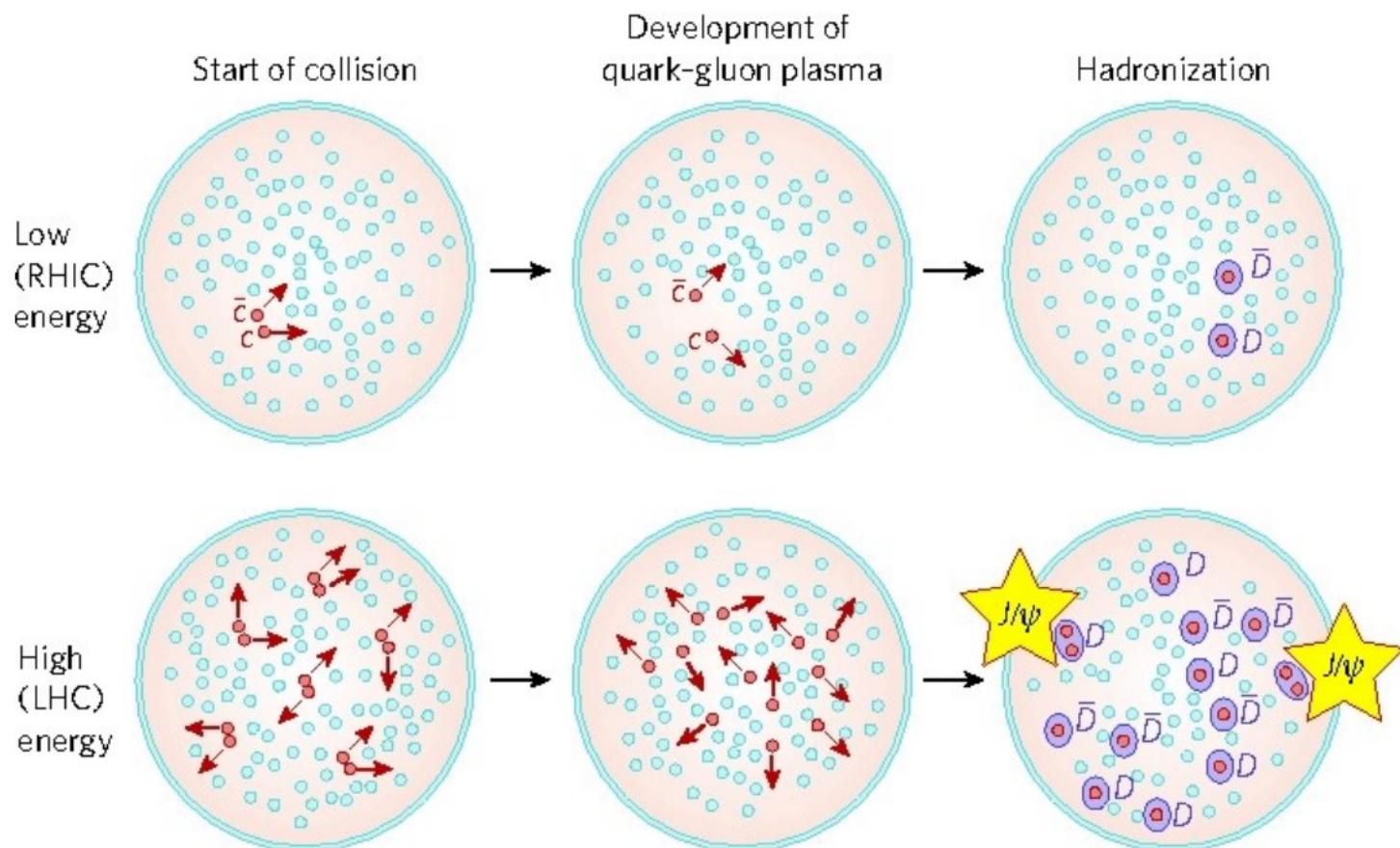
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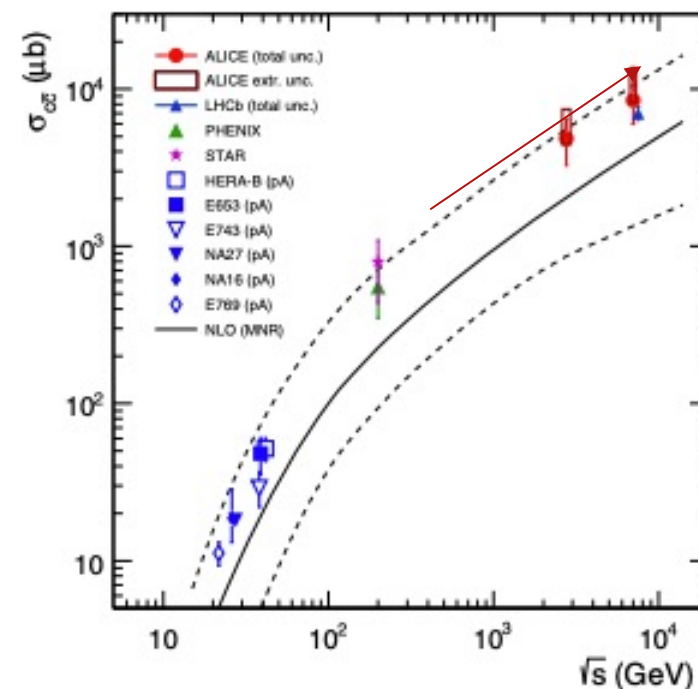
RHICからLHCへ：再結合の発見

21

P. Braun-Munzinger and J. Stachel, Nature 448 (2007)



$$p + p \rightarrow c + \bar{c} + X$$

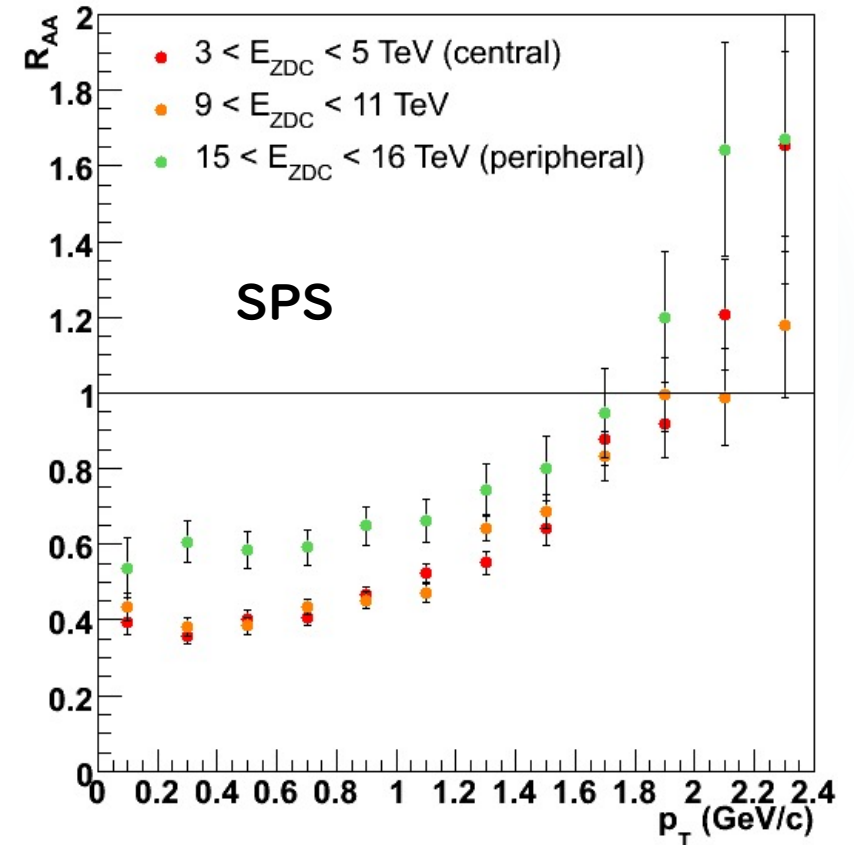
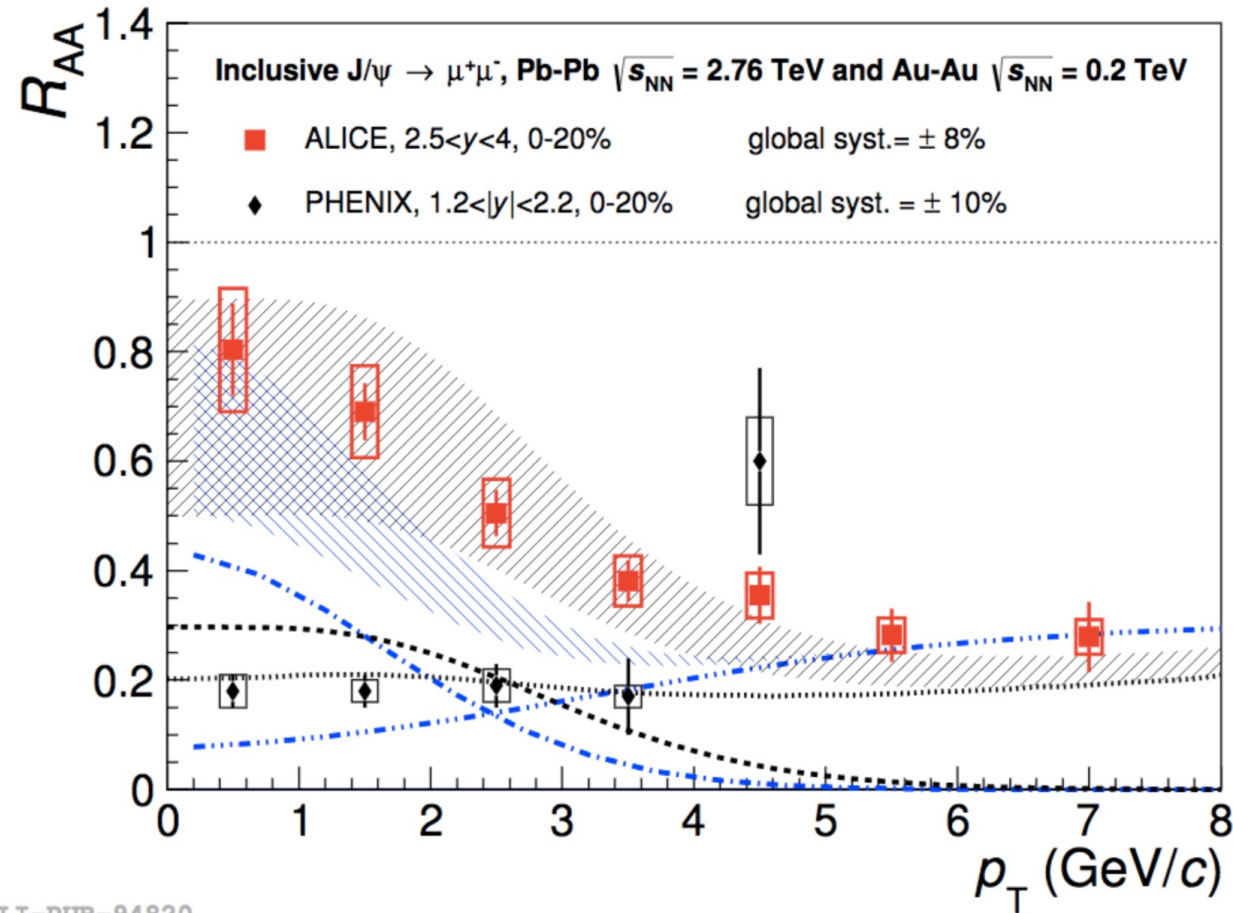


