Study of Hadron Correlations and Interactions Using a Dynamical Model

SOPHIA U

SOPHIA HADRON PHYSICS GROUP

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References

- KK, Ph.D. Thesis, Sophia University (2024)
  https://digital-archives.sophia.ac.jp/repository/view/repository/20243600403
- KK, T. Hirano, EPJ Web Conf. **316**, 03009 (2025)

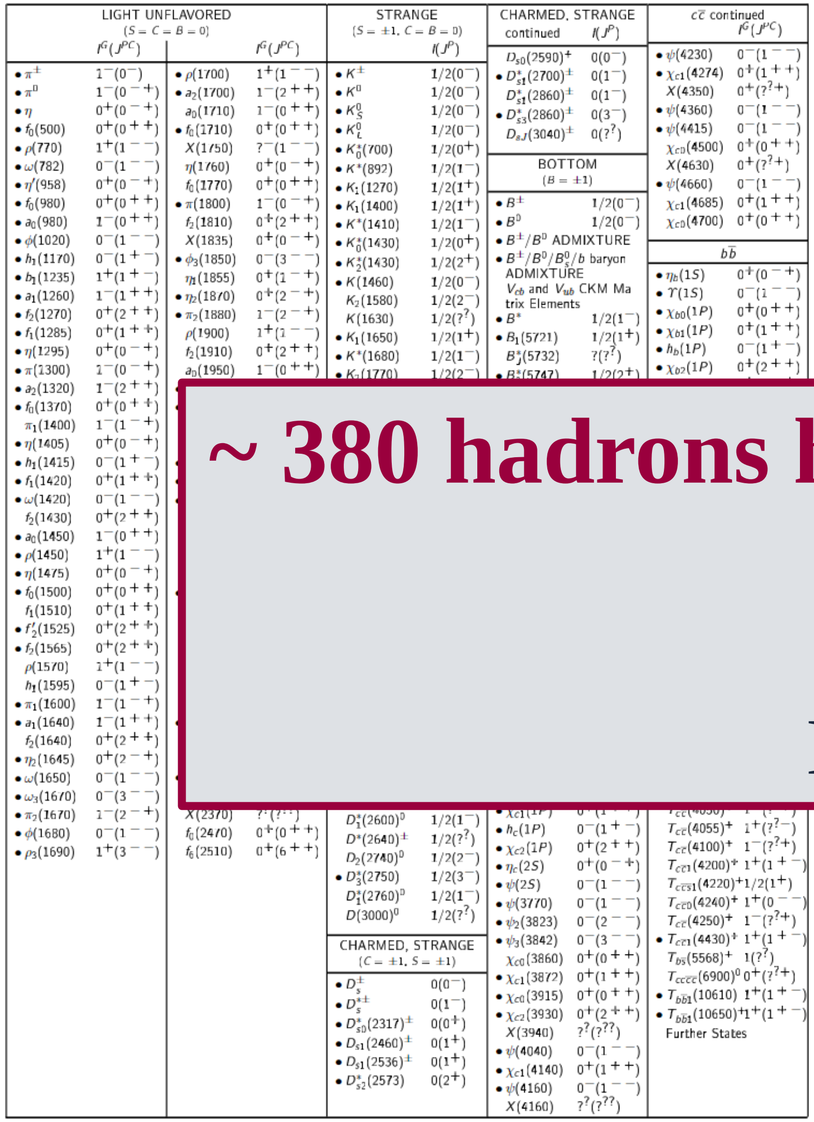
Contents

- Introduction
- Basics of Femtoscopy
- pϕ Femtoscopy

Contents

Various Hadrons

S. Navas et al. (Particle Data Group), Phys. Rev. D **110**, 030001 (2024)



LIGHT UNFLAVORED ($S=C=B=0$)	$I^G(J^{PC})$		$I^G(J^{PC})$	STRANGE ($S=\pm1, C=B=0$)	$I(J^P)$	CHARMED, STRANGE continued	$I(J^P)$	$c\bar{c}$ continued	$I^G(J^{PC})$
• $\pi^{\pm}$	$1^-(0^-)$	• $\rho(1700)$	$1^+(1^{--})$	• $K^{\pm}$	$1/2(0^-)$	$D_{s0}(2590)^+$	$0(0^-)$	• $\psi(4230)$	$0^-(1^{--})$
• π^0	$1^-(0^{-+})$	• $a_2(1700)$	$1^-(2^{++})$	• K^0	$1/2(0^-)$	• $D^*_{s1}(2700)^{\pm}$	$0(1^-)$	• $\chi_{c1}(4274)$	$0^+(1^{++})$
• η	$0^+(0^{-+})$	$a_0(1710)$	$1^-(0^{++})$	• K^0_S	$1/2(0^-)$	$D^*_{s1}(2860)^{\pm}$	$0(1^-)$	$X(4350)$	$0^+(?^{?+})$
• $f_0(500)$	$0^+(0^{++})$	• $f_0(1710)$	$0^+(0^{++})$	• K^0_L	$1/2(0^-)$	• $D^*_{s3}(2860)^{\pm}$	$0(3^-)$	• $\psi(4360)$	$0^-(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	$X(1750)$	$?^-(1^{--})$	• $K^*_0(700)$	$1/2(0^+)$	$D_{sJ}(3040)^{\pm}$	$0(?^?)$	• $\psi(4415)$	$0^-(1^{--})$
• $\omega(782)$	$0^-(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$	• $K^*(892)$	$1/2(1^-)$	BOTTOM ($B=\pm1$)		$\chi_{c0}(4500)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$f_0(1770)$	$0^+(0^{++})$	• $K_1(1270)$	$1/2(1^+)$	• $B^{\pm}$	$1/2(0^-)$	$X(4630)$	$0^+(?^{?+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$	• $K_1(1400)$	$1/2(1^+)$	• B^0	$1/2(0^-)$	• $\psi(4660)$	$0^-(1^{--})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$	• $K^*(1410)$	$1/2(1^-)$	• $B^{\pm}/B^0$ ADMIXTURE		$\chi_{c1}(4685)$	$0^+(1^{++})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$0^+(0^{-+})$	• $K^*_0(1430)$	$1/2(0^+)$	• $B^{\pm}/B^0/B^0_s/b$ baryon ADMIXTURE		$\chi_{c0}(4700)$	$0^+(0^{++})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$	• $K^*_2(1430)$	$1/2(2^+)$	V_{cb} and V_{ub} CKM Matrix Elements		$b\bar{b}$	
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_1(1855)$	$0^+(1^{-+})$	• $K(1460)$	$1/2(0^-)$	• B^*	$1/2(1^-)$	• $\eta_b(1S)$	$0^+(0^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\eta_2(1870)$	$0^+(2^{-+})$	$K_2(1580)$	$1/2(2^-)$	• $B_1(5721)$	$1/2(1^+)$	• $\Upsilon(1S)$	$0^-(1^{--})$
• $f_2(1270)$	$0^+(2^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$	$K(1630)$	$1/2(?^?)$	$B^*_J(5732)$	$?(?^?)$	• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $f_1(1285)$	$0^+(1^{++})$	$\rho(1900)$	$1^+(1^{--})$	• $K_1(1650)$	$1/2(1^+)$	• $B^*_2(5747)$	$1/2(2^+)$	• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	$f_2(1910)$	$0^+(2^{++})$	• $K^*(1680)$	$1/2(1^-)$			• $h_b(1P)$	$0^-(1^{+-})$
• $\pi(1300)$	$1^-(0^{-+})$	$a_0(1950)$	$1^-(0^{++})$	• $K_2(1770)$	$1/2(2^-)$			• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $a_2(1320)$	$1^-(2^{++})$								
• $f_0(1370)$	$0^+(0^{++})$								
$\pi_1(1400)$	$1^-(1^{-+})$								
• $\eta(1405)$	$0^+(0^{-+})$								
• $h_1(1415)$	$0^-(1^{+-})$								
• $f_1(1420)$	$0^+(1^{++})$								
• $\omega(1420)$	$0^-(1^{--})$								
$f_2(1430)$	$0^+(2^{++})$								
• $a_0(1450)$	$1^-(0^{++})$								
• $\rho(1450)$	$1^+(1^{--})$								
• $\eta(1475)$	$0^+(0^{-+})$								
• $f_0(1500)$	$0^+(0^{++})$								
$f_1(1510)$	$0^+(1^{++})$								
• $f'_2(1525)$	$0^+(2^{++})$								
• $f_2(1565)$	$0^+(2^{++})$								
$\rho(1570)$	$1^+(1^{--})$								
$h_1(1595)$	$0^-(1^{+-})$								
• $\pi_1(1600)$	$1^-(1^{-+})$								
• $a_1(1640)$	$1^-(1^{++})$								
$f_2(1640)$	$0^+(2^{++})$								
• $\eta_2(1645)$	$0^+(2^{-+})$								
• $\omega(1650)$	$0^-(1^{--})$								
• $\omega_3(1670)$	$0^-(3^{--})$	$X(2370)$	$?^?(?^{??})$	$D^*_1(2600)^0$	$1/2(1^-)$	$\chi_{c1}(1P)$		$T_{c\bar{c}}(4050)$	
• $\pi_2(1670)$	$1^-(2^{-+})$	$f_0(2470)$	$0^+(0^{++})$	$D^*(2640)^{\pm}$	$1/2(?^?)$	• $h_c(1P)$	$0^-(1^{+-})$	$T_{c\bar{c}}(4055)^+$	$1^+(?^{?-})$
• $\phi(1680)$	$0^-(1^{--})$	$f_6(2510)$	$0^+(6^{++})$	$D_2(2740)^0$	$1/2(2^-)$	• $\chi_{c2}(1P)$	$0^+(2^{++})$	$T_{c\bar{c}}(4100)^+$	$1^-(?^{?+})$
• $\rho_3(1690)$	$1^+(3^{--})$			• $D^*_3(2750)$	$1/2(3^-)$	• $\eta_c(2S)$	$0^+(0^{-+})$	$T_{c\bar{c}1}(4200)^+$	$1^+(1^{+-})$
				$D^*_1(2760)^0$	$1/2(1^-)$	• $\psi(2S)$	$0^-(1^{--})$	$T_{c\bar{c}s1}(4220)^+$	$1/2(1^+)$
				$D(3000)^0$	$1/2(?^?)$	• $\psi(3770)$	$0^-(1^{--})$	$T_{c\bar{c}0}(4240)^+$	$1^+(0^{--})$
				CHARMED, STRANGE ($C=\pm1, S=\pm1$)		• $\psi_2(3823)$	$0^-(2^{--})$	$T_{c\bar{c}}(4250)^+$	$1^-(?^{?+})$
				• $D^{\pm}_s$	$0(0^-)$	• $\psi_3(3842)$	$0^-(3^{--})$	• $T_{c\bar{c}1}(4430)^+$	$1^+(1^{+-})$
				• $D^{*\pm}_s$	$0(1^-)$	$\chi_{c0}(3860)$	$0^+(0^{++})$	$T_{b\bar{s}}(5568)^+$	$1(?^?)$
				• $D^*_{s0}(2317)^{\pm}$	$0(0^+)$	• $\chi_{c1}(3872)$	$0^+(1^{++})$	$T_{c\bar{c}c\bar{c}}(6900)^0$	$0^+(?^{?+})$
				• $D_{s1}(2460)^{\pm}$	$0(1^+)$	• $\chi_{c0}(3915)$	$0^+(0^{++})$	• $T_{b\bar{b}1}(10610)$	$1^+(1^{+-})$
				• $D_{s1}(2536)^{\pm}$	$0(1^+)$	• $\chi_{c2}(3930)$	$0^+(2^{++})$	• $T_{b\bar{b}1}(10650)^+$	$1^+(1^{+-})$
				• $D^*_{s2}(2573)$	$0(2^+)$	$X(3940)$	$?^?(?^{??})$	Further States	
						• $\psi(4040)$	$0^-(1^{--})$		
						• $\chi_{c1}(4140)$	$0^+(1^{++})$		
						• $\psi(4160)$	$0^-(1^{--})$		
						$X(4160)$	$?^?(?^{??})$		

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Λ_c^+	$1/2^+$	****	Λ_b^0	$1/2^+$	***
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	****	Σ^0	$1/2^+$	****	$\Lambda_c(2595)^+$	$1/2^-$	***	$\Lambda_b(5912)^0$	$1/2^-$	***
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Lambda_c(2625)^+$	$3/2^-$	***	$\Lambda_b(5920)^0$	$3/2^-$	***
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Lambda_c(2765)^+$		*	$\Lambda_b(6070)^0$	$1/2^+$	***
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1580)$	$3/2^-$	*	$\Lambda_c(2860)^+$	$3/2^+$	***	$\Lambda_b(6146)^0$	$3/2^+$	***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	***	$\Sigma(1620)$	$1/2^-$	*	$\Lambda_c(2880)^+$	$5/2^+$	***	$\Lambda_b(6152)^0$	$5/2^+$	***
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1660)$	$1/2^+$	***	$\Lambda_c(2910)^+$		*	Σ_b	$1/2^+$	***
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1670)$	$3/2^-$	****	$\Lambda_c(2940)^+$	$3/2^-$	***	Σ_b^*	$3/2^+$	***
$N(1700)$	$3/2^-$	***	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1750)$	$1/2^-$	***	$\Sigma_c(2455)$	$1/2^+$	****	$\Sigma_b(6097)^+$		***
$N(1710)$	$1/2^+$	****	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1775)$	$5/2^-$	****	$\Sigma_c(2520)$	$3/2^+$	***	$\Sigma_b(6097)^-$		***
$N(1720)$	$3/2^+$	****	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1780)$	$3/2^+$	*	$\Sigma_c(2800)$		***	Ξ_b^-	$1/2^+$	***
$N(1860)$	$5/2^+$	**	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1880)$	$1/2^+$	**	Ξ_c^+	$1/2^+$	***	Ξ_b^0	$1/2^+$	***
$N(1875)$	$3/2^-$	***	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1900)$	$1/2^-$	**	Ξ_c^0	$1/2^+$	****	$\Xi'_b(5935)^-$	$1/2^+$	***
$N(1880)$	$1/2^+$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1910)$	$3/2^-$	***	$\Xi_c'^+$	$1/2^+$	***	$\Xi_b(5945)^0$	$3/2^+$	***
$N(1895)$	$1/2^-$	****	$\Delta(2200)$	$7/2^-$	***	$\Sigma(1915)$	$5/2^+$	****	$\Xi_c'^0$	$1/2^+$	***	$\Xi_b(5955)^-$	$3/2^+$	***
$N(1900)$	$3/2^+$	****	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1940)$	$3/2^+$	*	$\Xi_c(2645)$	$3/2^+$	***	$\Xi_b(6087)^0$	$3/2^-$	***
											***	$\Xi_b(6095)^0$	$3/2^-$	***
											***	$\Xi_b(6100)^-$	$3/2^-$	***
											*	$\Xi_b(6227)^-$		***
											**	$\Xi_b(6227)^0$		***
											**	$\Xi_b(6327)^0$		***
											***	$\Xi_b(6333)^0$		***
											***	Ω_b^-	$1/2^+$	***
											***	$\Omega_b(6316)^-$		***
											*	$\Omega_b(6330)^-$		***
											***	$\Omega_b(6340)^-$		***
											***	$\Omega_b(6350)^-$		***
											***			
											***	$P_{c\bar{c}}(4312)^+$		*
											***	$P_{c\bar{c}s}(4338)^0$	$1/2^-$	*
											***	$P_{c\bar{c}}(4380)^+$		*
											***	$P_{c\bar{c}}(4440)^+$		*
											***	$P_{c\bar{c}}(4457)^+$		*
											***	$P_{c\bar{c}s}(4459)^0$		*
			$\Lambda(1890)$	$3/2^+$	****	$\Xi(1950)$		***	Ξ_{cc}^+		*			
			$\Lambda(2000)$	$1/2^-$	*	$\Xi(2030)$	$\geq\frac{5}{2}^?$	***	Ξ_{cc}^{++}		***			
			$\Lambda(2050)$	$3/2^-$	*	$\Xi(2120)$		*						
			$\Lambda(2070)$	$3/2^+$	*	$\Xi(2250)$		**						
			$\Lambda(2080)$	$5/2^-$	*	$\Xi(2370)$		**						
			$\Lambda(2085)$	$7/2^+$	**	$\Xi(2500)$		*						
			$\Lambda(2100)$	$7/2^-$	****									
			$\Lambda(2110)$	$5/2^+$	***	Ω^-	$3/2^+$	****						
			$\Lambda(2325)$	$3/2^-$	*	$\Omega(2012)^-$	$?^-$	***						
			$\Lambda(2350)$	$9/2^+$	***	$\Omega(2250)^-$		***						
			$\Lambda(2585)$		*	$\Omega(2380)^-$		**						
						$\Omega(2470)^-$		**						

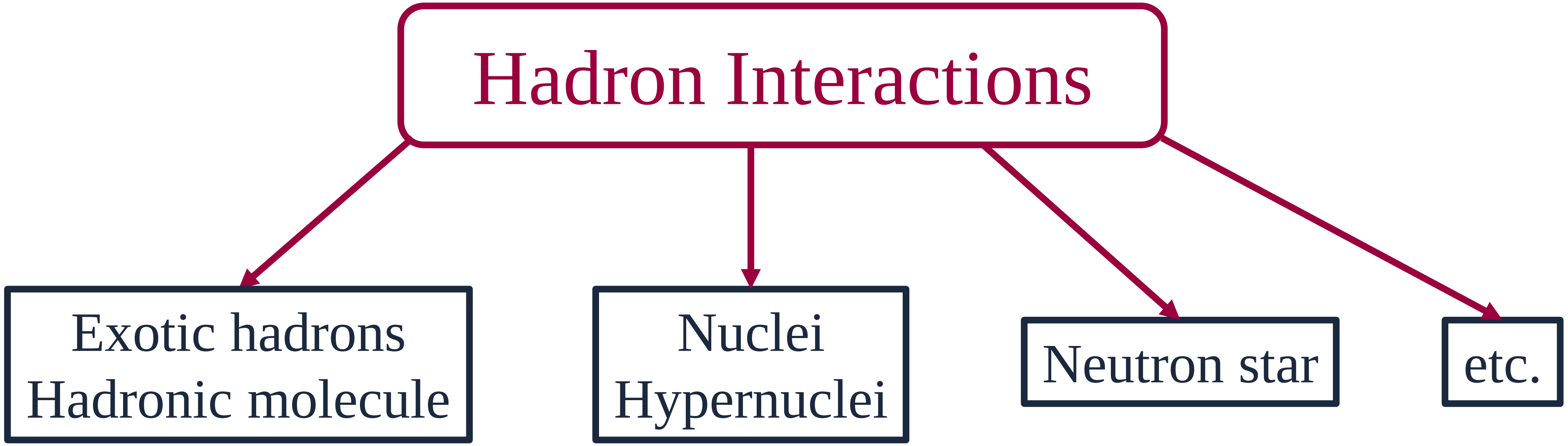
~ 380 hadrons have been discovered so far

Mesons

Baryons

Exotic hadrons

Fundamental inputs to various systems



Precise understanding of low-energy hadron interactions

Attraction or Repulsion?   Strength?   Bound state?

Interdisciplinary importance

Hadron Interactions from Experiments

■ Scattering experiments

Phase shift

- Target particles must be stable
- Beam particles must fly ~cm

NN, πN, K^{+}N, K^{-}N, ΣN, etc.

■ Hypernuclei, Exotic nuclei

Binding energy

- Medium effects

Λ, Σ, Ξ, ΛΛ hypernuclei, etc.

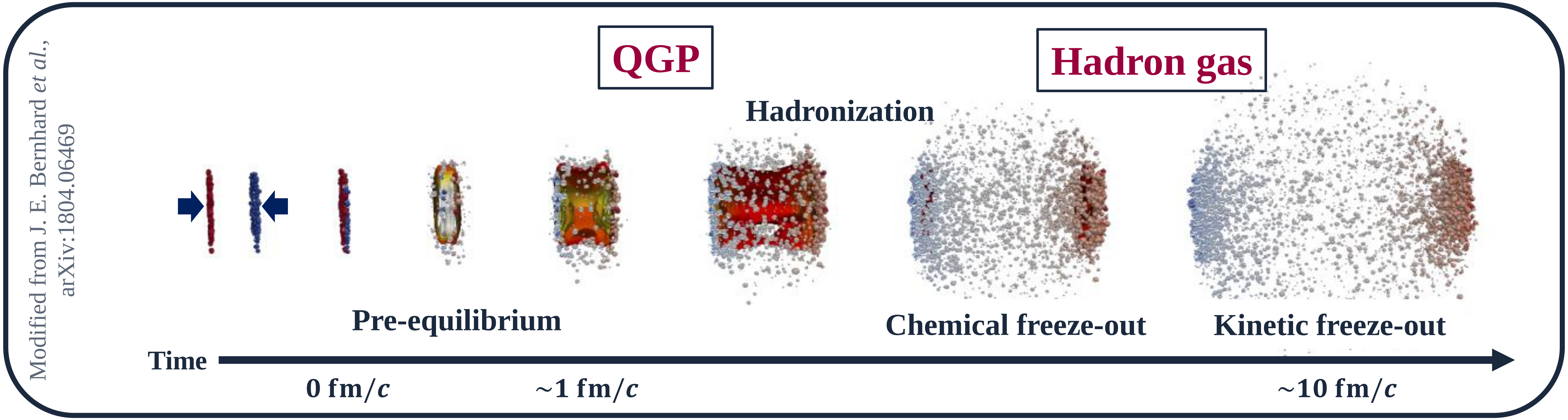
■ Femtoscopy

Correlation function of hadron pairs in **high-energy nuclear collisions**

In principle, applicable to all identifiable hadrons

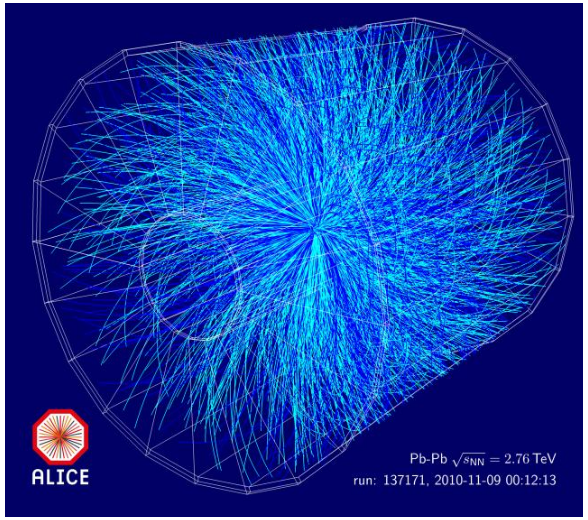
Low-energy hadron interactions from high-energy nuclear collisions!

High-Energy Heavy-Ion Collisions



Modified from J. E. Bernhard *et al.*, arXiv:1804.06469

Observables: **Momentum dist. of residual hadrons**



https://home.cern/resources/image/experiments/alice-images-gallery

Success of **hydrodynamics-based models**

"Perfect fluidity of QGP" (2005)

https://www.bnl.gov/newsroom/news.php?a=110303

In high-multiplicity pp & pA collisions

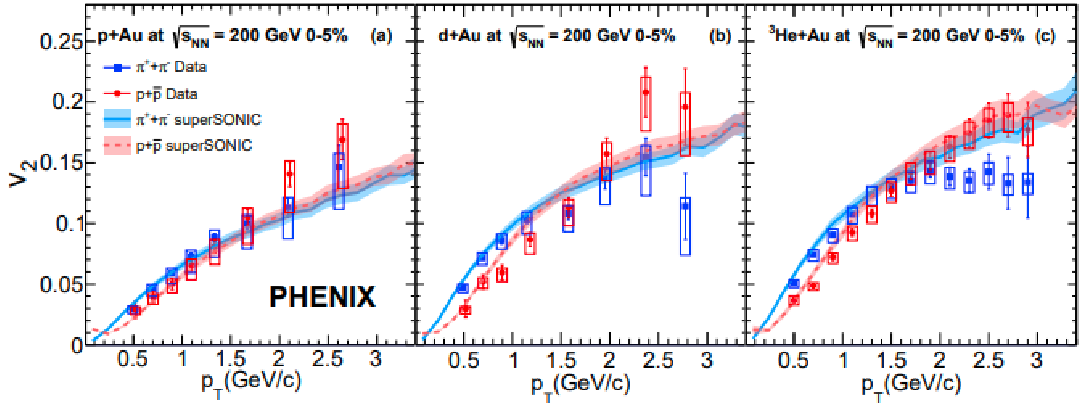
From experiments

- Hydro-like collectivity
- Thermal strangeness production

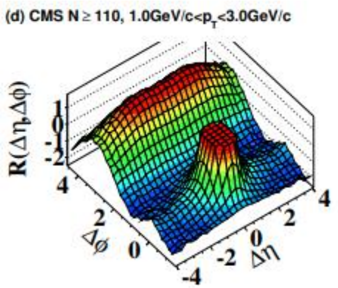
Hydrodynamics-based models

based on the core-corona picture

- **EPOS4** K. Werner, PRC **108**, 064903 (2023)
- **DCCI2** Y. Kanakubo *et al.*, PRC **105**, 024905 (2022)

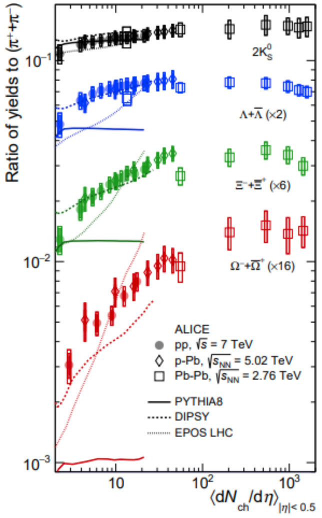


Flow harmonics PHENIX, PRC **97**, 064904 (2018)



Long range correlation

CMS, JHEP **09**, 091 (2010)



Strangeness enhancement

ALICE, Nature Phys. **13**, 535 (2017)

AA collisions and high-multiplicity pp, pA collisions

- **A diverse “hadron factory”**

  Abundant hadrons are generated almost at the same time

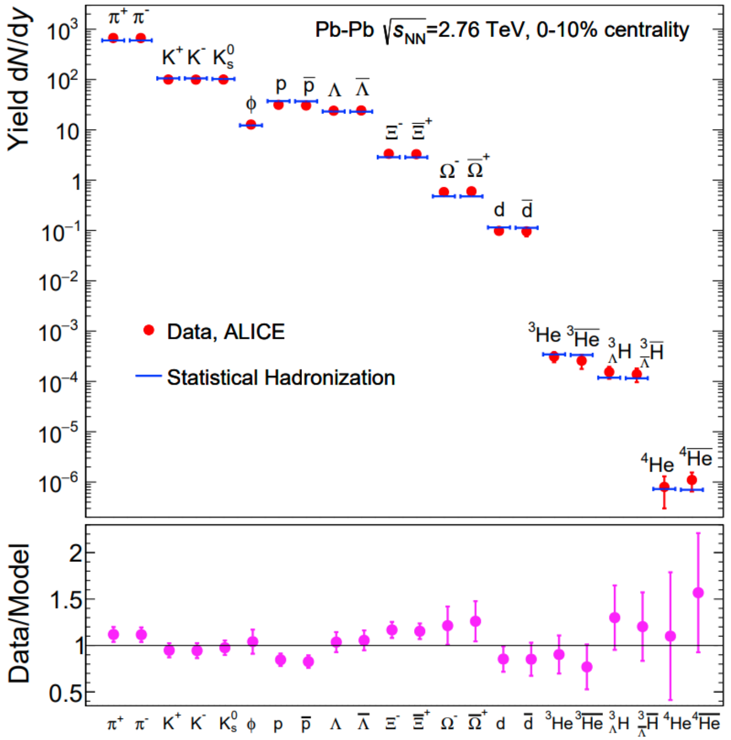
- **Chaotic hadron emission**

  Statistical models can well describe hadron yield ratio

- **Various collision systems, huge statistics**

- **Near 4π detector, precise pID, etc.**

A. Andronic *et al.*,
Nature **561**, 321 (2018)



Excellent testing ground for hadron physics!

Contents

Correlation Function (CF)

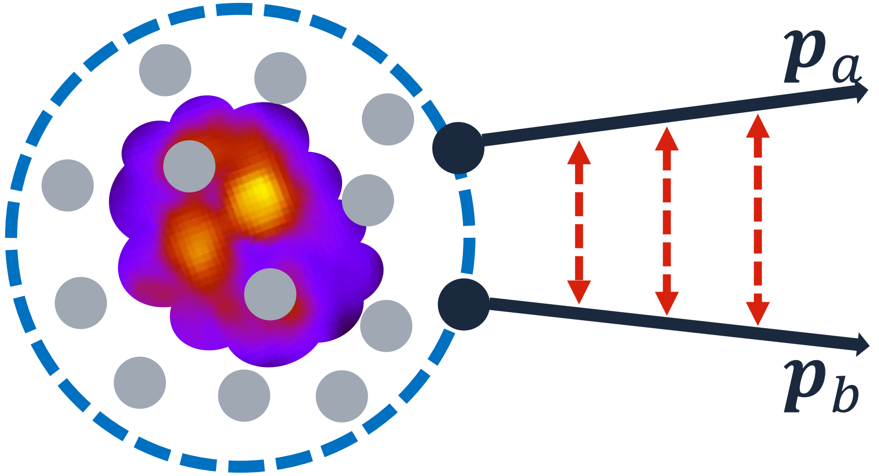
$$C(\boldsymbol{q}, \boldsymbol{P}) \coloneqq \frac{N_{\text{pair}}(\boldsymbol{p}_a, \boldsymbol{p}_b)}{N_a(\boldsymbol{p}_a)\, N_b(\boldsymbol{p}_b)}$$

$$P^\mu = p_a^\mu + p_b^\mu$$

$$q^\mu = \frac{1}{2}\left[p_a^\mu - p_b^\mu - \frac{(p_a - p_b)\cdot P}{P^2} P^\mu\right]$$

$N_{\text{pair}}(\boldsymbol{p}_a, \boldsymbol{p}_b)$: Two-particle momentum dist.

$N_a(\boldsymbol{p}_a)$: One-particle momentum dist.



Hadron CF provides insights into

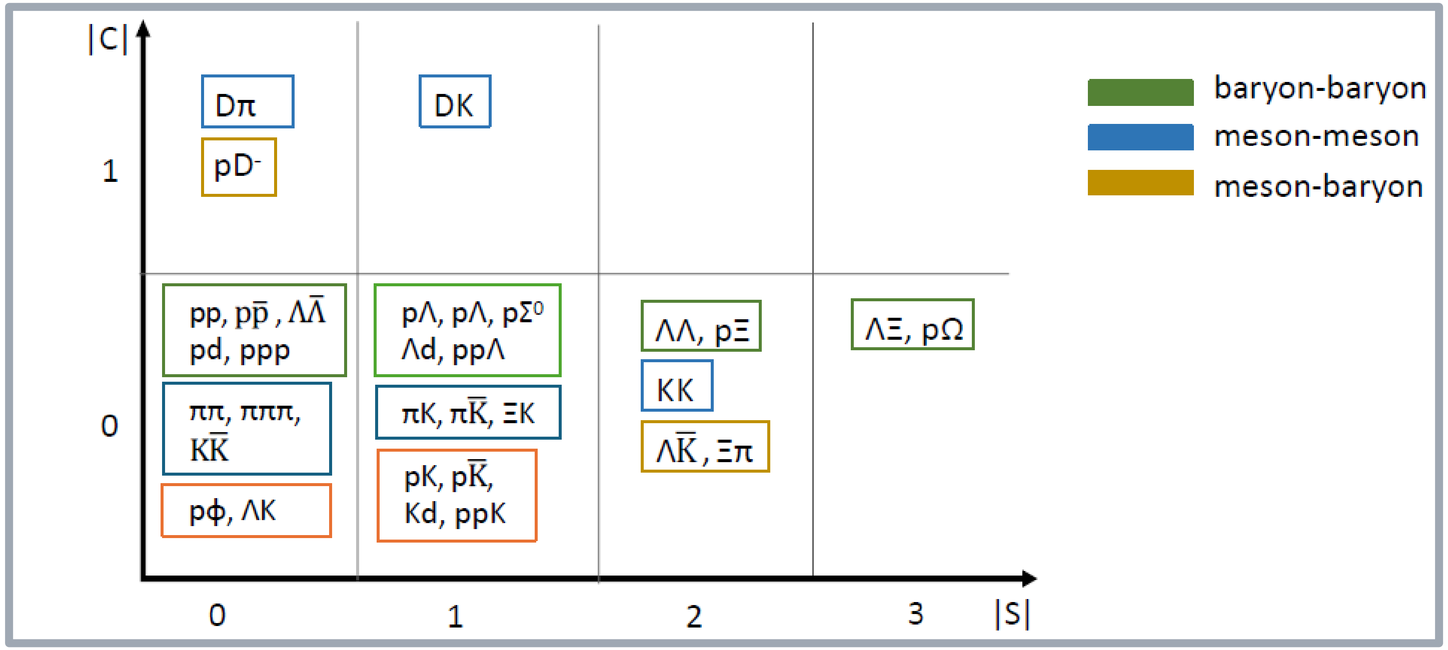
- **Space-time structure of matter**
- **Final state interactions, FSI**

Typical behavior at low-q, (for non-identical pair)