	$\sqrt{s_{NN}}$	$N_{c\bar{c}}$	$N_{b\bar{b}}$
RHIC	200 GeV	24	0.03
LHC	5500 GeV	240	1

荷電粒子の多重度は数倍しか変化していない

RHICからLHCへ： R_{AA} vs. p_T

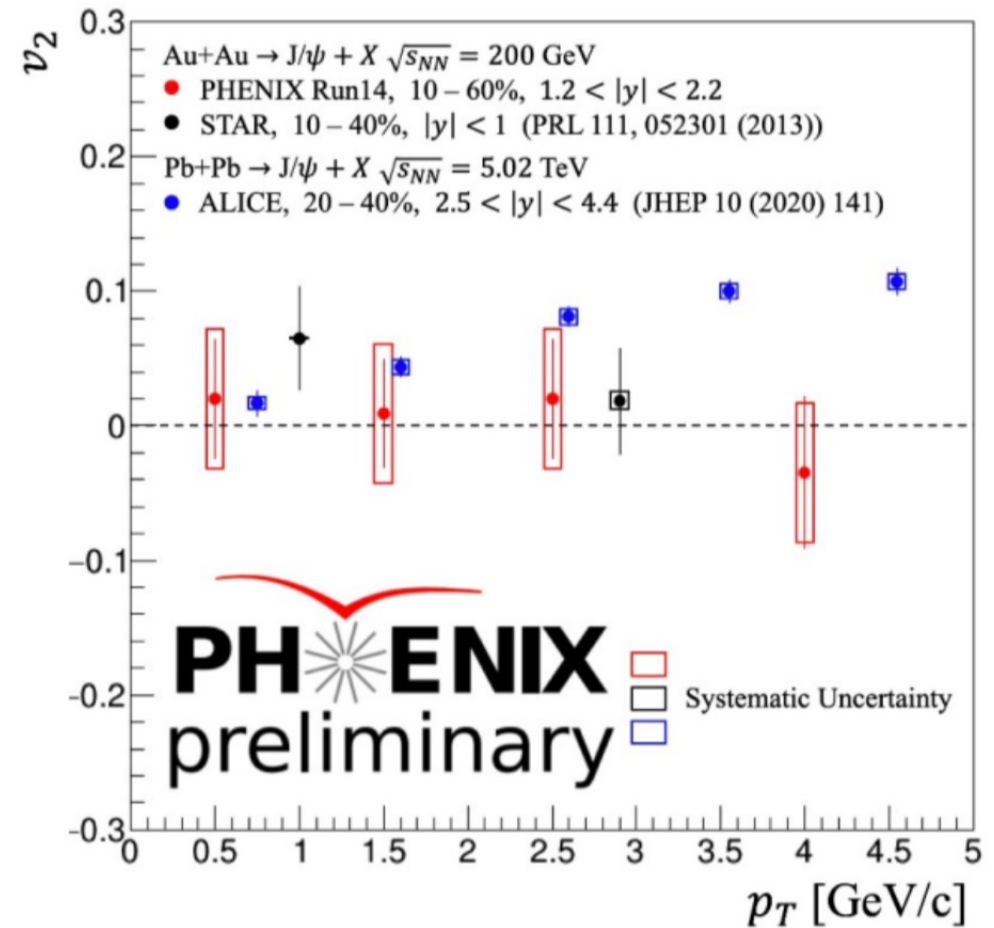
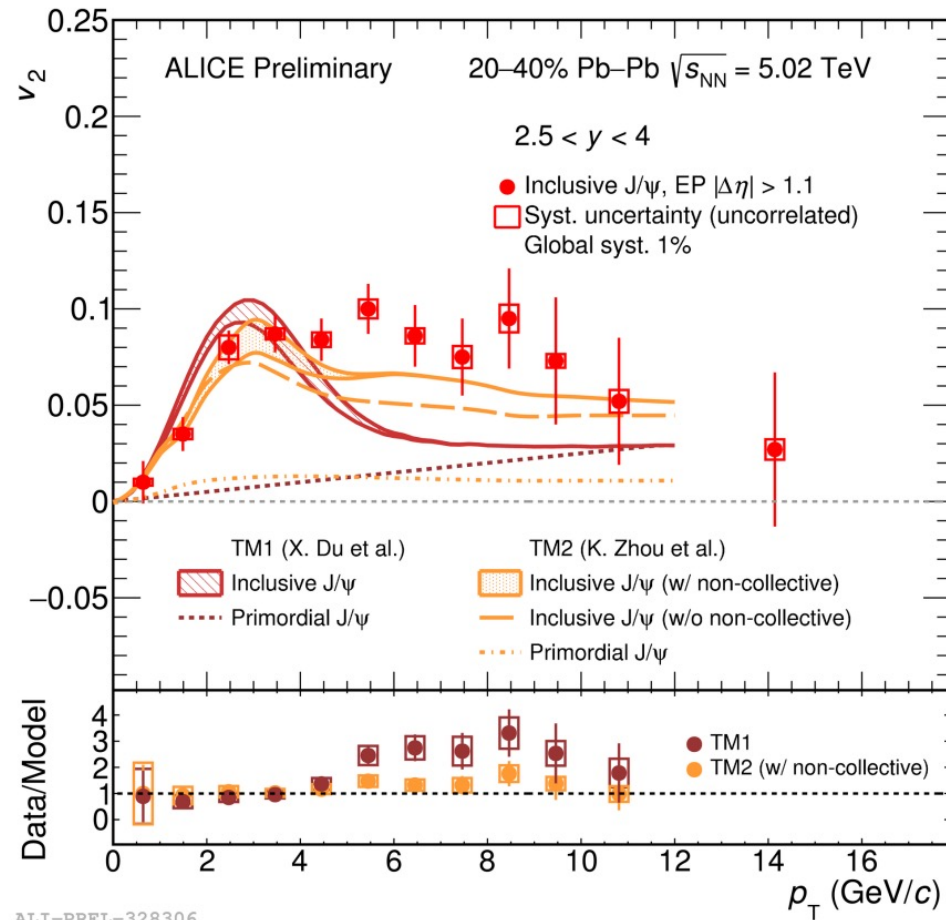
22



低い p_T の収量が増大（低運動量チャームが支配的なので）
RHICともSPSとも違う傾向（RHICでは再結合の影響はない）

RHICからLHCへ： v_2

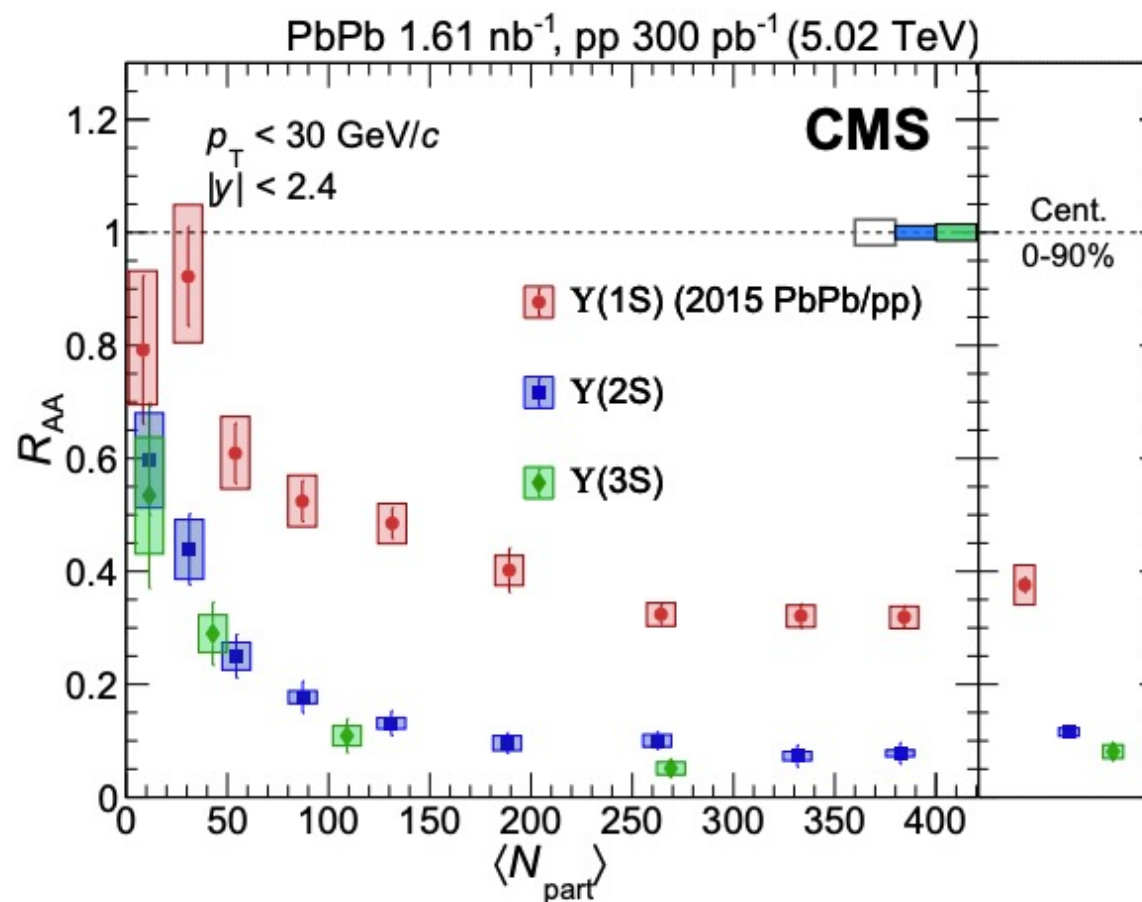
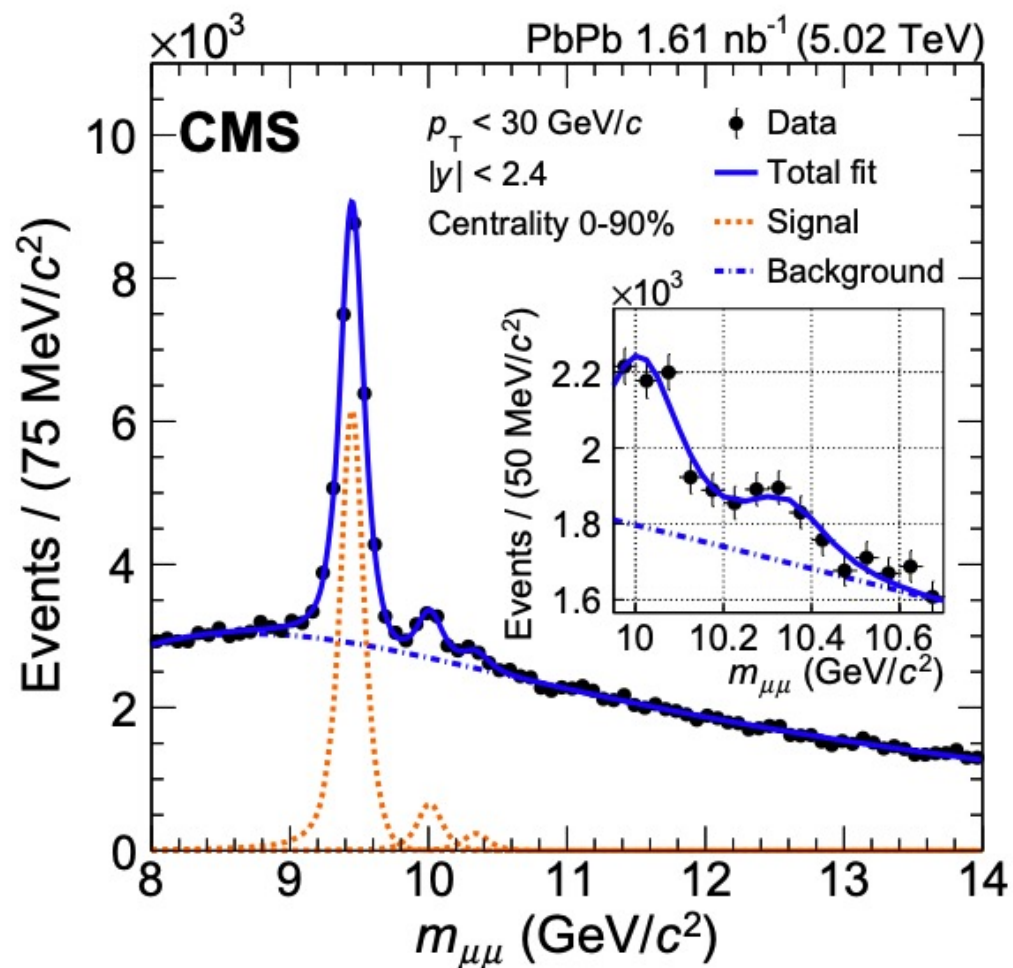
23