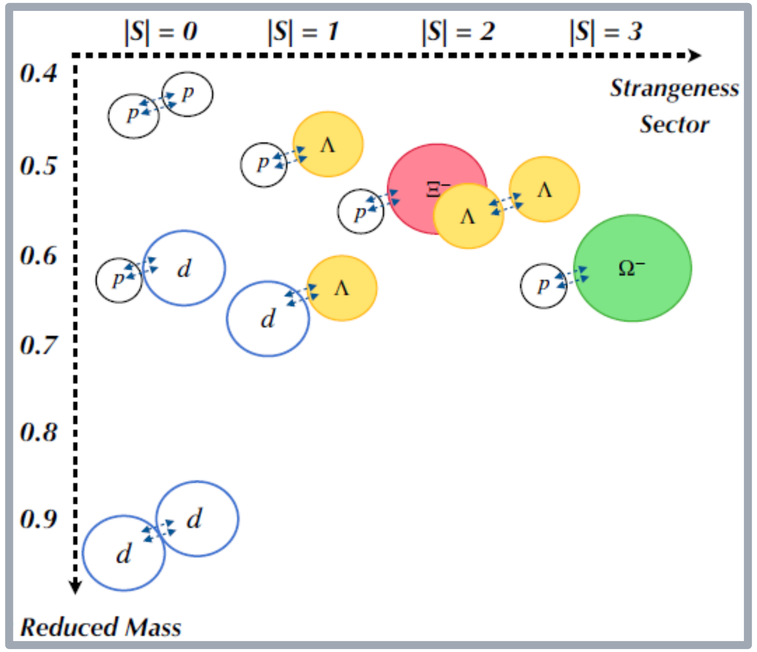
Attraction $\rightarrow C(q) > 1$ **Repulsion** $\rightarrow C(q) < 1$

Various Correlation Measurements

ALICE R. Del Grande, talk at SQM 2024



STAR K. Mi, talk at WHBM 2025



- ALICE: CFs mostly in **high-multiplicity pp collisions**
- STAR: CFs in **AA collisions**

Also, many other collaborations…

Koonin-Pratt Formula

S. E. Koonin, PLB **70**, 43 (1977); S. Pratt, PRL **53**, 1219 (1984)

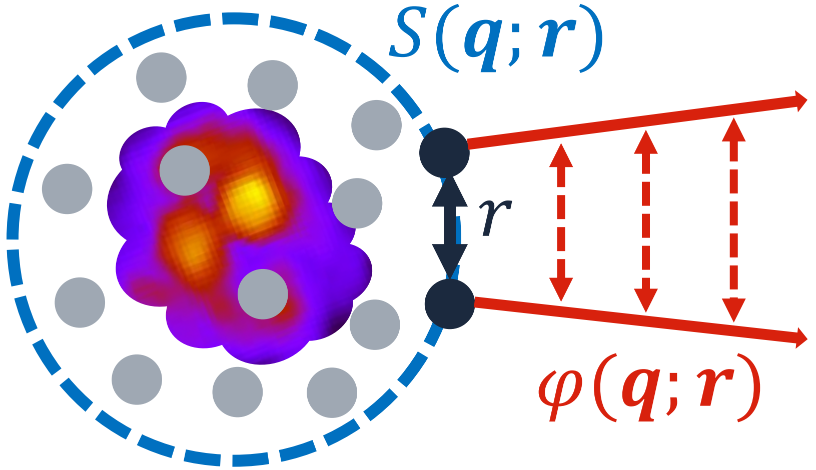
Assumptions
- **Chaotic source**: Pairs are emitted independently
- **Isolated system**: Negligible effects of surrounding hadron gas
- Smoothness approximation
- (Equal time approximation)

$$C(\boldsymbol{q},\boldsymbol{P}) = \frac{\int d^4x_a d^4x_b\, S_a(\boldsymbol{p}_a;x_a)S_b(\boldsymbol{p}_b;x_b)|\varphi(\boldsymbol{q};\boldsymbol{r})|^2}{\int d^4x_a\, S_a(\boldsymbol{p}_a;x_a)\int d^4x_b\, S_b(\boldsymbol{p}_b;x_b)}$$

Pair Rest Frame ($\boldsymbol{P}=0$)

Integrate out CM

$$C(\boldsymbol{q}) = \int d^3r\ S(\boldsymbol{q};\boldsymbol{r})\ |\varphi(\boldsymbol{q};\boldsymbol{r})|^2$$



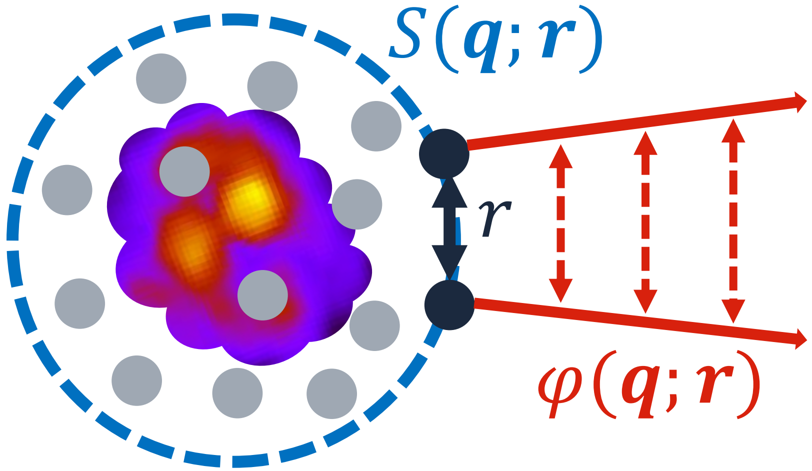
CF ⇐ **Source Func.** & **Relative WF**

"Femtoscopy" (Femtometer + Spectroscopy)

Based on Koonin-Pratt formula

$$C(\boldsymbol{q}) = \int d^3r \; S(\boldsymbol{q};\boldsymbol{r}) \; |\varphi(\boldsymbol{q};\boldsymbol{r})|^2$$

CF ⇐ **SF** & **Relative WF**



From **measured** CF,

- **Input:** WF
  → **Output:** SF a.k.a. "**HBT-GGLP Interferometry**"
- **Input:** SF
  → **Output:** Hadron interaction

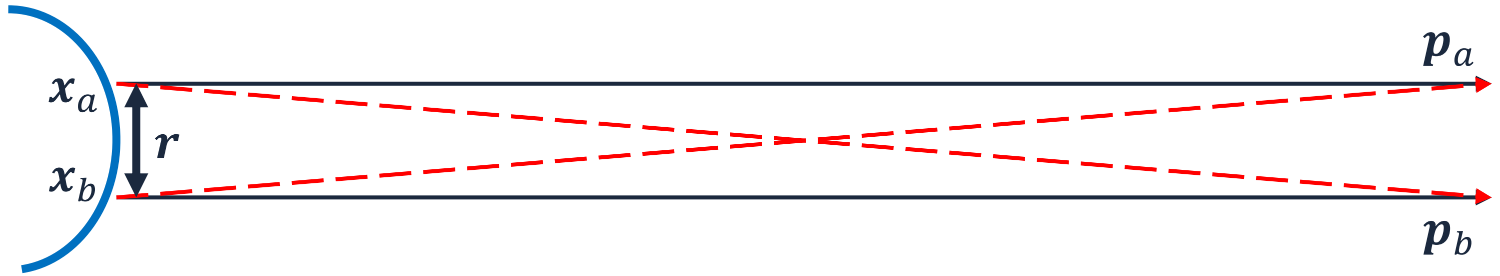
HBT-GGLP Interferometry

Two-photon momentum correlation from Sirius

R. Hanbury Brown and R. Q. Twiss, Nature **178**, 1046 (1956)

Two-pion momentum correlation in $p\bar{p}$ collision

G. Goldhaber, S. Goldhaber, W.-Y. Lee, and A. Pais, Phys. Rev. **120**, 300 (1960)



$\boldsymbol{P} = \boldsymbol{p}_a + \boldsymbol{p}_b,\ \boldsymbol{q} = \frac{1}{2}(\boldsymbol{p}_a - \boldsymbol{p}_b)$

$\boldsymbol{R} = \frac{1}{2}(\boldsymbol{x}_a + \boldsymbol{x}_b),\ \boldsymbol{r} = \boldsymbol{x}_a - \boldsymbol{x}_b$

$\boldsymbol{\Sigma}$: Variance-covariance matrix

Size & shape of SF

SF: **Assuming Gaussian one-particle SF**

$$S_i(\boldsymbol{x}) \propto \exp\left(-\frac{1}{2}\boldsymbol{x}^\top\boldsymbol{\Sigma}^{-1}\boldsymbol{x}\right) \Rightarrow S_a(\boldsymbol{x}_a)S_b(\boldsymbol{x}_b) \propto \exp(-\boldsymbol{R}^\top\boldsymbol{\Sigma}^{-1}\boldsymbol{R})\exp\left(-\frac{1}{4}\boldsymbol{r}^\top\boldsymbol{\Sigma}^{-1}\boldsymbol{r}\right)$$

WF: **Neglecting interaction, Symmetrization**

$$\Psi(\boldsymbol{p}_a, \boldsymbol{p}_b, \boldsymbol{x}_a, \boldsymbol{x}_b) \propto \frac{1}{\sqrt{2}}\left(e^{i\boldsymbol{p}_a\cdot\boldsymbol{x}_a + i\boldsymbol{p}_b\cdot\boldsymbol{x}_b} + e^{i\boldsymbol{p}_a\cdot\boldsymbol{x}_b + i\boldsymbol{p}_b\cdot\boldsymbol{x}_a}\right) = e^{i\boldsymbol{P}\cdot\boldsymbol{X}}\sqrt{2}\cos(\boldsymbol{q}\cdot\boldsymbol{r})$$

CF: $$C(\boldsymbol{q}) = \int d^3r \frac{1}{(4\pi)^{2/3}|\boldsymbol{\Sigma}|^{1/2}}\exp\left(-\frac{1}{4}\boldsymbol{r}^\top\boldsymbol{\Sigma}^{-1}\boldsymbol{r}\right)\left|\sqrt{2}\cos(\boldsymbol{q}\cdot\boldsymbol{r})\right|^2 = 1 + \exp(-4\boldsymbol{q}^\top\boldsymbol{\Sigma}\boldsymbol{q})$$

Spin-Averaged Correlation Function

Experimental CF = Weighted average of CFs in each ${}^{2S+1}L_J$ channel

$$|\varphi|^2 = \sum_{\text{states}(S,L,J)} \omega_{(S,L,J)} \left|\varphi^{(S,L,J)}\right|^2$$

$$\omega_{(S,L,J)} = \frac{2S+1}{(2s_a+1)(2s_b+1)} \frac{2J+1}{(2L+1)(2S+1)}$$

Koonin-Pratt formula

Spin-independent SF

$$C^{\text{tot}}(q) = \sum_{\text{states}(S,L,J)} \omega_{(S,L,J)}\, C^{(S,L,J)}(q)$$

⇒ **Comparable with exp. CF**

Wave Function with Final State Interaction

Focusing on low-q regions under assumptions of chaotic source and isolated system

→ Time-independent Schrödinger eq. with central force

Partial-wave expansion

$$\varphi(\boldsymbol{q};\boldsymbol{r}) = \sum_{l=0}^{\infty}(2l+1)i^l\varphi_l(q;r)P_l(\cos\theta)$$

φ_l: Radial WF
$P_l(\cos\theta)$: Legendre Polynomials

For each ${}^{2S+1}L_J$ channel,

$$\left[-\frac{\partial^2}{\partial r^2}-\frac{2}{r}\frac{\partial}{\partial r}+\frac{l(l+1)}{r^2}+2\mu V(r)\right]\varphi_l(q;r) = q^2\varphi_l(q;r)$$

$$\mu = \frac{m_a m_b}{m_a+m_b}$$

Rewriting Koonin-Pratt Formula

For non-identical pair w/o Coulomb interaction,

Spherical SF
$S(q;r)$

Only *s*-wave scattering

$$\varphi(\boldsymbol{q};\boldsymbol{r}) = \exp(i\boldsymbol{q}\cdot\boldsymbol{r}) - j_0(qr) + \varphi_0(q;r)$$

Plane wave | **Plane wave (*s*-wave)** | **WF w/ FSI (*s*-wave)**

$$C(\boldsymbol{q}) = \int d^3r\, S(\boldsymbol{q};\boldsymbol{r})\, |\varphi(\boldsymbol{q};\boldsymbol{r})|^2$$

$$= 1 + \int_0^\infty dr\, 4\pi r^2 S(q;r)\, [|\varphi_0(q;r)|^2 - |j_0(qr)|^2]$$

SF w/ Jacobian

WF Change Increase/Decrease of WF by **FSI**

Cf. Generalization to higher partial waves in K. Murase and T. Hyodo, JSPC **3**, 100017 (2025)

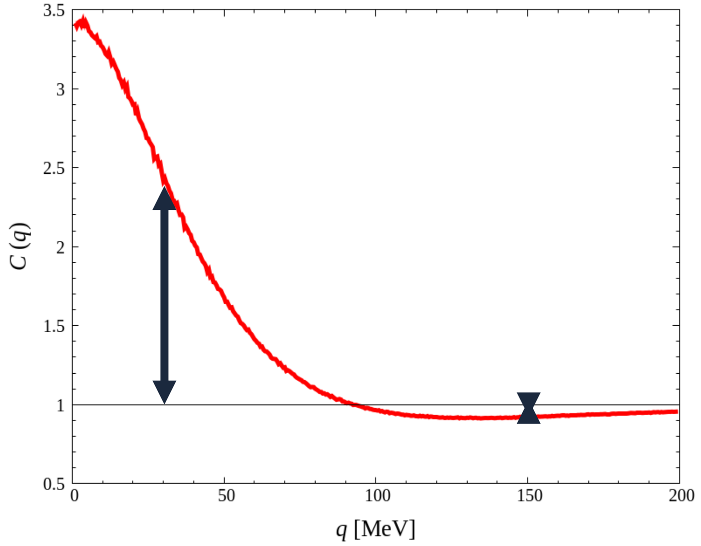
Interpretation of Correlation Function

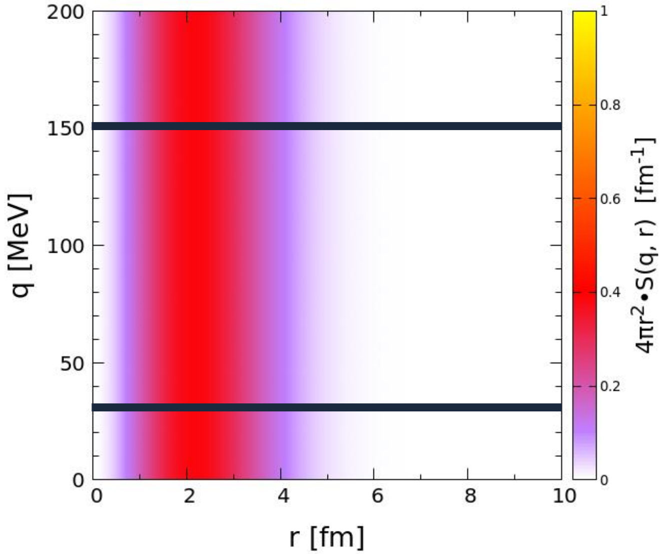
$$C(q) = 1 + \int_0^\infty dr \; 4\pi r^2 S(q;r) \; [|\varphi_0(q;r)|^2 - |j_0(qr)|^2]$$

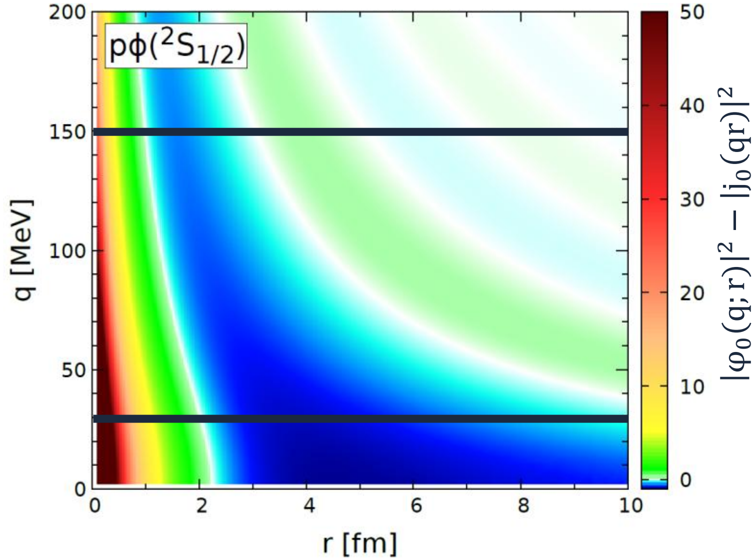
SF
with Jacobian

WF Change
Increase/Decrease in WF by FSI

Deviation of $C(q)$ from 1 = **How much SF "picks up" WF change**







Lednický-Lyuboshits Model

R. Lednický and V. L. Lyuboshits, Yad. Fiz. **35**, 1316 (1981)

$$C(q) = 1 + \int_0^\infty dr 4\pi r^2 S(q;r)[|\varphi_0(q;r)|^2 - |j_0(qr)|^2]$$