大きなフローの発見 (inclusive) → 大きすぎる？
 RHICではゼロと無矛盾 (RHICでは再結合の影響は小さい)

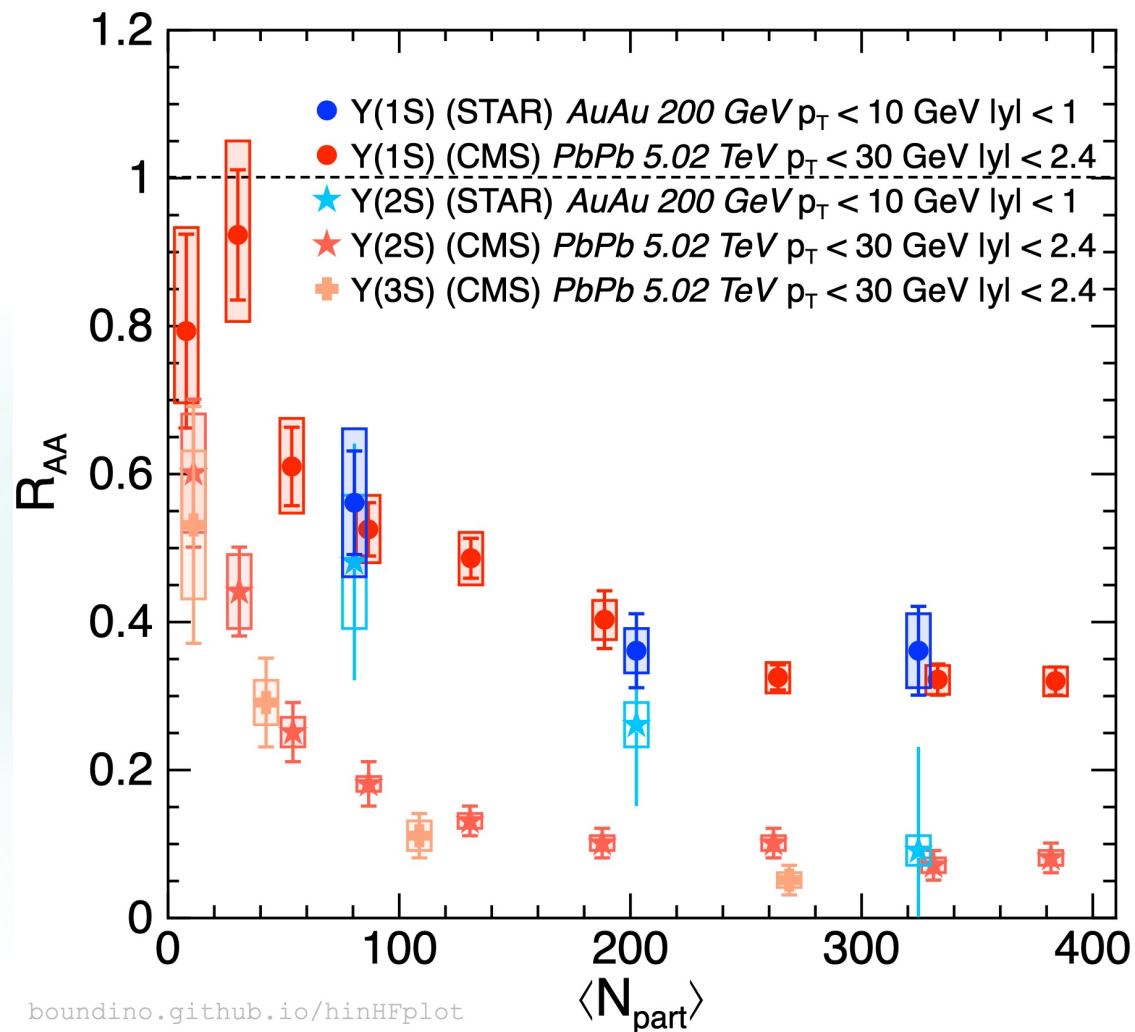
RHICからLHCへ：Yの測定

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RHICからLHCへ： Υ の測定

25



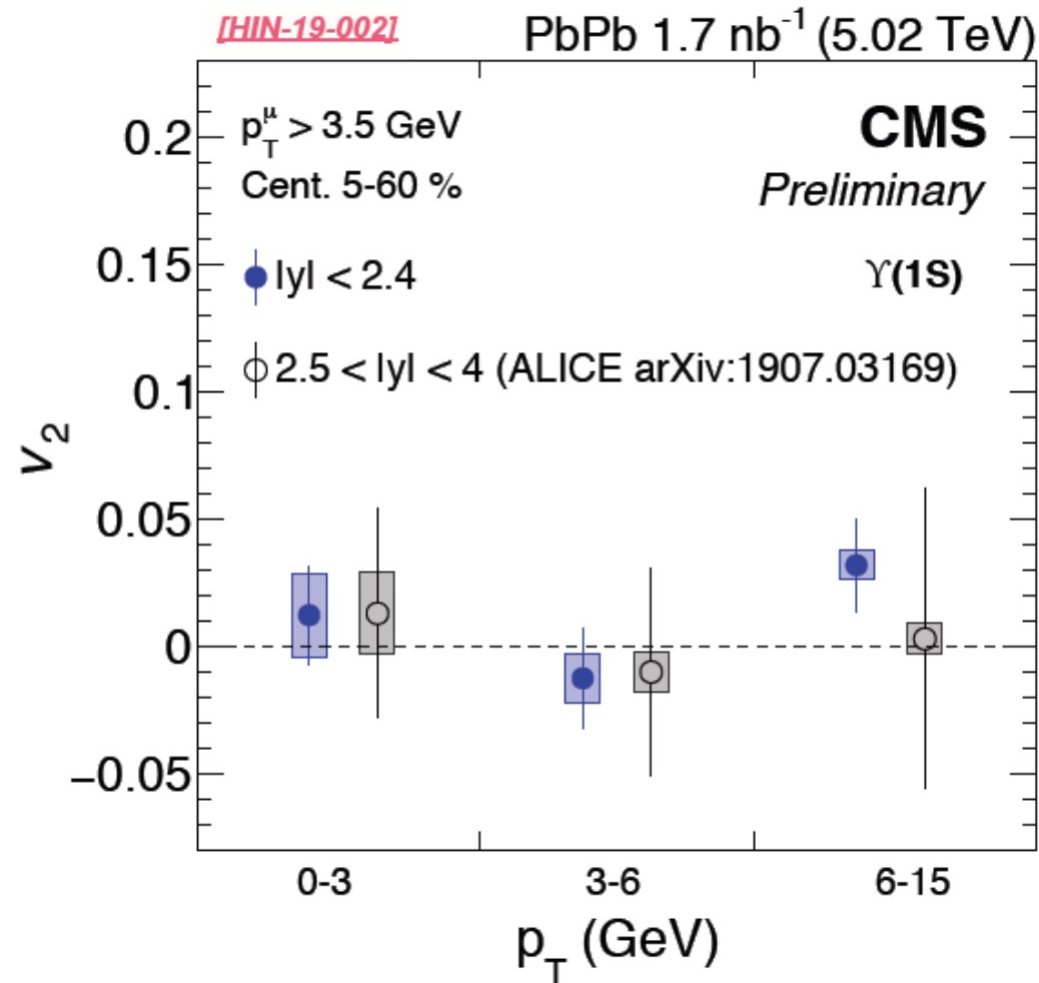
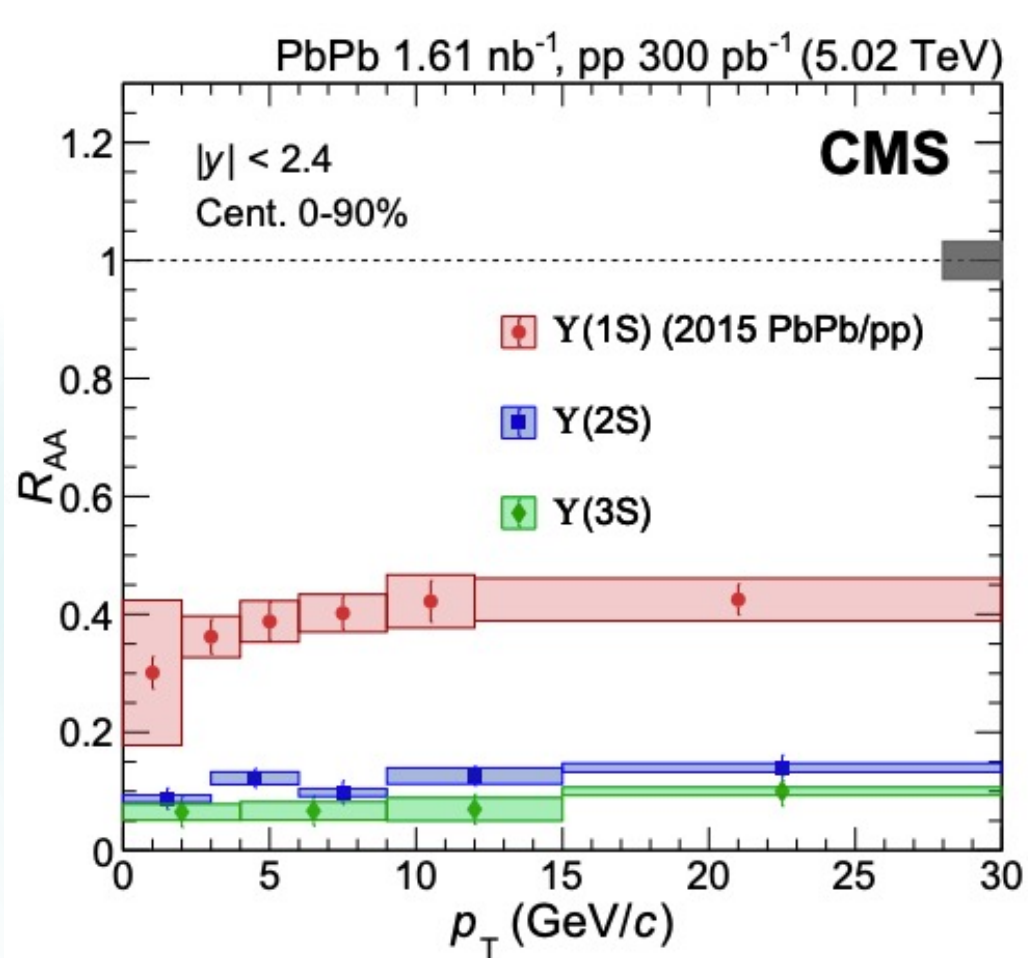
[boundino.github.io/hinHFplot](https://github.com/boundino/hinHFplot)

- $\Upsilon(nS)$ RAA at **PbPb 5.02 TeV** ~ **AuAu 0.2 TeV**:
 - Direct $\Upsilon(1S)$ not significantly suppressed in QGP?
 - Different cold nuclear matter effects.

RHICの今後は、sPHENIXに期待

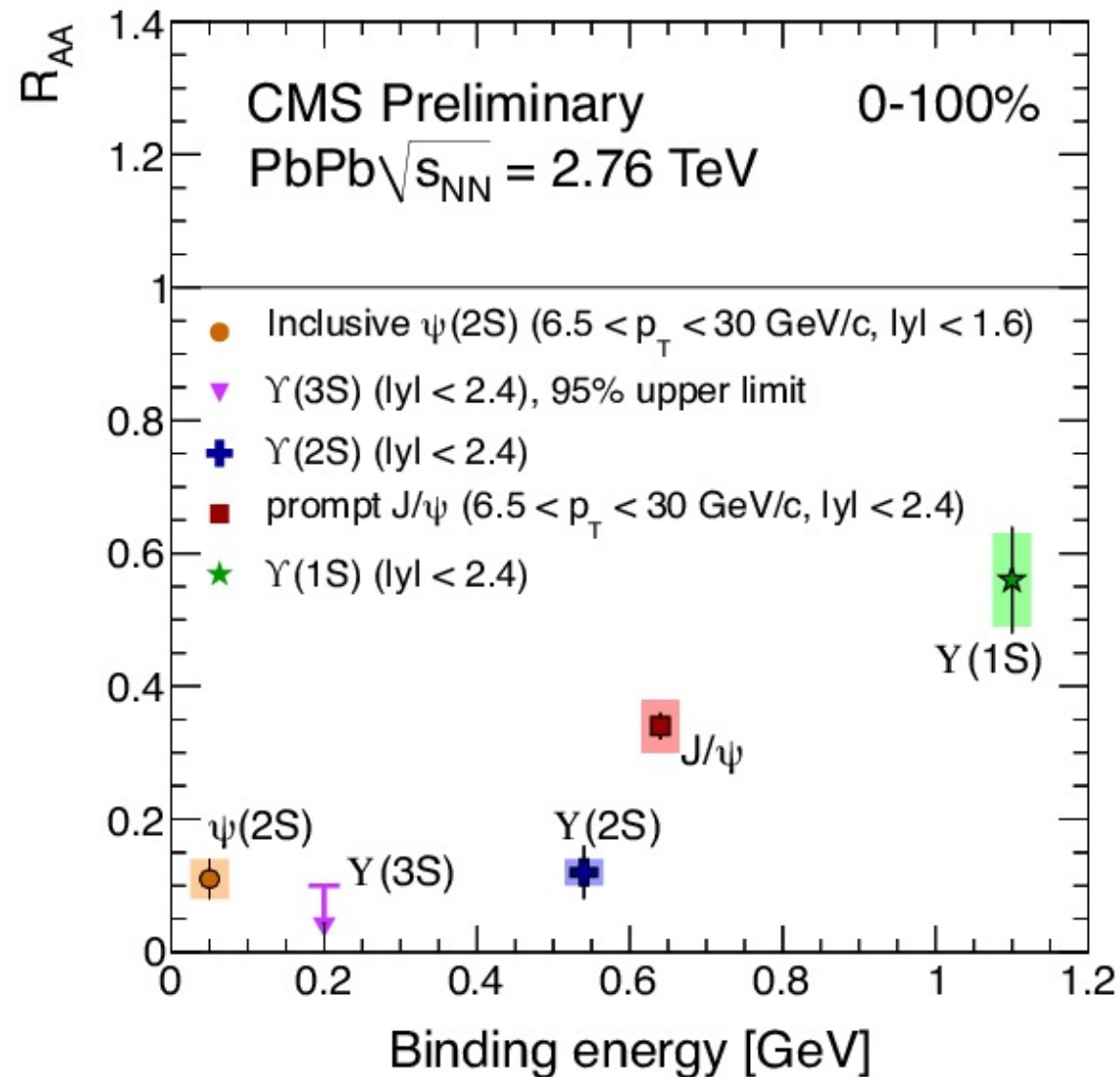
RHICからLHCへ： Υ の測定

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Suppression vs. binding energy

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量子開放系アプローチ

28

▶ 確率的ポテンシャル

- ▶ デバイカラー遮蔽といった静的な機構に加えて、クォーコニウムと媒質中のグルーオンや軽いクォークとの散乱による動的な機構（熱揺らぎ）を考慮

▶ マスター方程式による重クォーク束縛状態の時空発展

- ▶ 確率的ポテンシャルの虚部からLindblad方程式の散乱項を導出
- ▶ 最終的にクォーコニウムの量子状態の時間発展を追跡

量子開放系アプローチ

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Alexander Rothkopf, ALICE 3 WORKSHOP, 2023/10/18

- Lindblad equation takes the standard form:

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -i[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \sum_{i=1}^{N_{LB}} \gamma_i \left(\hat{L}_i \rho_{Q\bar{Q}} \hat{L}_i^\dagger - \frac{1}{2} \hat{L}_i \hat{L}_i^\dagger \rho_{Q\bar{Q}} - \frac{1}{2} \rho_{Q\bar{Q}} \hat{L}_i \hat{L}_i^\dagger \right)$$

- In-medium Hamiltonian exhibits a **screened and real-valued potential**

$$H_{Q\bar{Q}} = \frac{\mathbf{p}^2}{M_Q} + V_{Q\bar{Q}}(R)$$

$$V_{Q\bar{Q}}(R) = -\alpha_S \frac{e^{-m_D R}}{R} = \text{Re}[V_{\text{EFT}}(R)]$$

- Explicit form of the Lindblad operators:

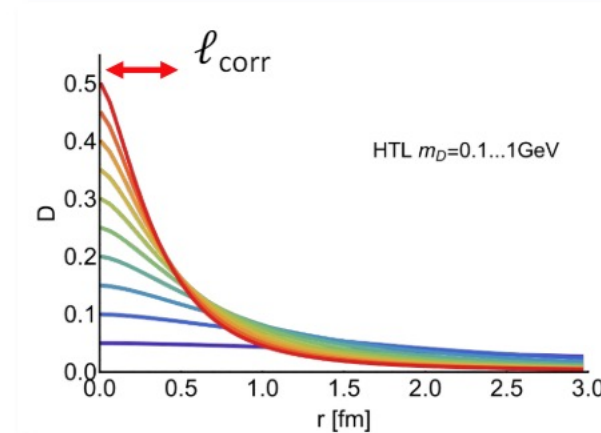
$$L_{\mathbf{k},a} = \sqrt{\frac{D(\mathbf{k})}{2}} \left[\underbrace{1}_{\text{fluctuations}} - \underbrace{\frac{\mathbf{k}}{4m_Q T} \cdot \left(\frac{1}{2} \mathbf{P}_{\text{CM}} + \mathbf{p} \right)}_{\text{dissipation}} \right] e^{i\mathbf{k} \cdot \mathbf{r}/2} (T^a \otimes 1)$$

medium acting on the quark

$$- \sqrt{\frac{D(\mathbf{k})}{2}} \left[\underbrace{1}_{\text{fluctuations}} - \underbrace{\frac{\mathbf{k}}{4m_Q T} \cdot \left(\frac{1}{2} \mathbf{P}_{\text{CM}} - \mathbf{p} \right)}_{\text{dissipation}} \right] e^{-i\mathbf{k} \cdot \mathbf{r}/2} (1 \otimes T^a)$$