Assumptions

- **Gaussian SF**: $S(q;r) \approx S(r) \propto \exp\left(-\frac{r^2}{4r_0^2}\right)$
- **Asymptotic WF** (+ effective range correction)

$$C(q) = 1 + \frac{|f_0(q)|^2}{2r_0^2} F_3\left(\frac{r_{\text{eff}}}{r_0}\right) + \frac{2\text{Re} f_0(q)}{\sqrt{\pi} r_0} F_1(2qr_0) - \frac{\text{Im} f_0(q)}{r_0} F_2(2qr_0)$$

$$f_0(q) = \frac{1}{q\cot\delta_0(q) - iq} \approx \frac{1}{-\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 - iq}, \quad F_1(x) = \frac{D_+(x)}{x} = \frac{e^{-x^2}}{x}\int_0^x dt\, e^{t^2}, \quad F_2(x) = \frac{1-e^{-x^2}}{x}, \quad F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}$$

CF becomes a function of a_0, r_{eff}, and r_0

Cf. Generalization to higher partial waves in K. Murase and T. Hyodo, JSPC **3**, 100017 (2025)

Hadron Interaction Study via Femtoscopy

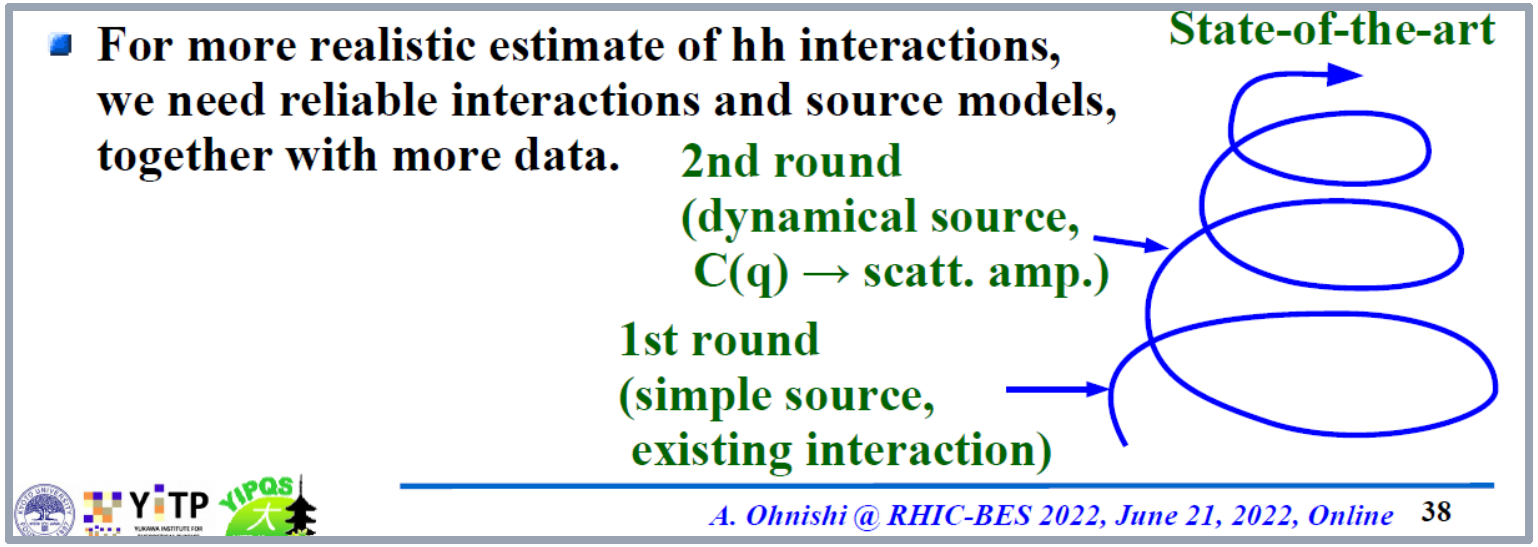
Recent active studies have demonstrated its effectiveness and significance

L. Fabbietti *et al.*, Ann. Rev. Nucl. Part. Sci. **71**, 377 (2021)

Assuming static (*q*-independent) Gaussian SF

Actual SF should reflect the complex dynamics of nuclear collisions

A. Ohnishi, talk at RHIC-BES On-line seminar IV (2022)



For precision study of hadron interactions,

Femtoscopy using **dynamical models**

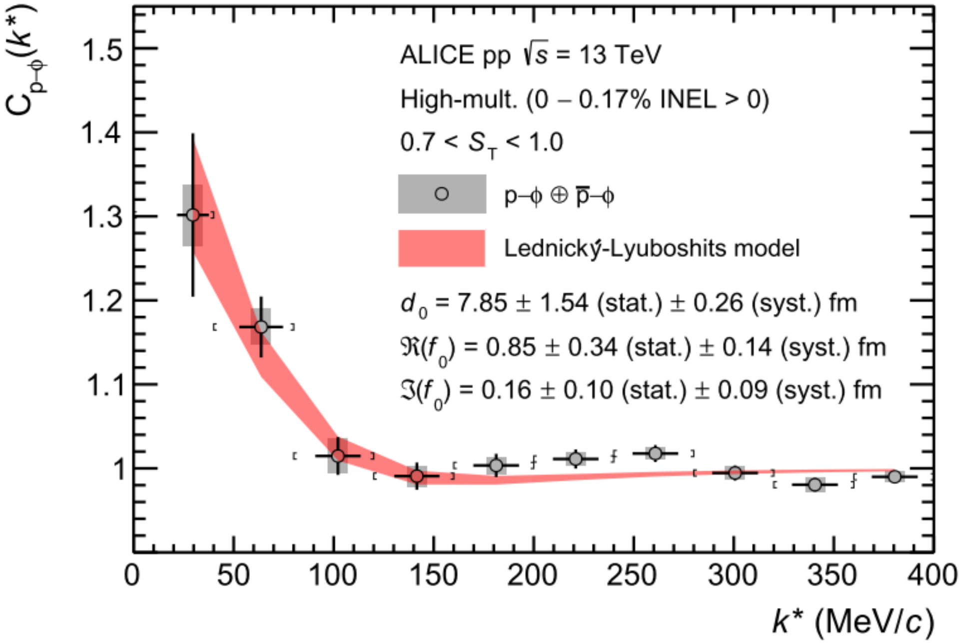
Contents

- Introduction
- Basics of Femtoscopy
- pϕ Femtoscopy
  - Overview
  - Source Function from a Dynamical Model
  - Correlation Function

Existing pϕ Femtoscopy 1

Experimental pϕ CF ALICE, PRL **127**, 172301 (2021)

High-multiplicity (0–0.17%) **p+p collisions at** $\sqrt{s} = 13$ TeV



Lednický-Lyuboshits Fit

Gaussian source size: $r_0 = 1.08$ fm

← Resonance Source Model

ALICE, PLB **811**, 135849 (2020)
[Corrigendum: PLB **861**, 139233 (2025)]

Scattering length: $a_0 \cong -0.85 - 0.16i$ fm

Effective range: $r_{\text{eff}} \cong 7.85$ fm

- Attractive pϕ interaction **as a spin-average**
- Small effects of channel-couplings in vacuum?

Existing pϕ Femtoscopy 2

Spin-channel-by-channel femtoscopy

E. Chizzali *et al.*, PLB **848**, 138358 (2023)

Gaussian source size: $r_0 = 1.08$ fm

$^4S_{3/2}$: HAL QCD potential Y. Lyu *et al.*, PRD 106, 074507 (2022)

$a_0^{(3/2)} \cong -1.43$ fm, $r_{\mathrm{eff}}^{(3/2)} \cong 2.36$ fm

Overall attraction without bound states

$^2S_{1/2}$: **Parametrized potential** ← **Constrain by experimental CF**

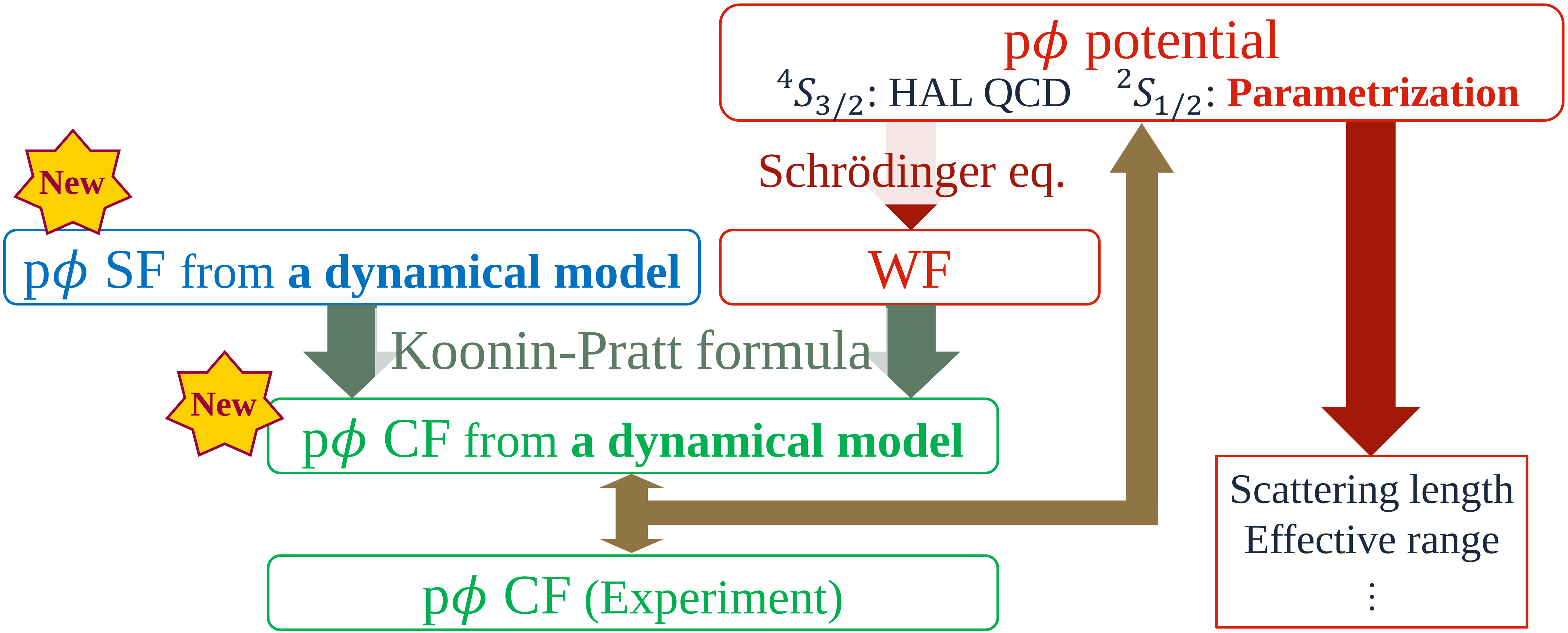
$a_0^{(1/2)} \cong 1.54 - i0.00$ fm, $r_{\mathrm{eff}}^{(1/2)} \cong 0.39 + i0.00$ fm

- **Strong attraction**
- **Small effects of channel-couplings**

Indication of a pϕ bound state

Overview of Present Study

This study: **pϕ femtoscopy using SF from a dynamical model**



$^4S_{3/2}$ Channel

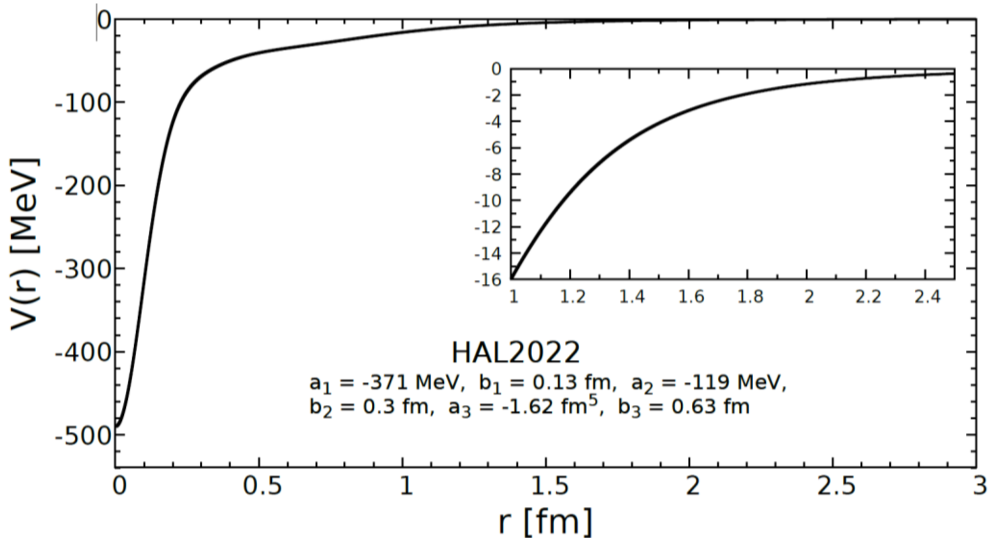
HAL QCD potential Y. Lyu *et al.*, PRD **106**, 074507 (2022)

Lattice QCD at nearly physical point ($m_\pi = 146.4$ MeV)

$$V^{(3/2)}(r) = \underbrace{a_1 e^{-(r/b_1)^2} + a_2 e^{-(r/b_2)^2}}_{\text{Short-range attraction}} + \underbrace{a_3 m_\pi^4 f(r;b_3)\frac{e^{-2m_\pi r}}{r^2}}_{\text{TPE}}$$

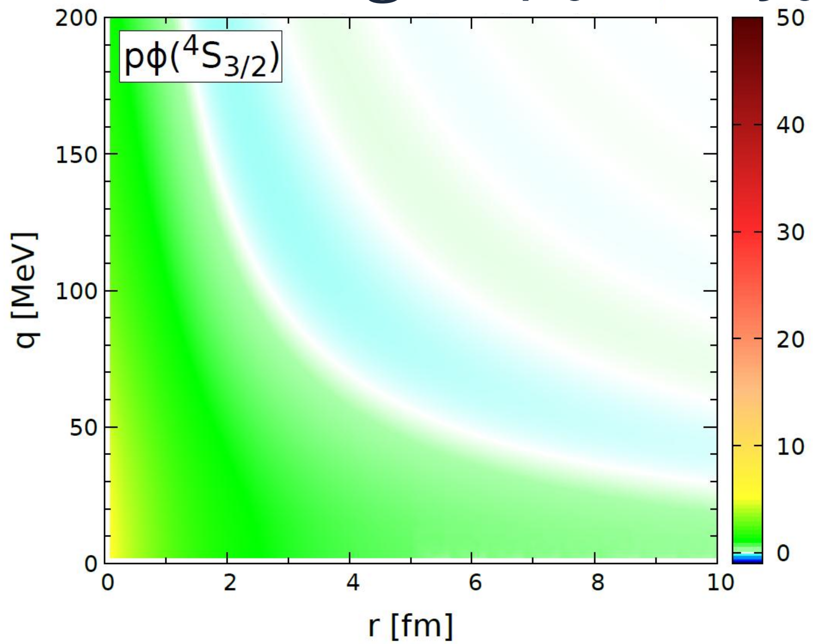
Argonne-type form factor:
$f(r;b_3) = \left[1 - e^{-(r/b_3)^2}\right]^2$

Parameter	Fitted value
a_1 [MeV]	−371±27
b_1 [fm]	0.13±0.01
a_2 [MeV]	−119±39
b_2 [fm]	0.30±0.05
a_3 [fm^5]	−1.62±0.23
b_3 [fm]	0.63±0.04



No bound state

WF change: $|\varphi_0|^2 - |j_0|^2$



Enhancement at small *qr*
due to overall attraction

$^2S_{1/2}$ Channel

Parametrized potential E. Chizzali *et al.*, PLB **848**, 138358 (2023)

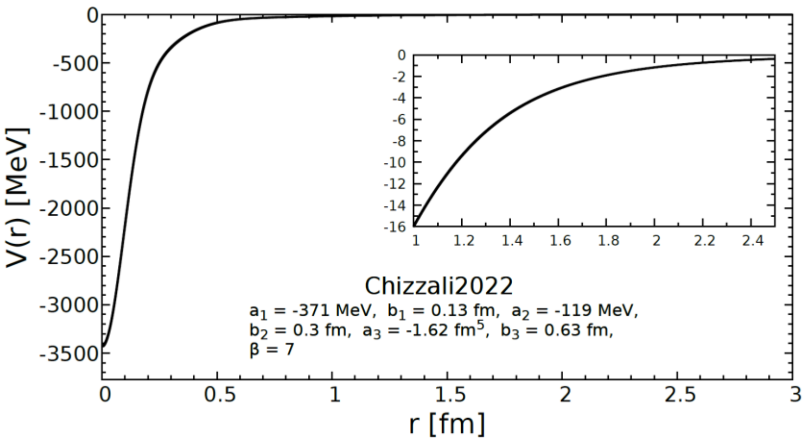
Channel-couplings are neglected for simplicity

$$V^{(1/2)}(r) = \beta\left[a_1 e^{-(r/b_1)^2} + a_2 e^{-(r/b_2)^2}\right] + a_3 m_\pi^4 f(r;b_3)\frac{e^{-2m_\pi r}}{r^2}$$

Short-range interaction — TPE

Only one adjustable parameter β

default: $\beta = 7$

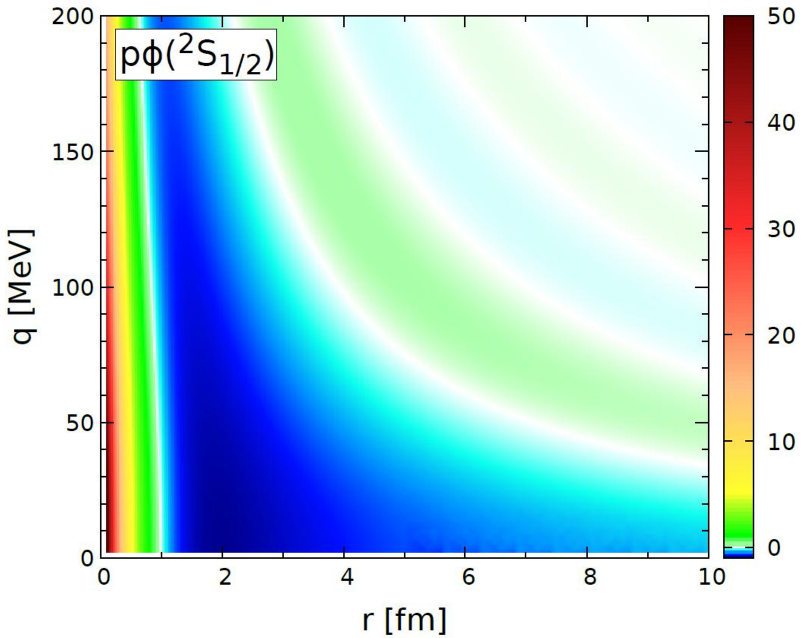


$a_0 = 1.99$ fm

$r_{\rm eff} = 0.46$ fm

A bound state

WF change: $|\varphi_0|^2 - |j_0|^2$

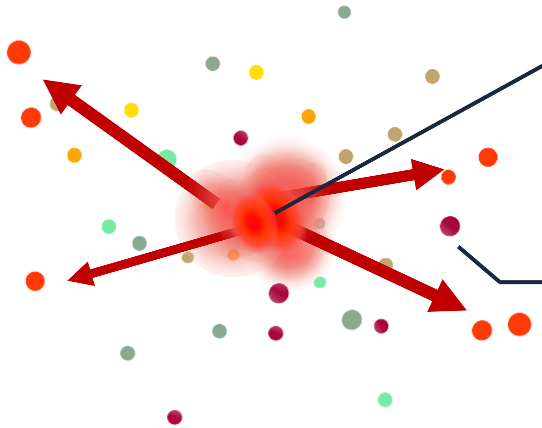


- **Strong enhancement at small qr**
- **"Negative valley" around a_0**
  ← A node of φ_0

Dynamical Core–Corona Initialization model (DCCI)

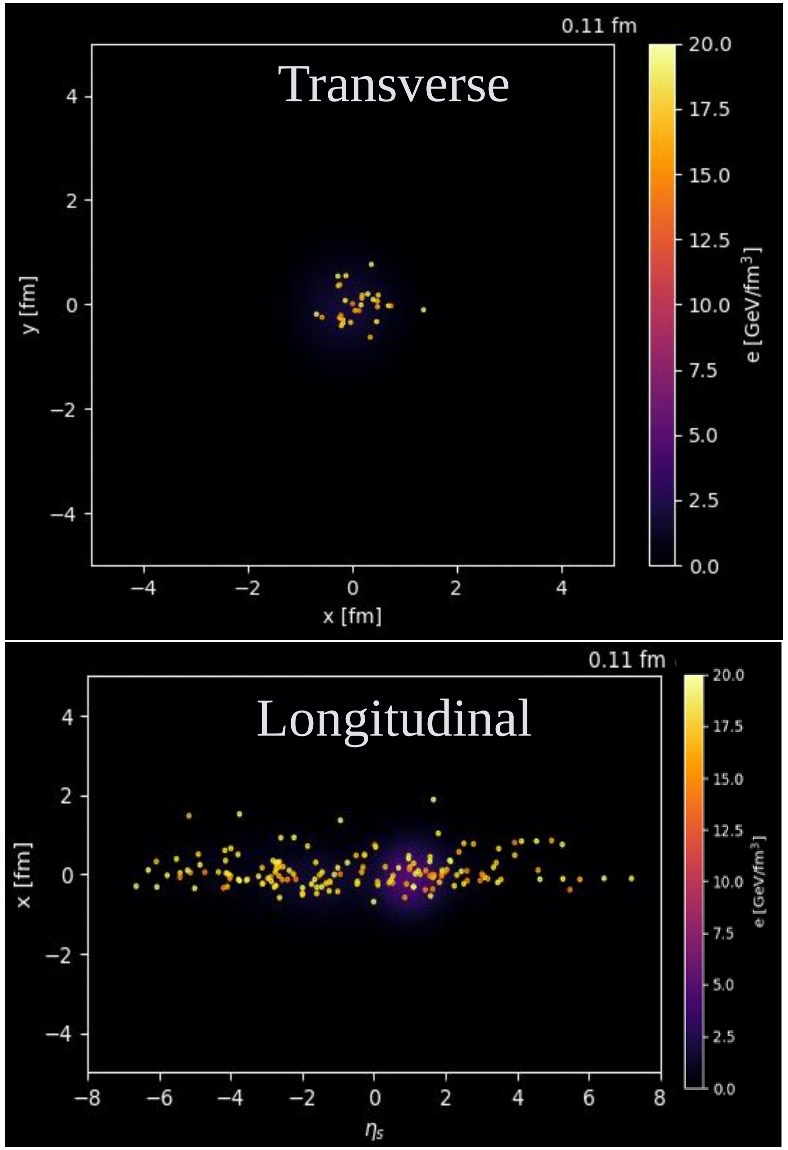
Y. Kanakubo, Y. Tachibana, and T. Hirano, PRC **105**, 024905 (2022)

A state-of-the-art dynamical model based on core–corona picture



Core: Equilibrated matter ~ QGP

Corona: Non-equilibrium partons



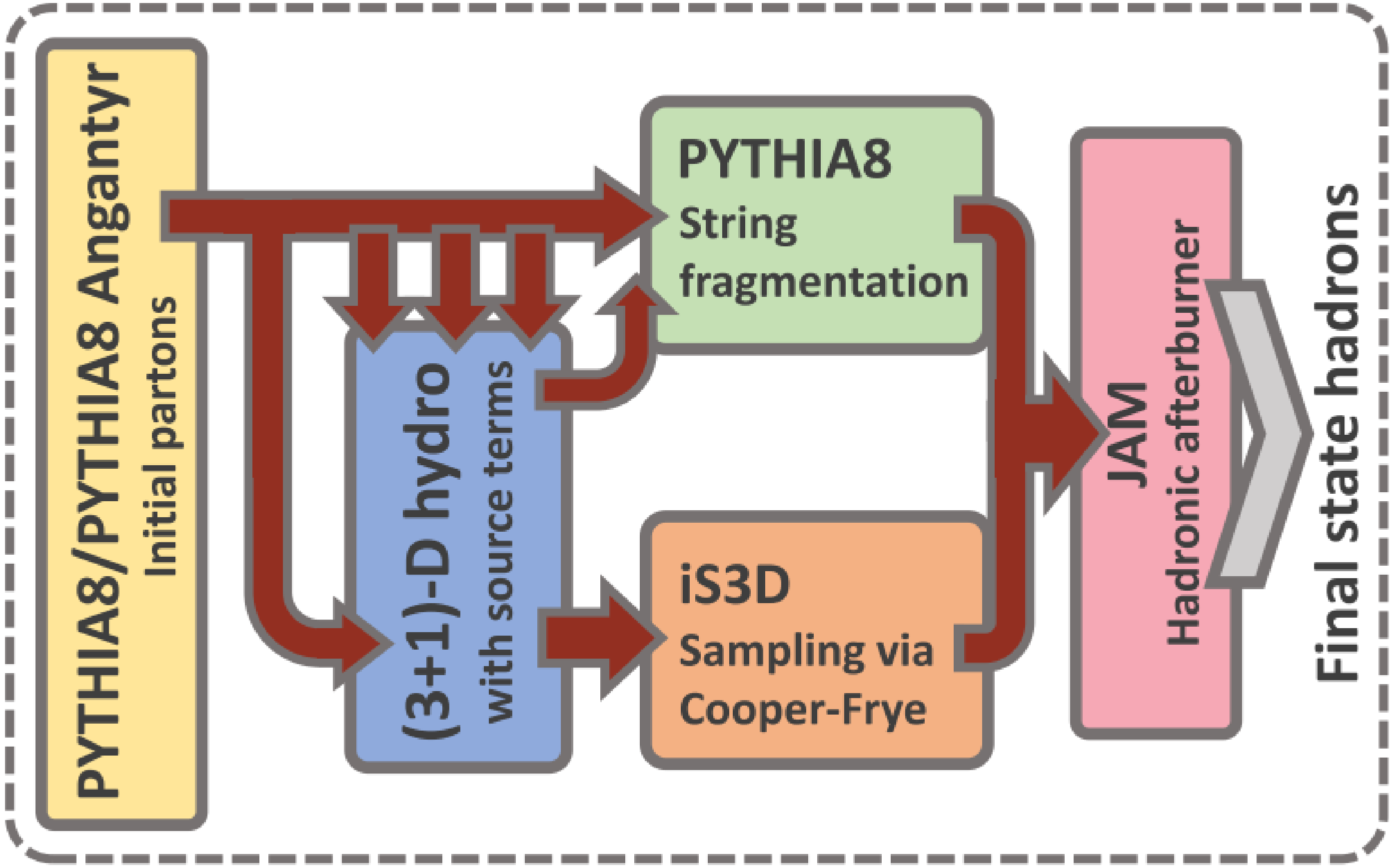
High-mult. p+p collisions at $\sqrt{s}$ = 7 TeV

Movies provided by Y. Kanakubo

DCCI2 Framework

Y. Kanakubo, Y. Tachibana, and T. Hirano, PRC **105**, 024905 (2022)

Describe the entire evolution of nuclear collision reactions



PYTHIA8

T. Sjöstrand *et al.*, Comput. Phys. Commun. **191**, 159 (2015)

pQCD + String Fragmentation

(3+1)-D hydro

Y. Tachibana and T. Hirano, PRC **90**, 021902 (2014)

Hydrodynamic eq. w/ source term

iS3D

M. McNelis *et al.*, Comput. Phys. Commun. **258**, 107604 (2021)

Grand canonical sampling of hadrons

JAM

Y. Nara *et al.*, PRC **61**, 024901 (2000)

Hadron transport based on QMD

Pros of Using DCCI

- MC event generator
  Dynamical hadron emission reflecting **whole collision reactions**

- Core-corona picture
  Unified description from pp to AA collisions
  Applicable to high-multiplicity pp collisions

- Soft from corona Y. Kanakubo *et al.*, PRC **106**, 054908 (2022)
  High experimental reproducibility of low-p_T hadron yields

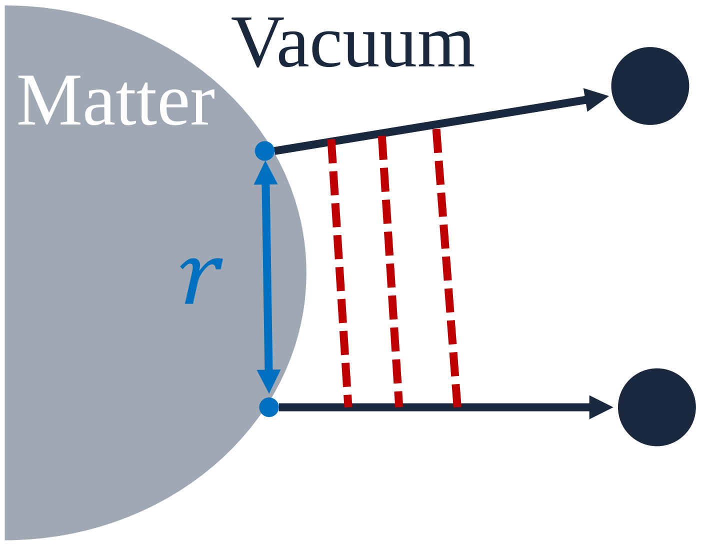
SF that reflects "realistic" collision dynamics
compared to static Gaussian SF

Source Function from Dynamical Models

SF = Phase space dist. of hadrons at their **"emission point"**

Isolated system after emission in Koonin-Pratt formula

Dist. at **"final interacting point (FIP)"** w/ surrounding hadron gas



SF from dynamical models at PRF

$$S(\boldsymbol{q};\boldsymbol{r}) = \frac{1}{N_{\mathrm{pair}}(\boldsymbol{q})}\frac{d^3N_{\mathrm{pair}}(\boldsymbol{q})}{d^3r}$$

$\boldsymbol{q}$: Relative momentum **at FIP**

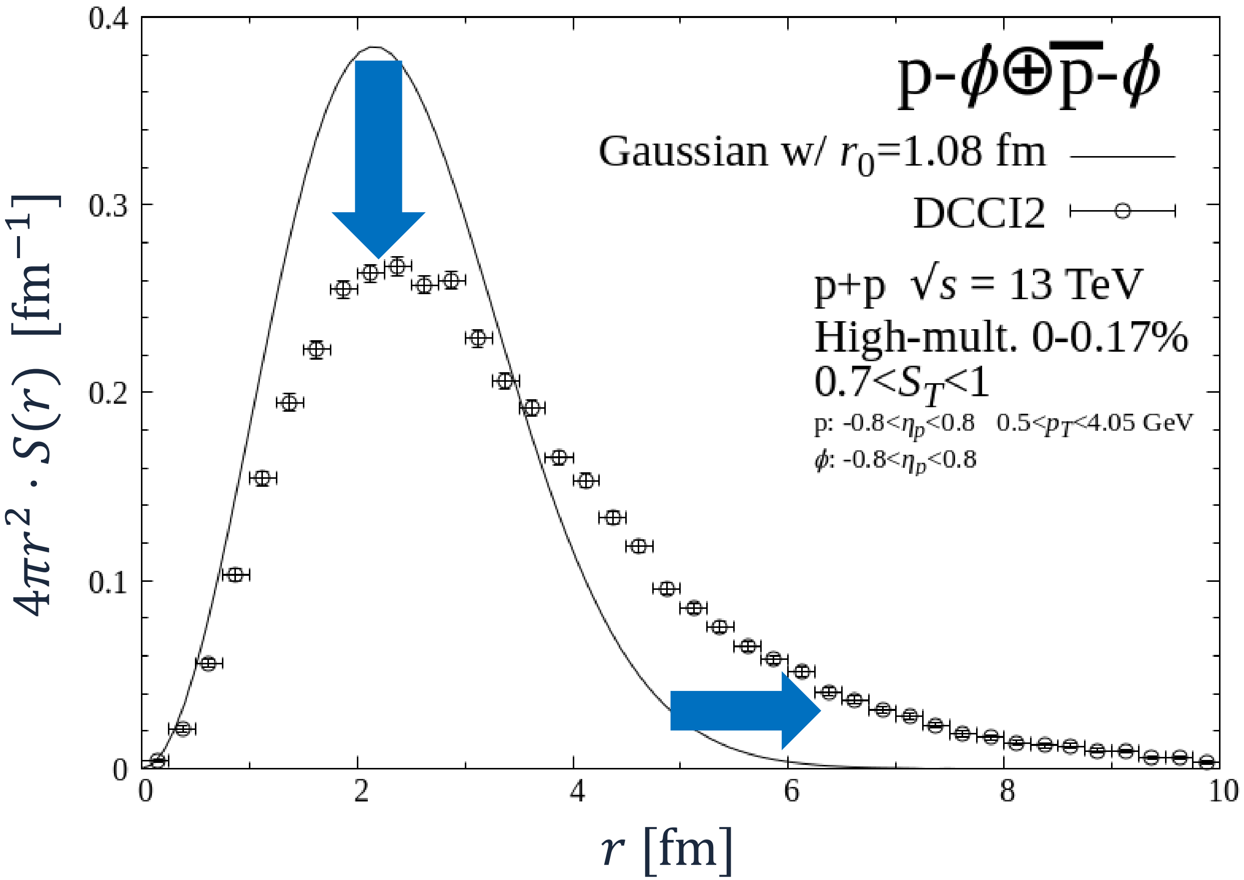
$\boldsymbol{r}$: Relative coordinate **at FIP**