medium acting on the anti-quark

$$\tilde{D}(\mathbf{k}) \sim \text{Fourier}(\text{Im}[V_{\text{EFT}}(R)])$$

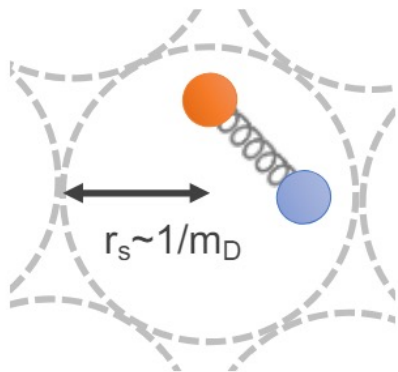


現代のScreening描像

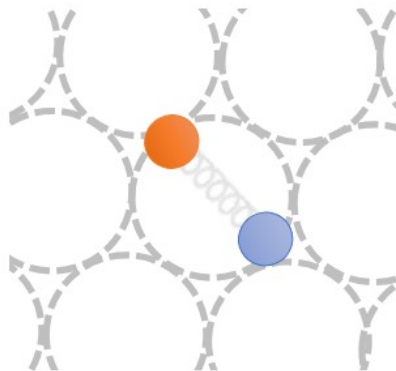
30

Alexander Rothkopf, ALICE 3 WORKSHOP, 2023/10/18

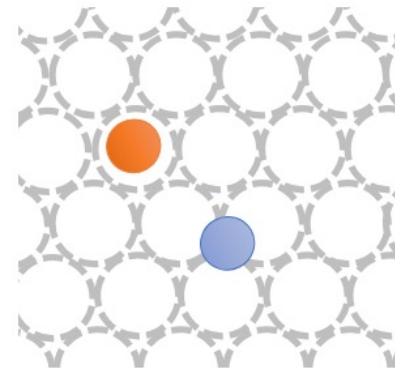
screening



binding force acts efficiently among Q and $\bar{Q}$

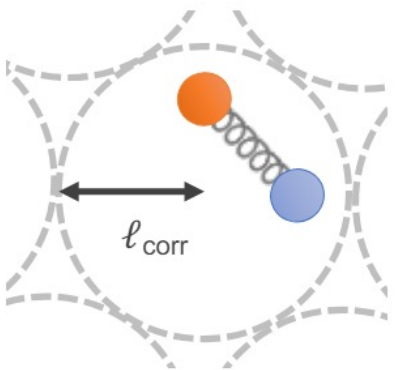


medium impedes gluons mediating force

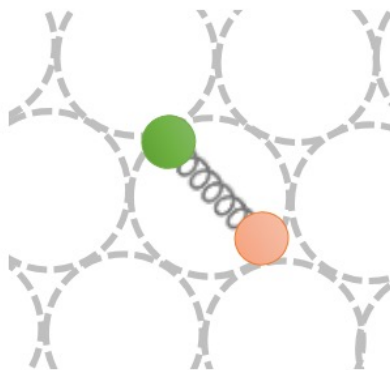


decoherence

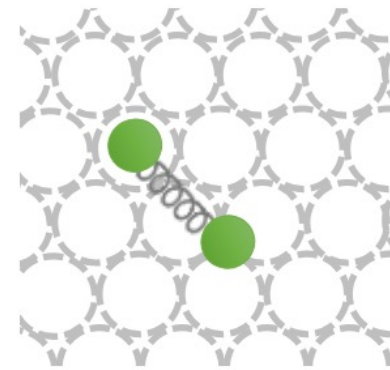
increasing temperature



color rotation acts coherently on Q and $\bar{Q}$



color rotation acts individually on Q and $\bar{Q}$



計算結果

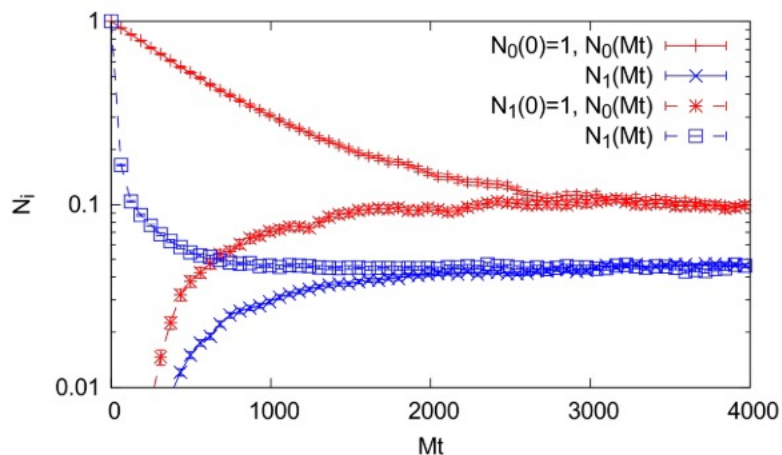
31

Alexander Rothkopf, ALICE 3 WORKSHOP, 2023/10/18

- Two independent approaches used to solve the color singlet 1d Lindblad equation

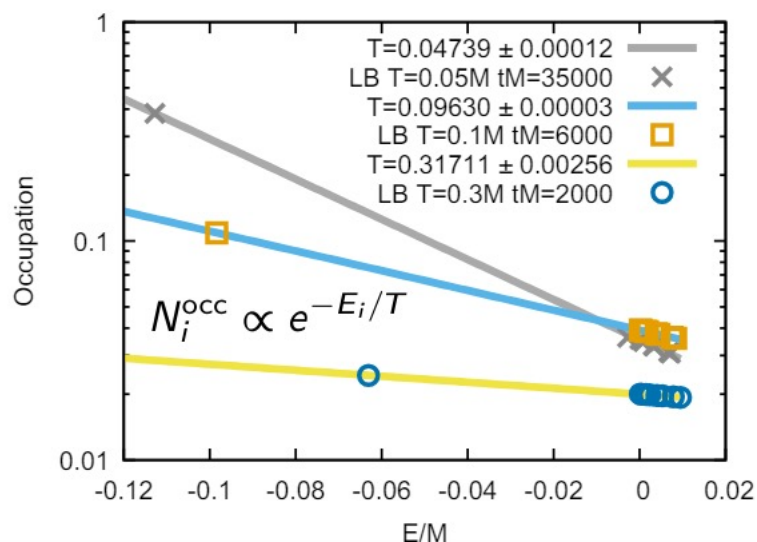
O. Ålund, Y. Akamatsu, F. Laurén, T. Miura, J. Nordström, A.R. JCP 425 (2021) 109917

T. Miura, Y. Akamatsu, M. Asakawa, A.R. PRD 101 (2020) 034011



- Start either with single ground state or single excited state and monitor survival probability

- Late-time results independent of initial conditions: steady-state



- Steady state characterized by Boltzmann distributed occupation numbers. Quarkonium temperature very close to medium temperature.

- First QQS real-time approach that can quantum thermalize quarkonium states

量子開放系に基づく様々な方法

Jiaxing Zhao,
SQM2023

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Duke-MIT Approach

- pNRQCD+OQS works in quantum optical limit $M \gg M_V \gg M_V^2 \gtrsim T \gtrsim m_D$
- A semi-classical (gradient) expansion and w/o quantum effect anymore
- Used for bottomonium.

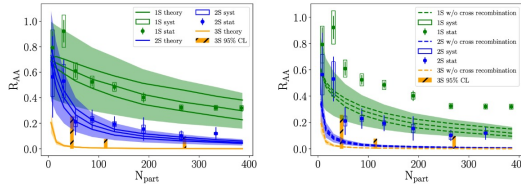
X. Yao, T. Mehen, W. Ke, Y. Xu, S. Bass, B. Müller, Phys.Rev.D 99 (2019) 9, 096028; JHEP 01 (2021) 046.

$$\rho_S(t) = \rho_S(0) + \sum_{a,b,c,d} \gamma_{ab,cd}(t) \left(L_{ab} \rho_S(0) L_{cd}^\dagger - \frac{1}{2} [L_{cd}^\dagger L_{ab}, \rho_S(0)] \right) - i \sum_{a,b} \sigma_{ab}(t) [L_{ab}, \rho_S(0)] + \mathcal{O}(H_I^3).$$

Lindblad equation

$$f_{nl}(\mathbf{x}, \mathbf{k}, t) \equiv \int \frac{d^3 k'}{(2\pi)^3} e^{i\mathbf{k}' \cdot \mathbf{x}} \left(\mathbf{k} + \frac{\mathbf{k}'}{2}, nl, 1 | \rho_S(t) | \mathbf{k} - \frac{\mathbf{k}'}{2}, nl, 1 \right)$$

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{x}, \mathbf{k}, t) + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_{nl}(\mathbf{x}, \mathbf{k}, t) = \mathcal{C}_{nl}^{(+)}(\mathbf{x}, \mathbf{k}, t) - \mathcal{C}_{nl}^{(-)}(\mathbf{x}, \mathbf{k}, t) \quad \text{Similar to the TAMU and Tsinghua model}$$



Importance of recombination from correlated $b\bar{b}$!

Gives a connection between the OQS and Boltzmann equation in the quantum optical limit!

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Munich-Kent Approach

- pNRQCD+OQS works in Quantum Brownian motion Regime $M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E_{bind}$
- Expansion of E_{bind}/T from LO to NLO; the quantum jumps are now implemented.
- Used for bottomonium.

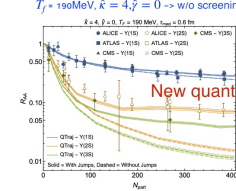
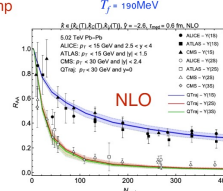
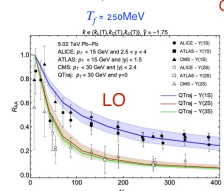
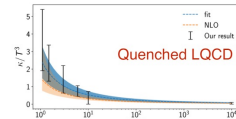
N. Brambilla, M. Escobedo, M. Strickland, A. Vairo, J. Weber, Phys.Rev.D 104 (2021) 9, 094049; JHEP 05 (2021) 136; JHEP 08 (2022) 303; Phys.Rev.D 108 (2023) 1, L011502.

$$\frac{d\rho(t)}{dt} = -i[H_{eff}, \rho(t)] + \sum_n \left(C_n \rho(t) C_n^\dagger - \frac{1}{2} \{ C_n^\dagger C_n, \rho(t) \} \right)$$

$$\rho(t) = \begin{pmatrix} \rho_{tt}(t) & 0 \\ 0 & \rho_{bb}(t) \end{pmatrix}, \quad \kappa = \frac{g^2}{18} \int_0^\infty dt \langle \{ \tilde{E}^{a,i}(t, 0), \tilde{E}^{a,i}(0, 0) \} \rangle, \quad \gamma = -i \frac{g^2}{18} \int_0^\infty dt \langle [\tilde{E}^{a,i}(t, 0), \tilde{E}^{a,i}(0, 0)] \rangle$$

$$C_0^\dagger = \sqrt{\frac{\kappa}{N_c^2 - 1}} r^\dagger \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2 - 1} & 0 \end{pmatrix}, \quad C_1^\dagger = \sqrt{\frac{(N_c^2 - 4)\kappa}{2(N_c^2 - 1)}} r^\dagger \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Strong coupling between heavy quark and medium; κ and γ are related to the thermal width Γ and mass shift δM of the bottomonium.



The new results with quantum jumps and w/o color screening agree well with the R_{AA} and double ratios!

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Nantes Approach

- NRQCD+OQS works in Quantum Brownian motion Regime $M \gg T \sim m_D \gtrsim E_{bind}$
- Expansion of τ_c/τ_s .
- Used for bottomonium and charmonium in 1D.

Stéphane Delorme, Tue. 2:20

J. Blaizot, M. Escobedo, JHEP 06, 034 (2018);
S. Delorme, T. Gousset, R. Katz, P.B. Gossiaux, Acta Phys. Pol. B Proc. Suppl. 16, 1-112 (2023);
Eur. Phys. J. A 58(10), 198 (2022); arXiv:2402.04488.

$$\frac{d\mathcal{D}_Q}{dt} = \mathcal{L}\mathcal{D}_Q, \quad \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$$

$$\mathcal{L}_0 \mathcal{D} = -i[H_Q, \mathcal{D}]$$

$$\mathcal{L}_1 \mathcal{D} = -\frac{i}{2} \int_{xx'} V(x-x') [n_x^a n_{x'}^a, \mathcal{D}]$$

$$\mathcal{L}_2 \mathcal{D} = \frac{1}{2} \int_{xx'} W(x-x') (\{n_x^a n_{x'}^a, \mathcal{D}\} - 2n_x^a \mathcal{D} n_{x'}^a) \quad \text{fluctuations}$$

$$\mathcal{L}_3 \mathcal{D} = -\frac{i}{4T} \int_{xx'} W(x-x') \left(\hat{n}_x^a \mathcal{D} n_{x'}^a - n_x^a \mathcal{D} \hat{n}_{x'}^a + \frac{1}{2} \{ \mathcal{D}, [\hat{n}_x^a, n_{x'}^a] \} \right) \quad \text{dissipation}$$

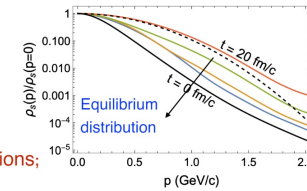
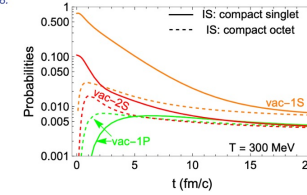
$$\mathcal{L}_4 \mathcal{D} = \frac{1}{32T^2} \int_{xx'} W(x-x') (\{ \hat{n}_x^a \hat{n}_{x'}^a, \mathcal{D} \} - 2\hat{n}_x^a \mathcal{D} \hat{n}_{x'}^a) \quad \text{preservation of positivity}$$

$$\frac{d}{dt} \begin{pmatrix} \mathcal{D}_1 \\ \mathcal{D}_o \end{pmatrix} = \mathcal{L} \begin{pmatrix} \mathcal{D}_1 \\ \mathcal{D}_o \end{pmatrix}, \quad \mathcal{L} = \begin{pmatrix} \mathcal{L}_{11} & \mathcal{L}_{1o} \\ \mathcal{L}_{o1} & \mathcal{L}_{oo} \end{pmatrix}$$

Beyond the dipole approximation;

The equations are solved with different initial states and medium configurations;

Equilibrium is checked.



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Osaka Approach

- NRQCD+OQS works in Quantum Brownian motion Regime $M \gg T \sim m_D \gtrsim E_{bind}$
- Weak coupling (strict) and go beyond the weak coupling (approximation)
- Used for bottomonium.

T. Miura, Y. Akamatsu, M. Asakawa, et al, PRD 87 (2013) 045016; PRD 91 (2015) 5, 056002; PRD97 (2018), 014003; Phys.Rev.D 106 (2022) 7, 074001.

$$\frac{d}{dt} \rho_r(t) = -i[H_{eff}^{(r)}, \rho_r(t)] + \sum_{ka} \left(2L_{ka}^{(r)} \rho_r L_{ka}^{(r)\dagger} - L_{ka}^{(r)\dagger} L_{ka}^{(r)} \rho_r - \rho_r L_{ka}^{(r)} L_{ka}^{(r)\dagger} \right),$$

$$H_{eff}^{(r)} = \frac{\vec{p}^2}{M} + V(\vec{r})(t \otimes t^{**}) - \frac{1}{4MT} \left\{ \vec{p}, \vec{\nabla} D(\vec{r}) \right\} (t^a \otimes t^{**}),$$

$$L_{ka}^{(r)} = \sqrt{\frac{\vec{D}(\vec{k})}{2L^3}} \left[1 - \frac{\vec{k}}{4MT} \cdot \left(\frac{1}{2} \vec{P}_{CM} + \vec{p} \right) \right] e^{i\frac{\vec{k} \cdot \vec{r}}{2}} (t^a \otimes 1)$$

$$- \sqrt{\frac{\vec{D}(\vec{k})}{2L^3}} \left[1 - \frac{\vec{k}}{4MT} \cdot \left(\frac{1}{2} \vec{P}_{CM} - \vec{p} \right) \right] e^{-i\frac{\vec{k} \cdot \vec{r}}{2}} (1 \otimes t^{**}).$$

Beyond the weak coupling and assume

the real and imaginary potential:

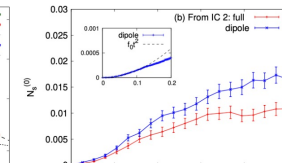
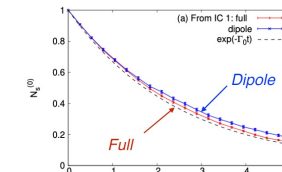
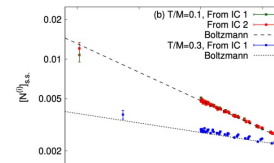
$$V(x) = -\frac{\alpha}{\sqrt{x^2 + x_c^2}} e^{-m_D|x|},$$

$$D(x) = \gamma \exp(-x^2/\ell_{corr}^2).$$

Beyond the dipole approximation;

The dipole approximation is an efficient alternative method, but it depends on the initial condition!

Equilibrium is satisfied.



頑張ってください

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まとめと展望

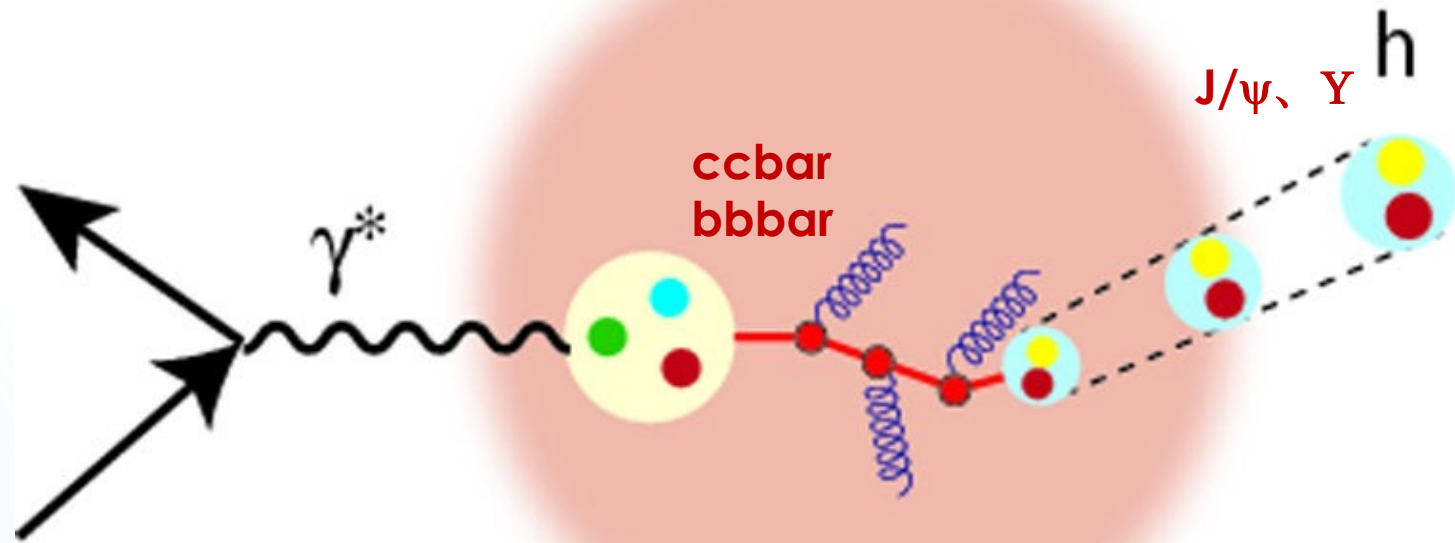
33

- ▶ 浅川さん、おめでとうございます。どうもありがとうございます。
- ▶ クォークonium：SPS, RHIC, LHCにわたる飛躍的な展開
 - ▶ SPS: 励起状態の抑制
 - ▶ RHIC: 直接生成J/ψの抑制
 - ▶ LHC: J/ψ再結合の発見、Y
 - ▶ J/ψのv2?
- ▶ 開放系による記述
 - ▶ スクリーニングと動的効果の同時取り扱い
- ▶ sPHENIXとLHC実験の高精度データ：Y、feed-down、小さい系

LHCからEIC:

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グルーオン飽和中のクォークonium?



- ▶ クォークoniumはグルーオン飽和中でどう振る舞う？

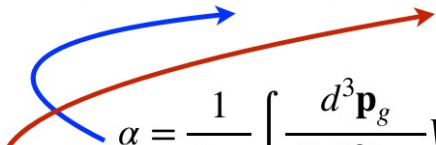
補足資料

Assume the quarkonium is a **classical particle**!

Charmonium are not fully dissociated. Dissociation and regeneration happen gradually in QGP.

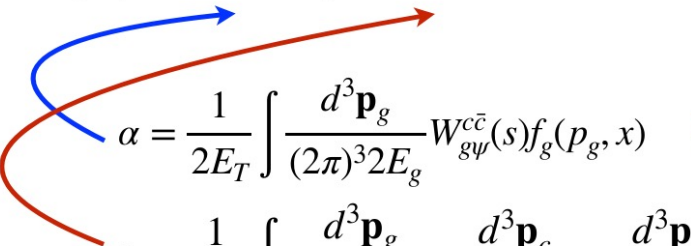
➔ Transport description (Boltzmann equation)

$$p^\mu \partial_\mu f_\psi = -\alpha E f_\psi + \beta E$$



$$\alpha = \frac{1}{2E_T} \int \frac{d^3 \mathbf{p}_g}{(2\pi)^3 2E_g} W_{g\psi}^{c\bar{c}}(s) f_g(p_g, x)$$

Gluon-dissociation



$$\beta = \frac{1}{2E_T} \int \frac{d^3 \mathbf{p}_g}{(2\pi)^3 2E_g} \frac{d^3 \mathbf{p}_c}{(2\pi)^3 2E_c} \frac{d^3 \mathbf{p}_{\bar{c}}}{(2\pi)^3 2E_{\bar{c}}} W_{c\bar{c}}^{g\psi}(s) f_c(p_c, x) f_{\bar{c}}(p_{\bar{c}}, x) (2\pi)^4 \delta^{(4)}(p + p_g - p_c - p_{\bar{c}})$$

Regeneration

Dissociation and regeneration are related to each other via the detailed balance.

➔ Transport description (Rate equation)

$$\frac{dN_\psi(\tau)}{d\tau} = -\Gamma_\psi [N_\psi(\tau) - N_\psi^{\text{eq}}(\tau)]$$