$N_{\mathrm{pair}}(\boldsymbol{q}) = d^3N_{\mathrm{pair}}/d^3q$: Number of pairs w/ $\boldsymbol{q}$ **at FIP**

Source Function

High-multiplicity 0-0.17% pp collisions at $\sqrt{s} = 13$ TeV

Plot: **DCCI2 SF**, Line: Gaussian SF $S(r) \propto \exp(-r^2/4r_0^2)$ w/ $r_0 = 1.08$ fm



Note: Event-mixing to increase statistics

Non-Gaussian long-tail

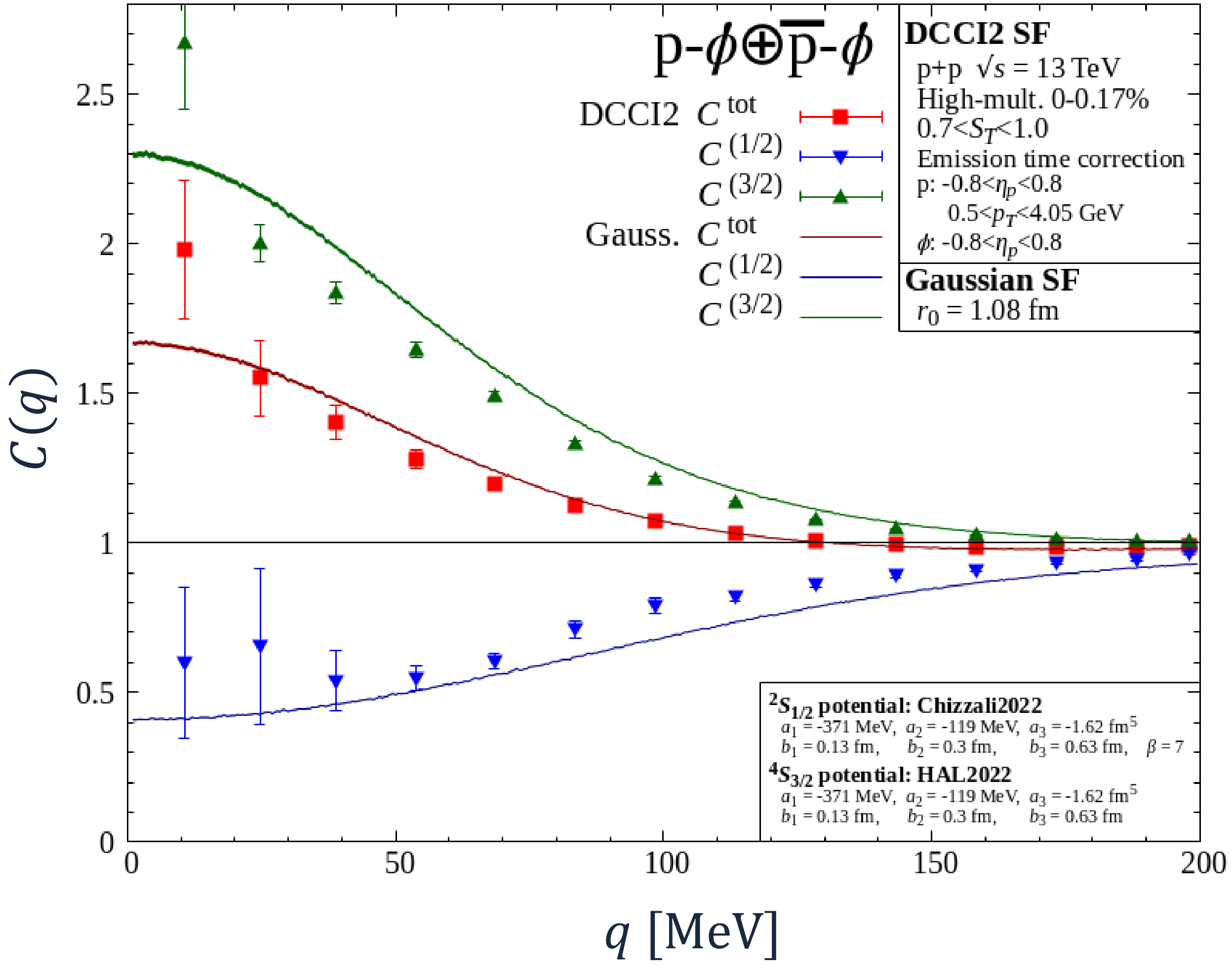
Mainly due to proton rescatterings with surrounding pion gas a.k.a. "**Pion wind**"

Hadronic rescatterings even in pp collisions

Correlation Function

Green: $C^{(3/2)}$, Blue: $C^{(1/2)}$, Red: $C^{\text{tot}} = \frac{2}{3}C^{(3/2)} + \frac{1}{3}C^{(1/2)}$

Plots: **DCCI2 SF**, Lines: Gaussian SF w/ $r_0 = 1.08$ fm



DCCI2 vs. Gaussian

- **Slightly weaker correlation**
  Due to non-Gaussian long-tail
- **Non-trivial behavior at small *q***

A small but statistically significant difference

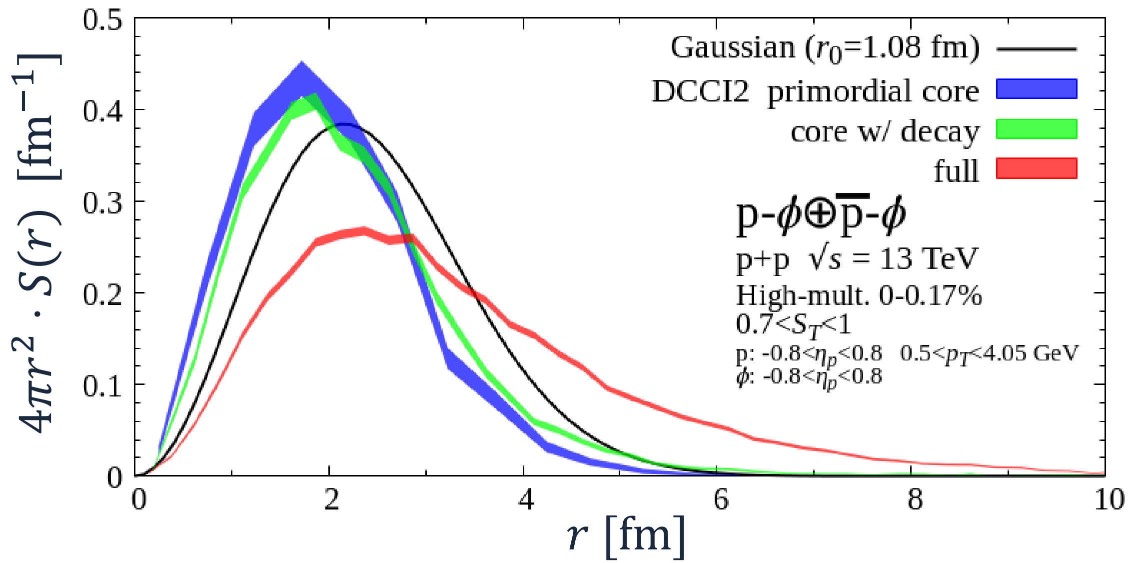
Effects of Decay and Rescattering

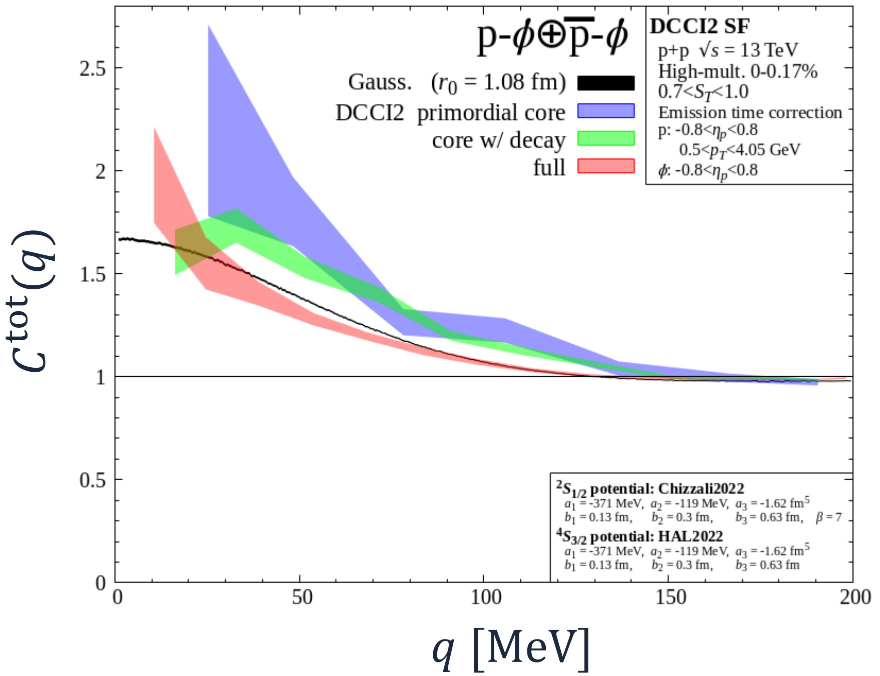
Primordial core

Direct p and ϕ from core at **hypersurface**

+ Decay → **Core w/ decay**

+ Decay + Rescattering + Corona → **Full** (Comparable w/ exp. data)



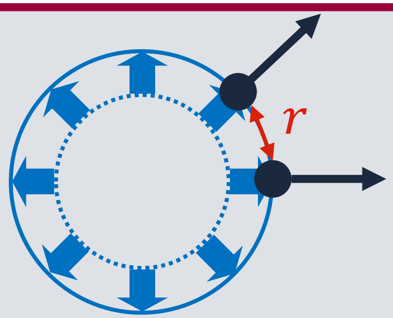


- Distribution at hypersurface ~ Gaussian
- Resonance decay → **A little long-tail**
- Hadronic rescattering → **Long-tail**

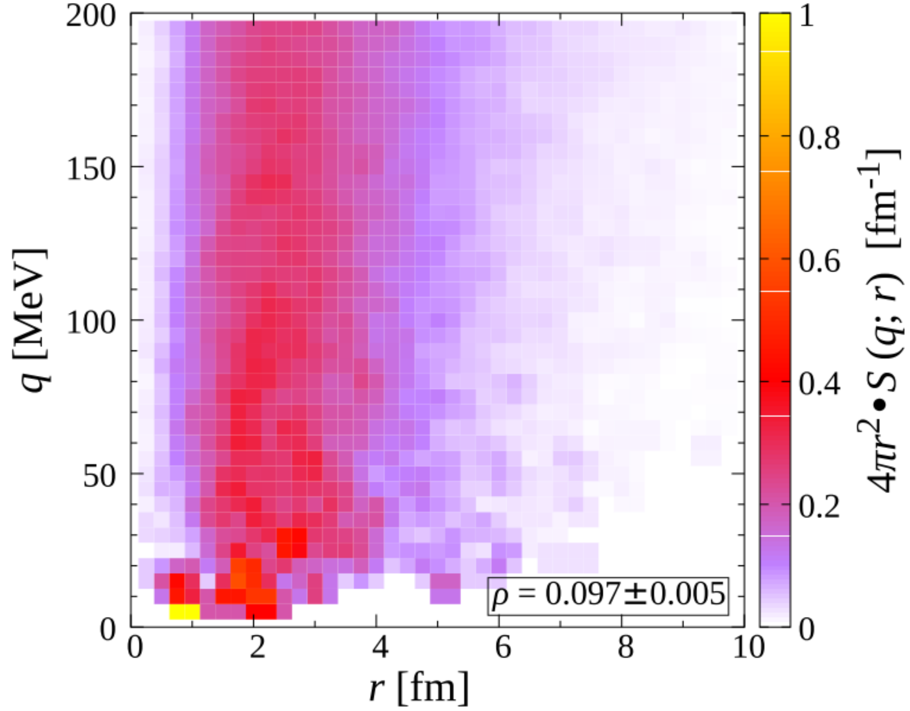
Larger effects of hadronic rescattering than resonance decay on SF and CF

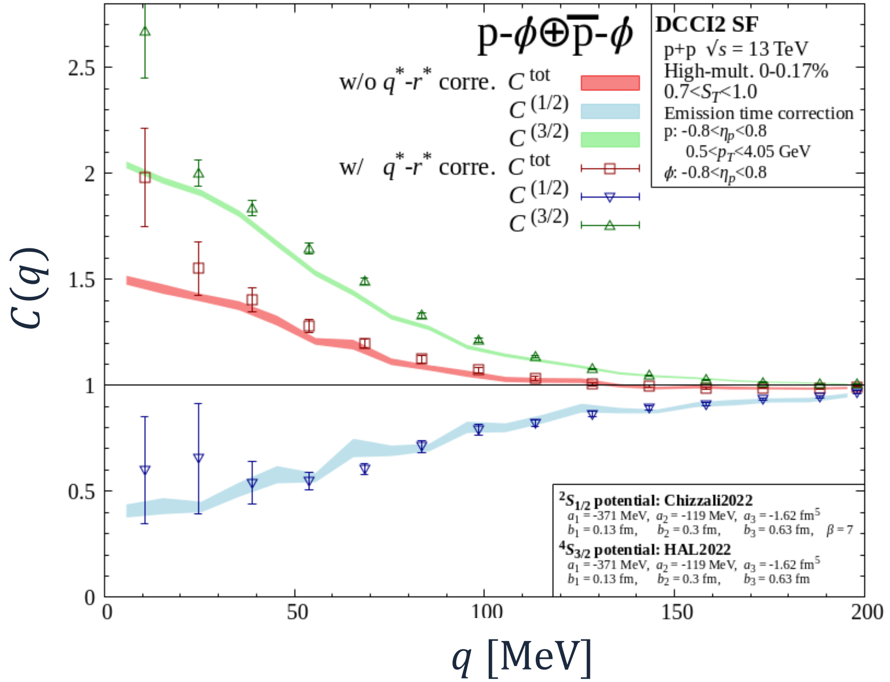
Effects of Collectivity

SF generally depends on *q* due to e.g., collectivity



Close in position space
⇕
Close in momentum space





Plots: W/ q–r correlation

Bands: W/o q–r correlation

- Slightly positive q–r correlation
- Significant small source at small q

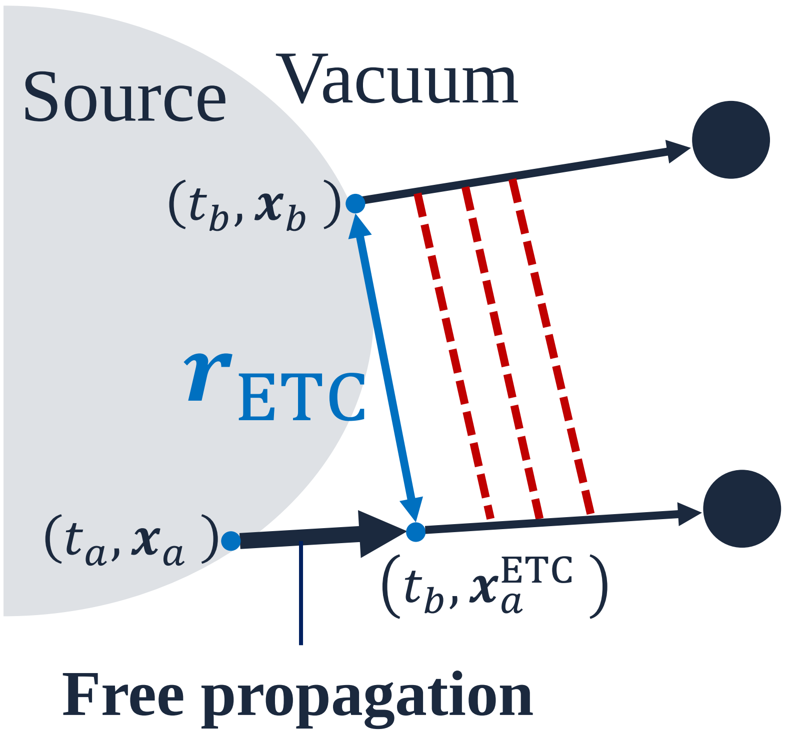
⇨ **CF at small q is more sensitive to the WF in the scattering region**