Dissociation rate



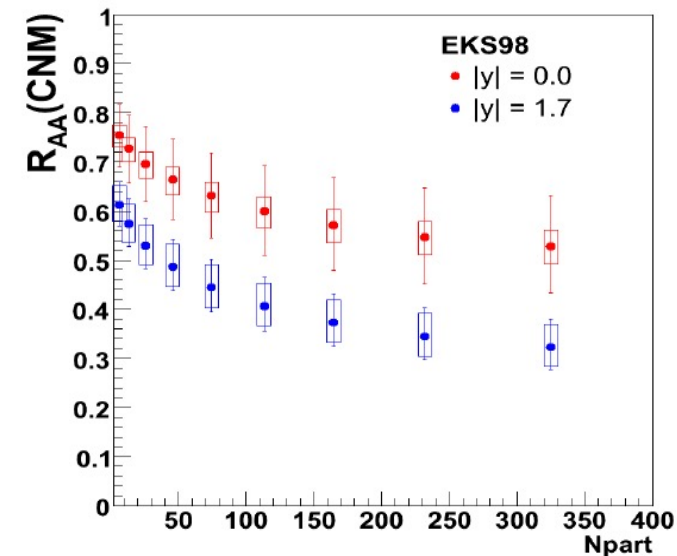
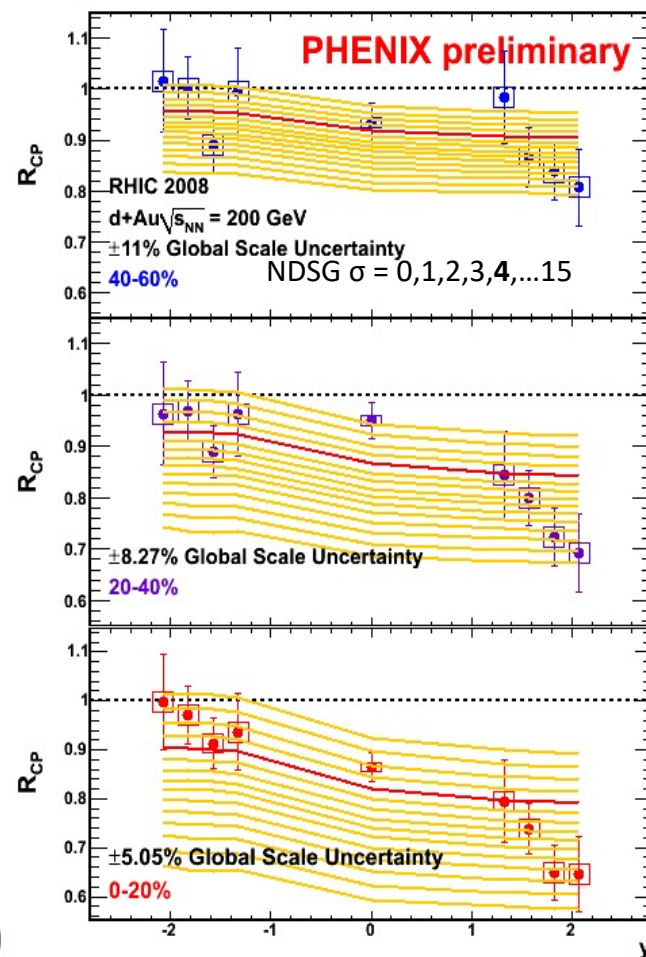
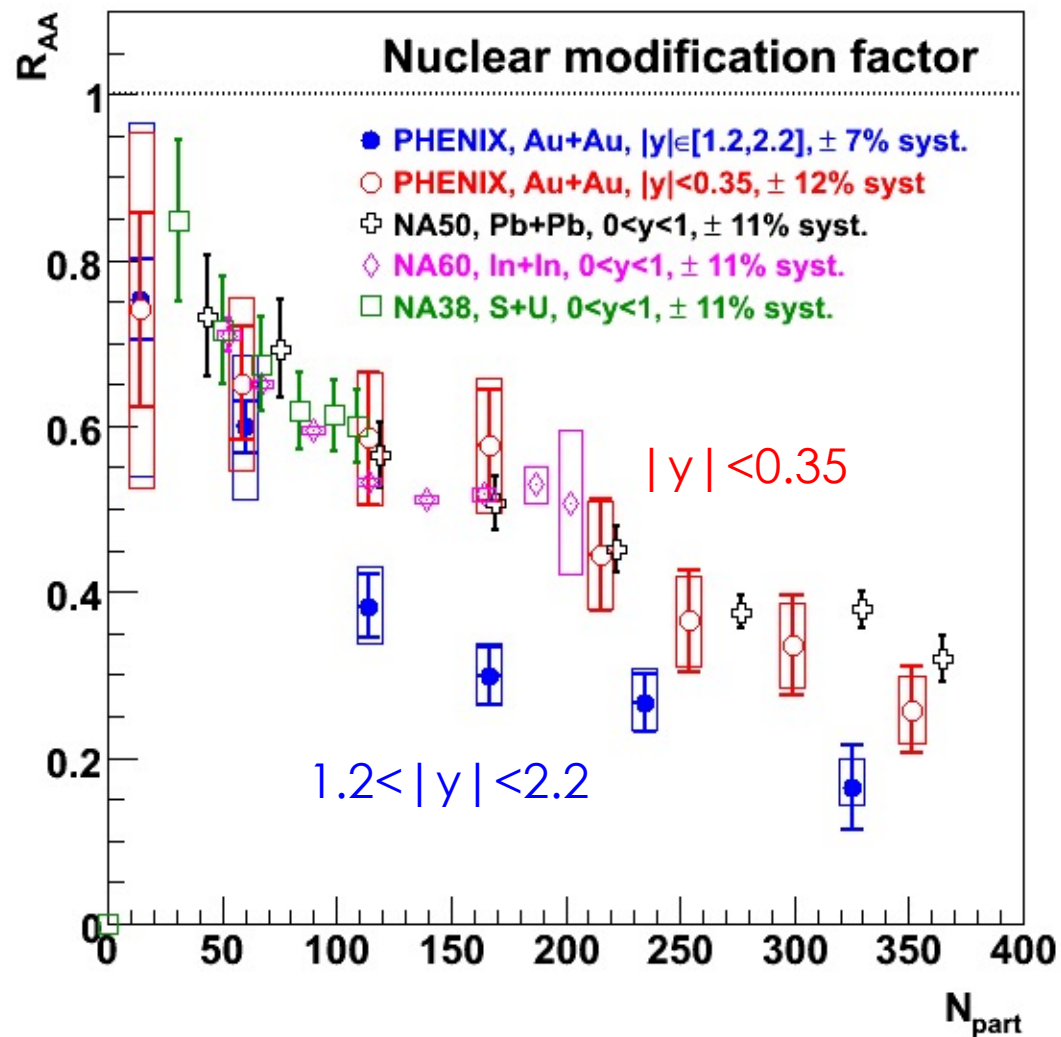
equilibrium limit of each state (Satisfied obviously.)

Include both gluon-dissociation
and NLO (quasifree) process

$$N_\psi^{\text{eq}} = g_c^2 N^{\text{eq}}$$

RHICでのJ/ψ R_{AA}

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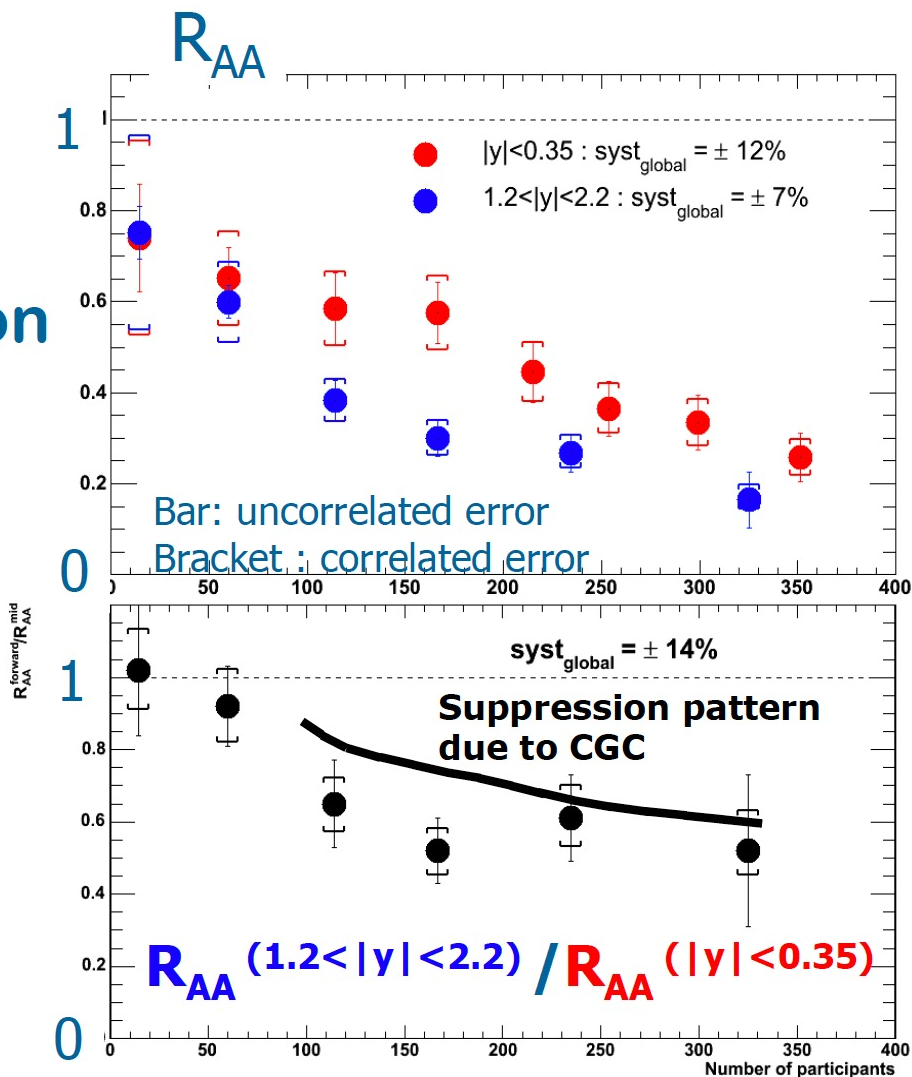
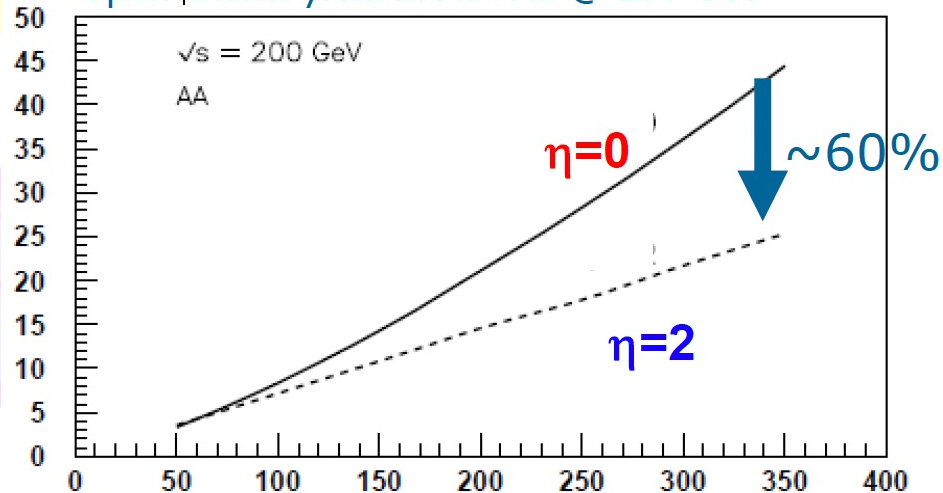
RHICでのJ/ψ R_{AA}

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- Stronger suppression at forward rapidity.
 - Gluon saturation (CGC)?
 - Leading to suppression of charm production

K. L. Tuchin hep-ph/0402298

Open charm yield in Au+Au @ 200 GeV



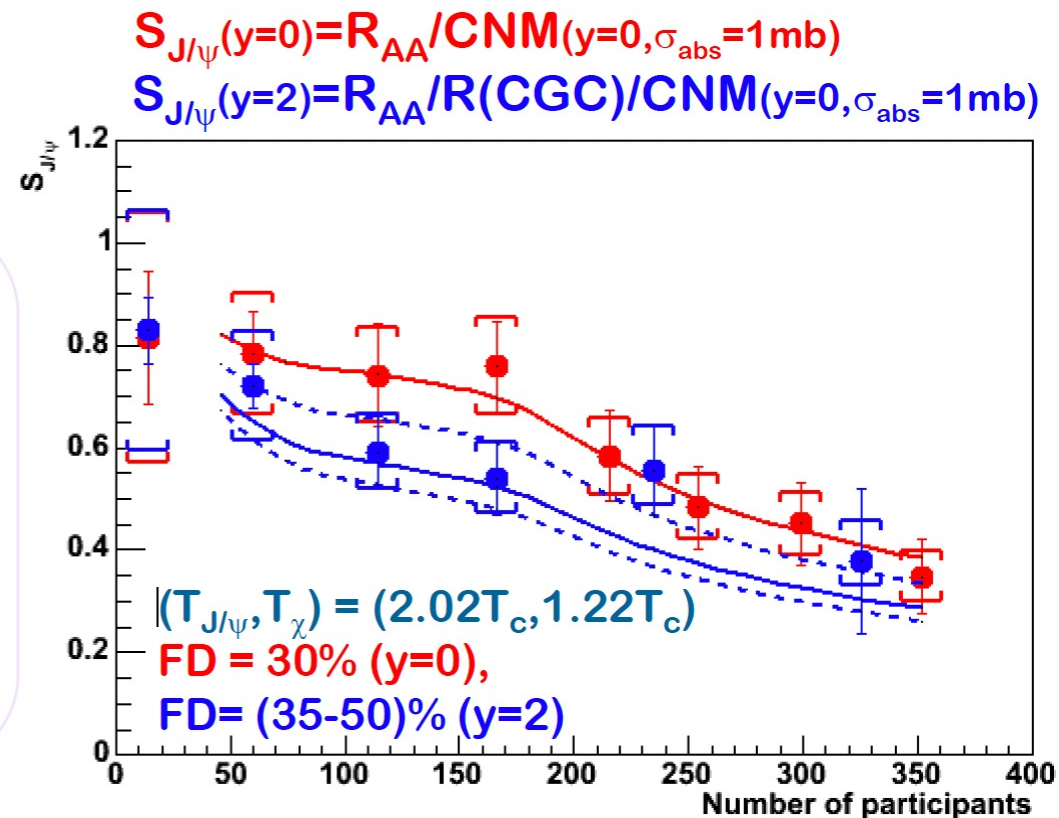
RHICでのJ/ψ R_{AA}

39

- Experimental $S_{J/\psi}^{\text{tot}}(y=2)$
 - CNM at $y=0$ & CGC suppression ($y=2/y=0$)

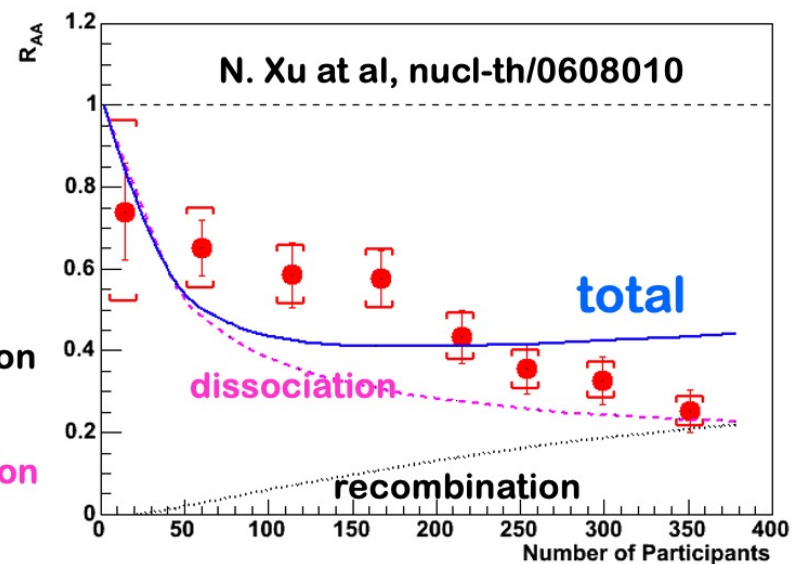
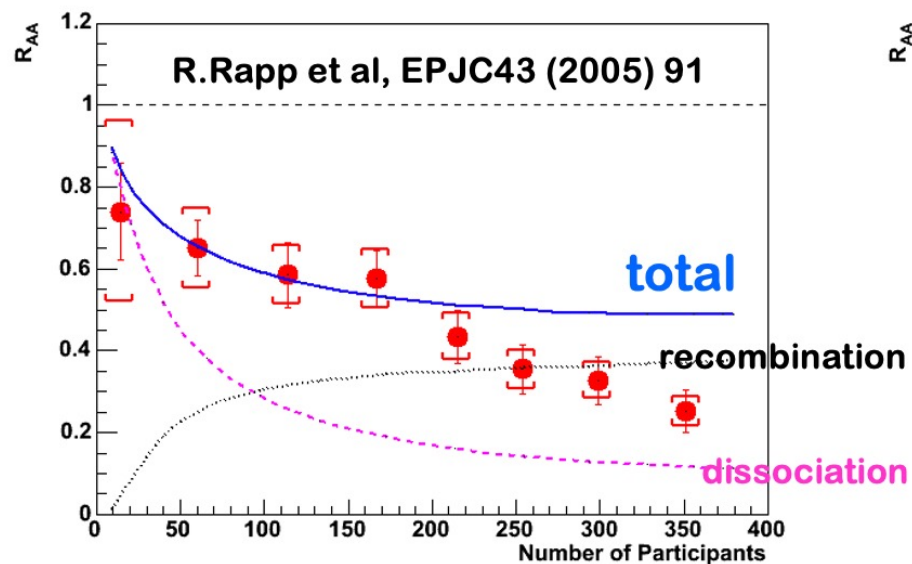
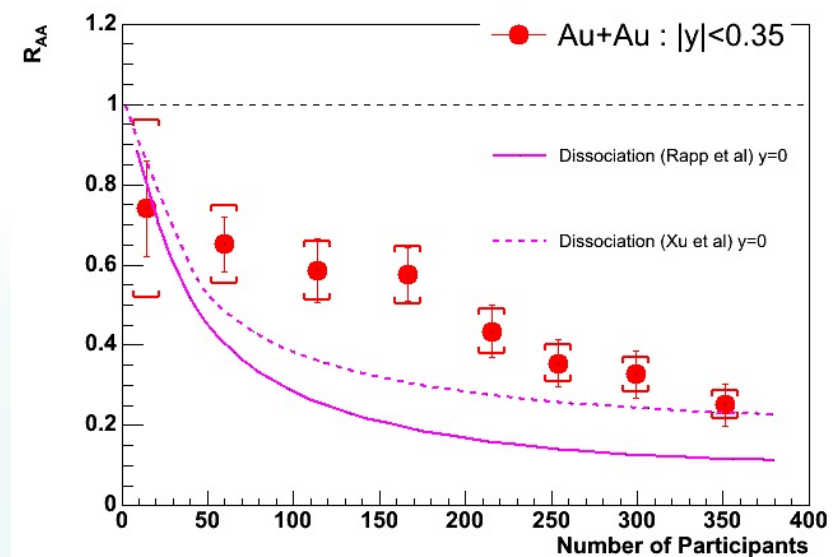
- Model $S_{J/\psi}^{\text{tot}}(y=2)$
 - Hydro at $y=2$

- Need larger feed-down fraction at $y=2$.
 - Onset of suppression at $N_{\text{part}} \sim 240$?
 - $2.02T_c$ is achieved at $N_{\text{part}} \sim 240$ at $y=2$?
- Further analysis is on going.



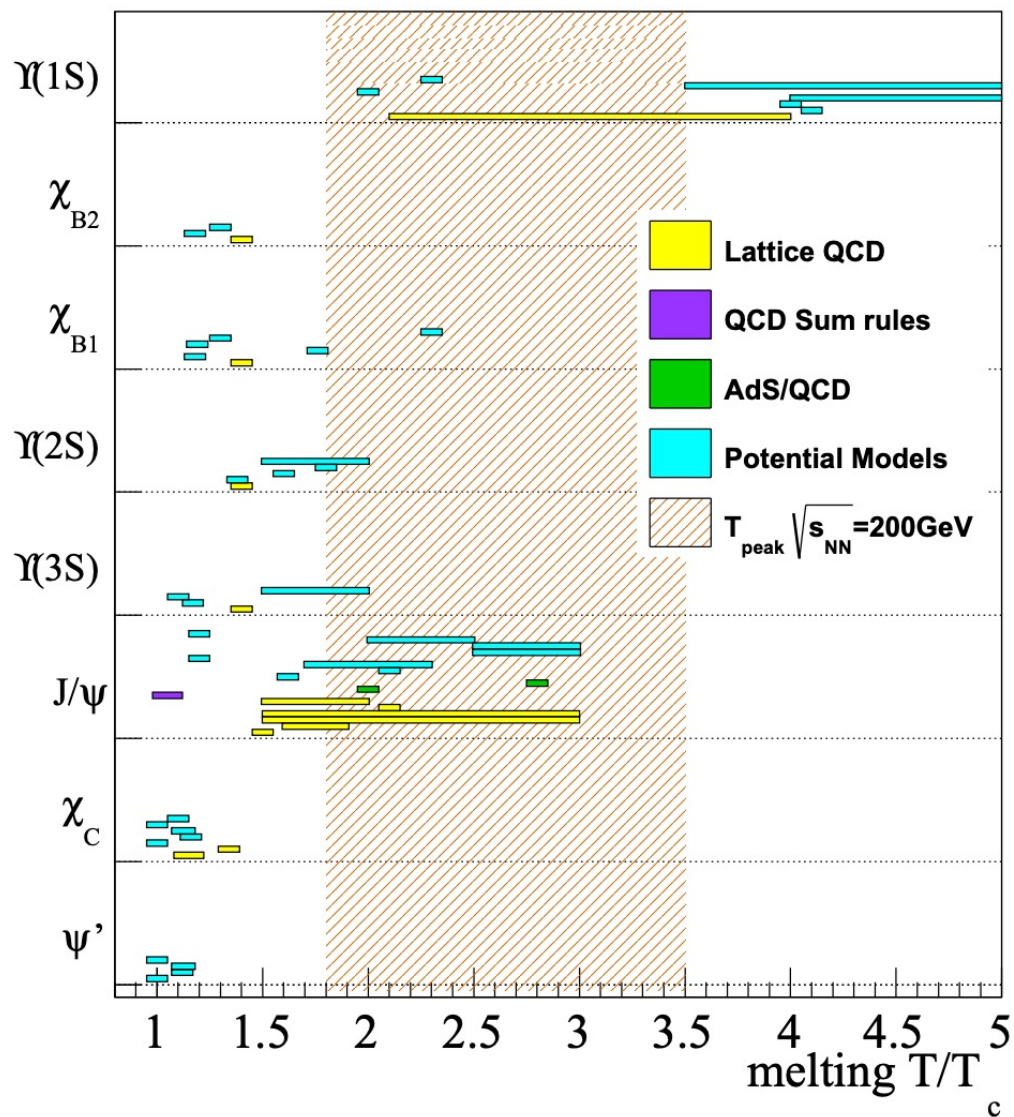
RHICでのJ/ψ R_{AA}

40



溶解温度の現状

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[PhysRevC.91.024913](#)