Effects of Dynamical Hadron Emission

Dynamical model → **Emission time difference:** $S(\boldsymbol{q}; r^0 \neq 0, \boldsymbol{r})$

Emission Time Correction (ETC)

Free propagation until the other's emission



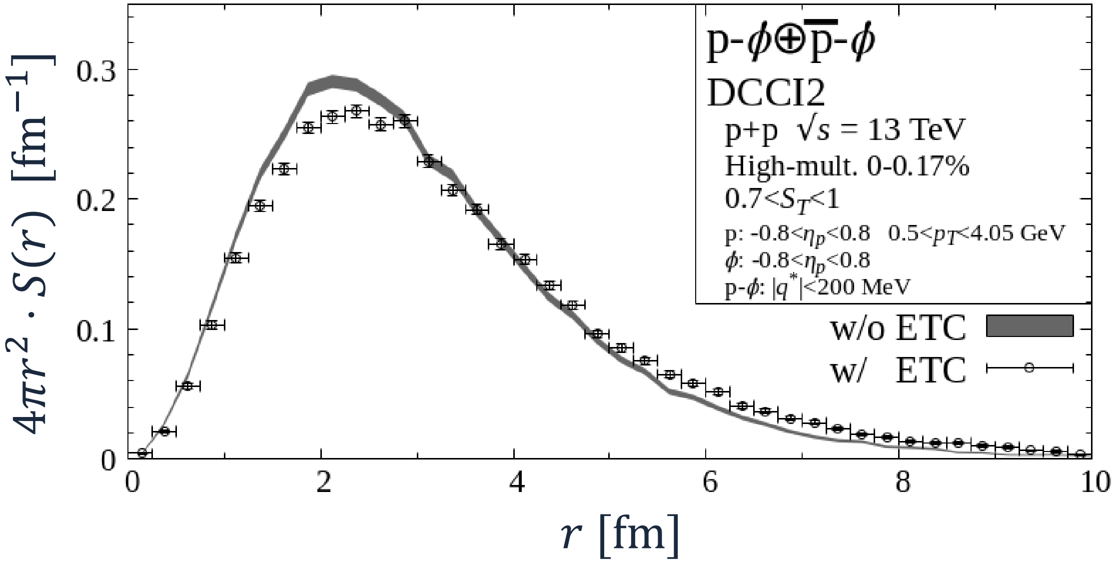
$S(\boldsymbol{q}; r^0 \neq 0, \boldsymbol{r})$ **ETC** → $S^{\mathrm{ETC}}(\boldsymbol{q}; \boldsymbol{r}_{\mathrm{ETC}})\delta(r^0)$

$$\boldsymbol{r}_{\mathrm{ETC}} = \boldsymbol{r} + \frac{\boldsymbol{p}_a}{E_a}(t_b - t_a)\theta(t_b - t_a) - \frac{\boldsymbol{p}_b}{E_b}(t_a - t_b)\theta(t_a - t_b)$$

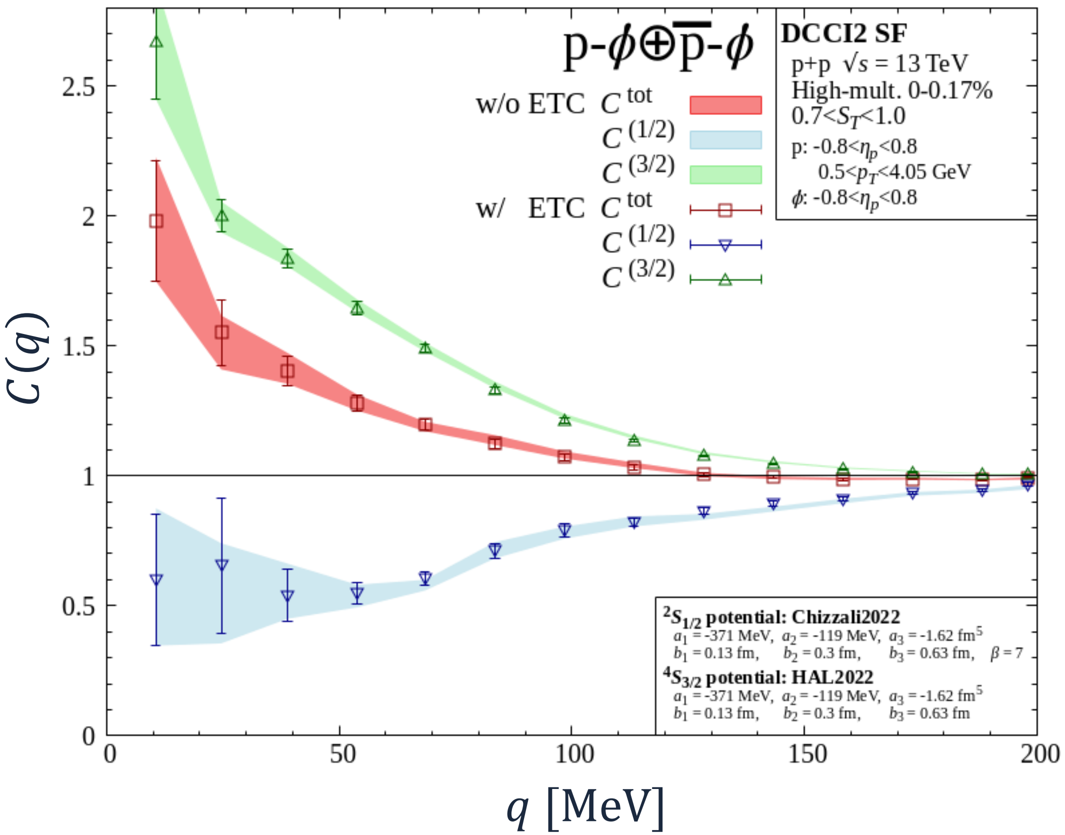
Correction

Effects of Dynamical Hadron Emission

Plots: w/ ETC,   Bands: w/o ETC



ETC slightly enlarges source size



No statistically significant effect on the pϕ CF in pp collisions

Compare with ALICE Data

$C^{(3/2)}$: Fixed, $C^{(1/2)}$: Change with β

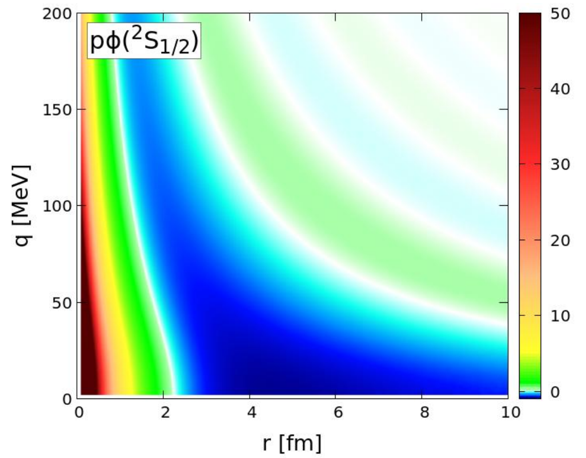
Compare $C^{\mathrm{tot}} = \frac{2}{3}C^{(3/2)} + \frac{1}{3}C^{(1/2)}$ with ALICE data

Weak **Attractive potential w/ a bound state** Strong

$\beta = 6$

$a_0 = 4.54$ fm

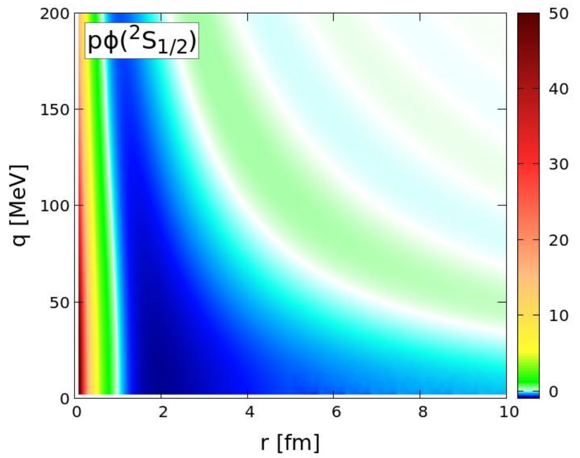
$E_B = 2.3$ MeV



$\beta = 7$

$a_0 = 1.99$ fm

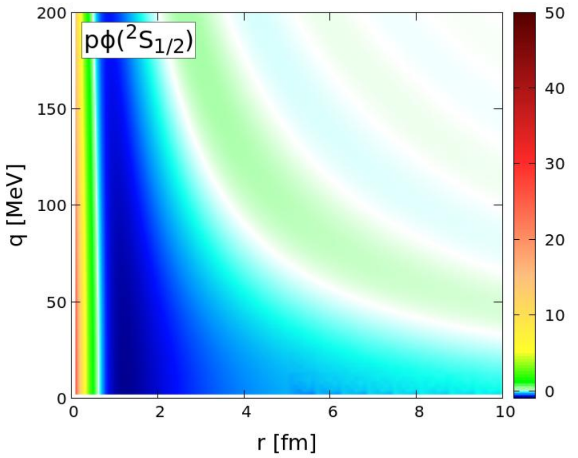
$E_B = 13.3$ MeV



$\beta = 8$

$a_0 = 1.23$ fm

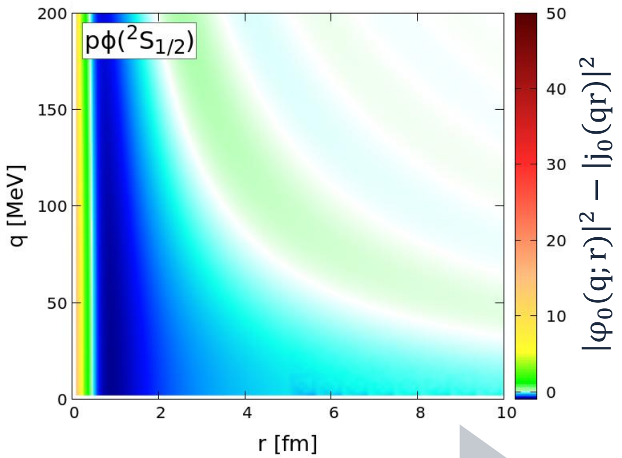
$E_B = 37.5$ MeV



$\beta = 9$

$a_0 = 0.85$ fm

$E_B = 93.1$ MeV



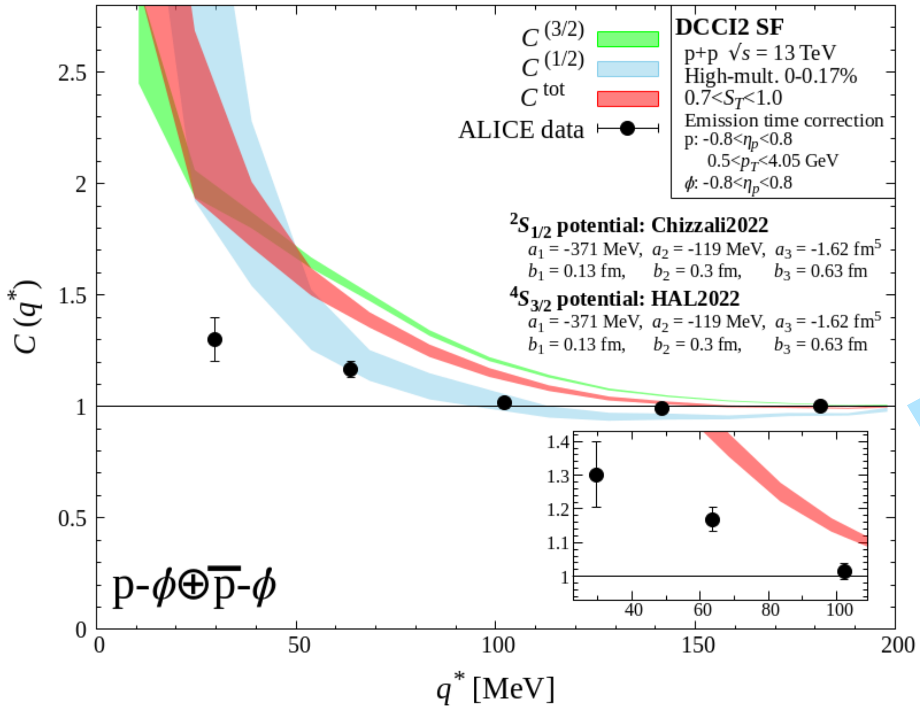
The negative valley moves towards the small r region

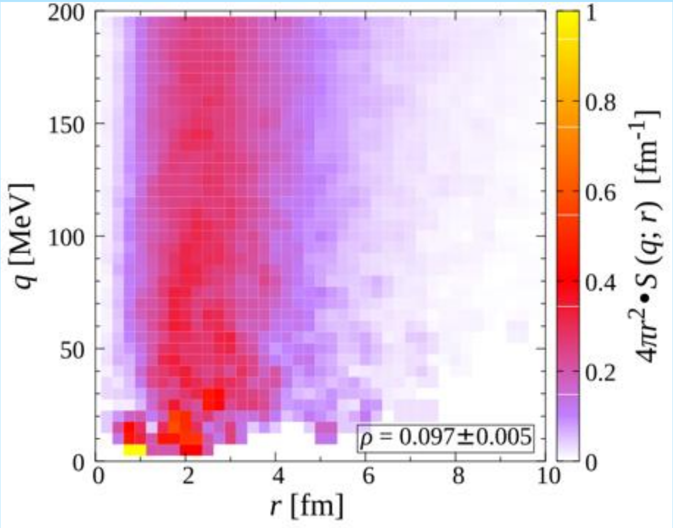
Compare with ALICE Data

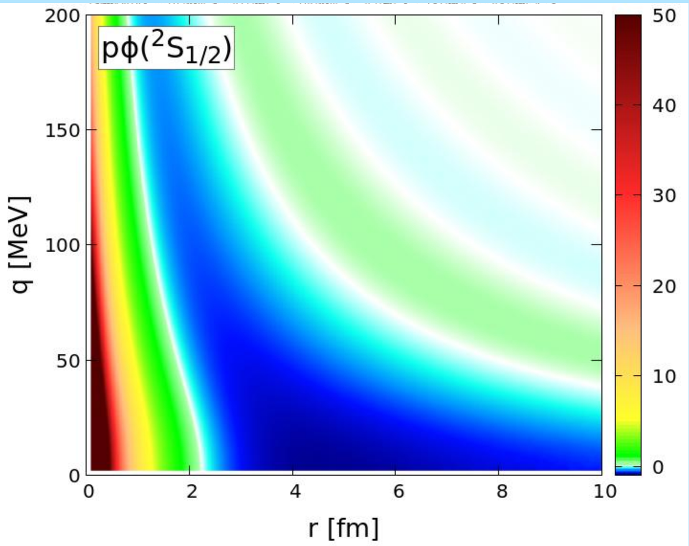
$C^{(3/2)}$: Fixed, $C^{(1/2)}$: Change with β

Compare $C^{\text{tot}} = \frac{2}{3}C^{(3/2)} + \frac{1}{3}C^{(1/2)}$ with ALICE data

$\boldsymbol{\beta = 6}$







SF picks up strong positive region of WF

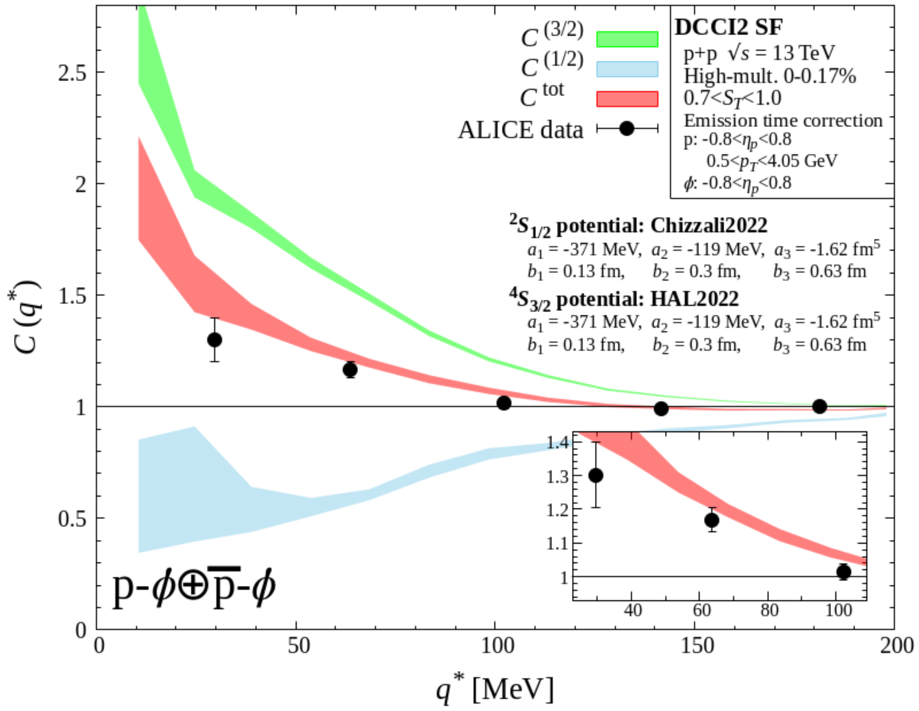
$C^{\text{tot}} > C^{\text{exp}}$

Compare with ALICE Data

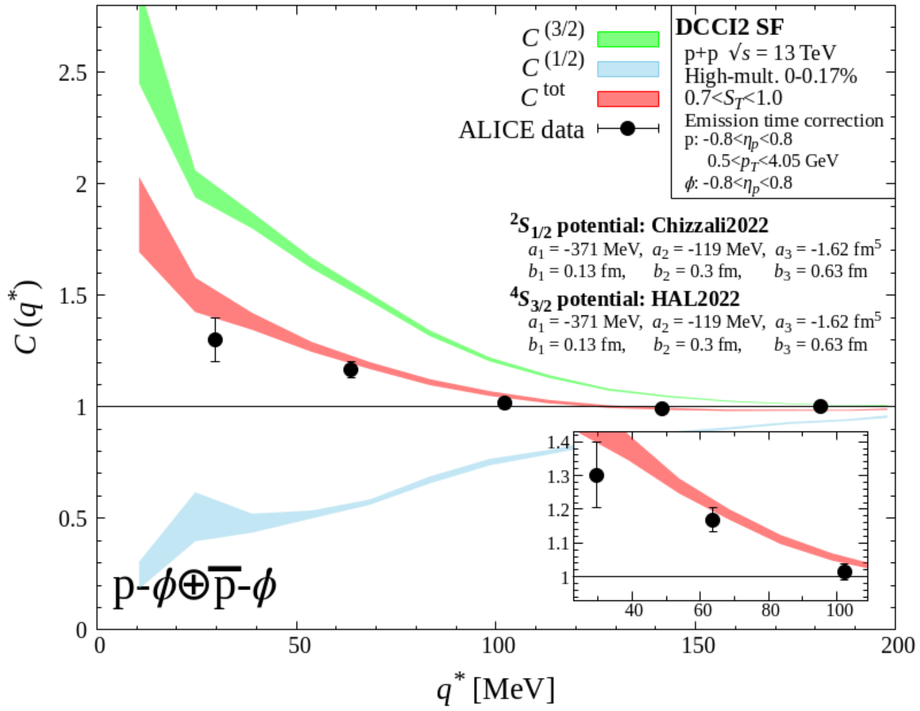
$C^{(3/2)}$: Fixed, $C^{(1/2)}$: Change with β

Compare $C^{\mathrm{tot}} = \frac{2}{3} C^{(3/2)} + \frac{1}{3} C^{(1/2)}$ with ALICE data

$\boldsymbol{\beta = 7}$



$\boldsymbol{\beta = 8}$



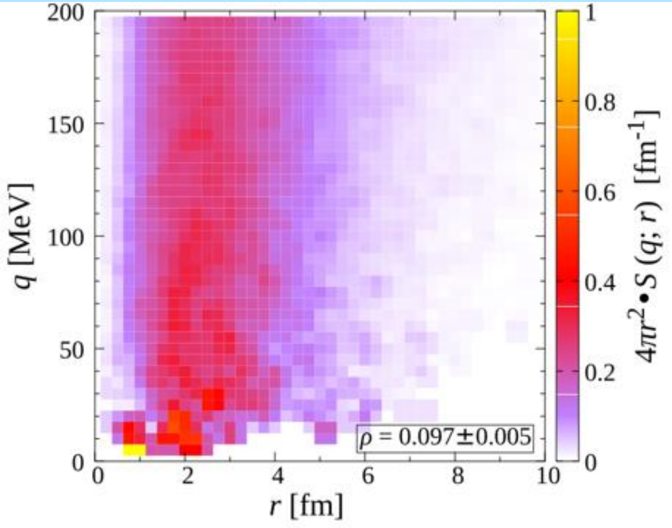
$C^{\mathrm{tot}} \approx C^{\mathrm{exp}}$

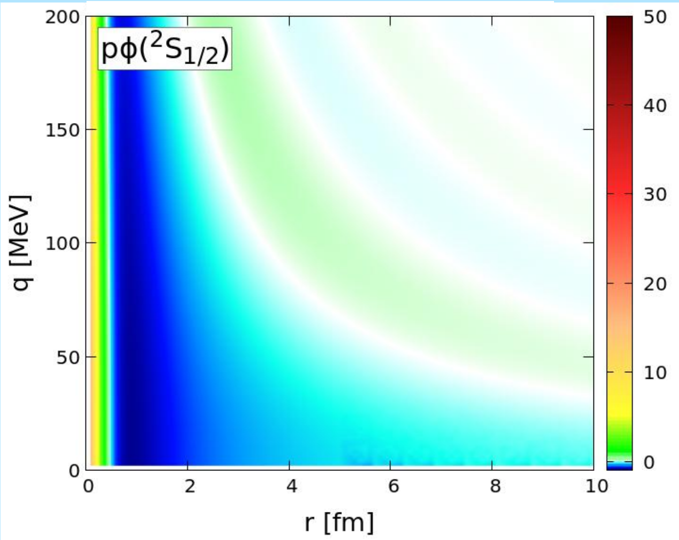
Compare with ALICE Data

$C^{(3/2)}$: Fixed, $C^{(1/2)}$: Change with β

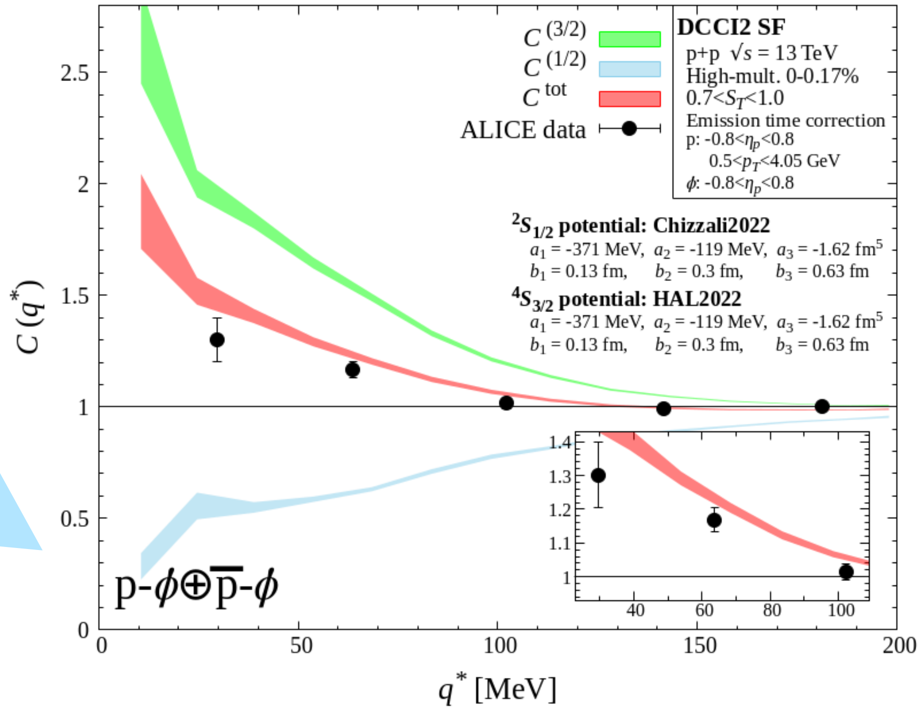
Compare $C^{\text{tot}} = \frac{2}{3} C^{(3/2)} + \frac{1}{3} C^{(1/2)}$ with ALICE data

$\boldsymbol{\beta = 9}$





SF cannot pick up negative valley efficiently

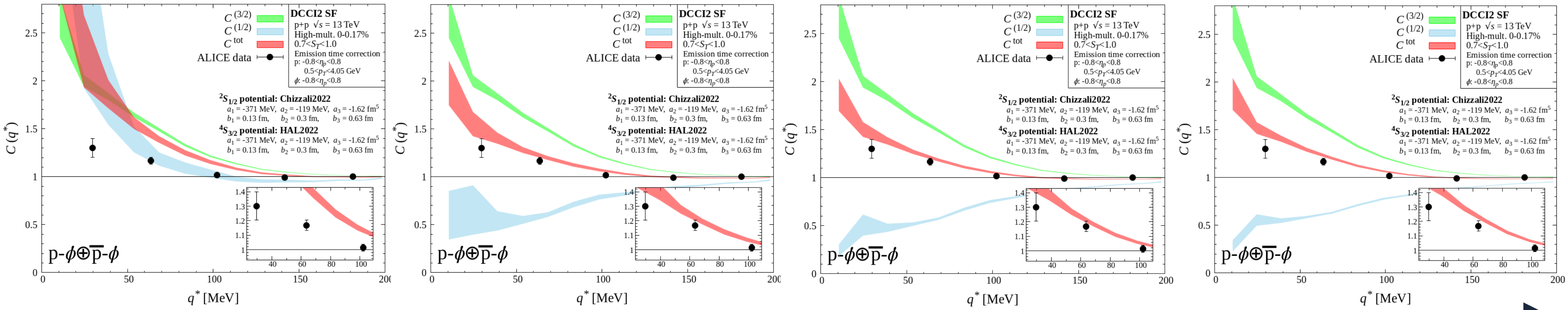


$C^{\text{tot}} > C^{\text{exp}}$

Compare with ALICE Data

$C^{(3/2)}$: Fixed, $C^{(1/2)}$: Change with β

Compare $C^{\text{tot}} = \frac{2}{3}C^{(3/2)} + \frac{1}{3}C^{(1/2)}$ with ALICE data



$\beta = 6$	$\beta = 7$	$\beta = 8$	$\beta = 9$
$a_0 = 4.54$ fm	$a_0 = 1.99$ fm	$a_0 = 1.23$ fm	$a_0 = 0.85$ fm
$E_B = 2.3$ MeV	$E_B = 13.3$ MeV	$E_B = 37.5$ MeV	$E_B = 93.1$ MeV

Overestimate | Agree within errors | Overestimate

pϕ femtoscopy using Source Function from a dynamical model (DCCI2)

Effects of Collision Dynamics

Small but statistically significant

- ✓ SF has non-Gaussian tail mainly due to **hadronic rescatterings**
- ✓ SF depends on relative momentum due to e.g., **collectivity**

Phenomenological Constraints on pϕ Interaction

- ✓ Indication of a bound state in $^2S_{1/2}$ channel ($E_B \cong$ 10–70 MeV)

Slightly different but qualitatively consistent with that using Gaussian SF

One needs to understand the effects of

- **Collectivity**
- **Feed-down and rescattering**
- **Kinematics**
- **Non-femtoscopic background**

etc.

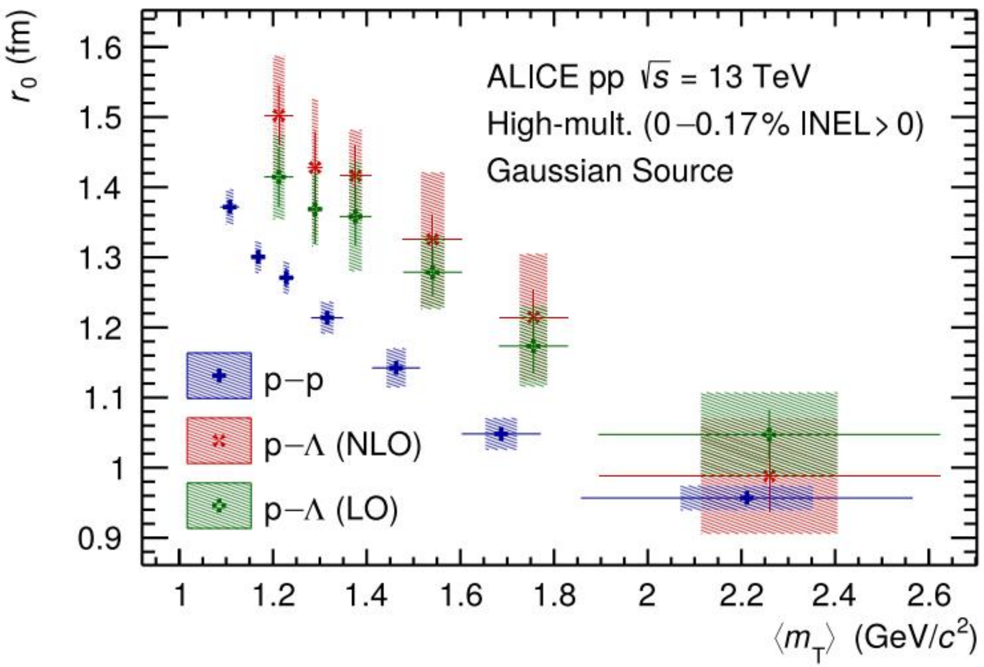
Importance of using Source Function that reflects collision dynamics

Backup

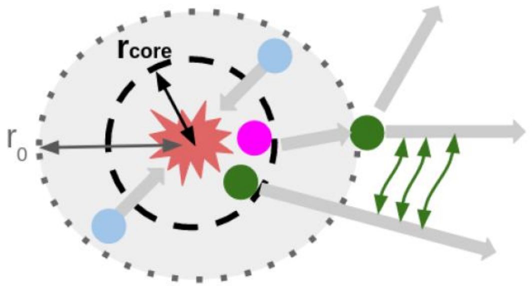
Source Size Estimation in ALICE

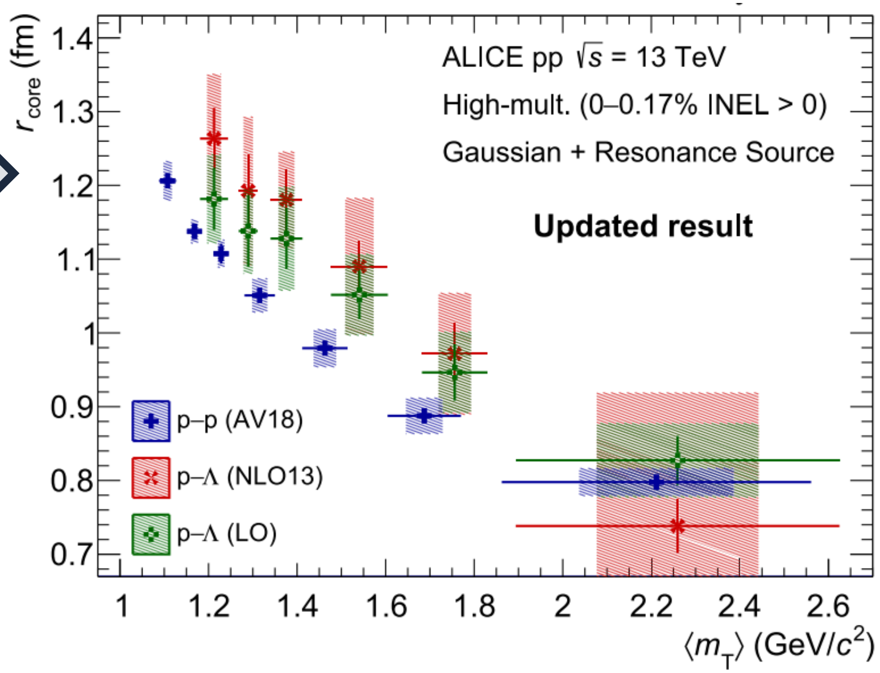
Resonance Source Model ALICE, PLB **811**, 135849 (2020) [Corrigendum: PLB **861**, 139233 (2025)]

Source size estimation from pp and pΛ CF in high-mult. p+p collisions



Subtract **feed-down** effects





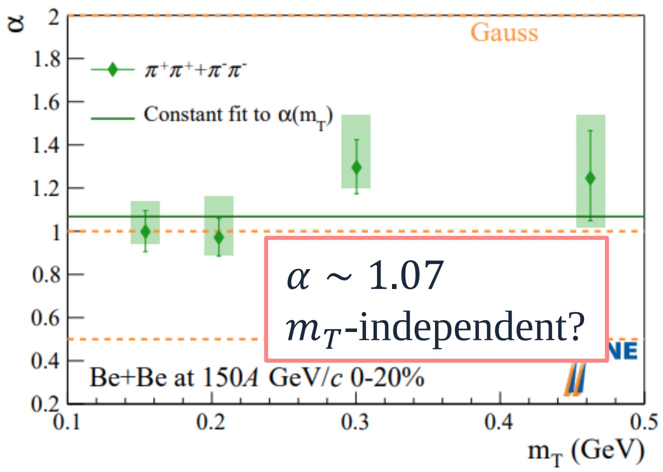
Approximately common m_T-scaling

Indication of common "core" source for all particles

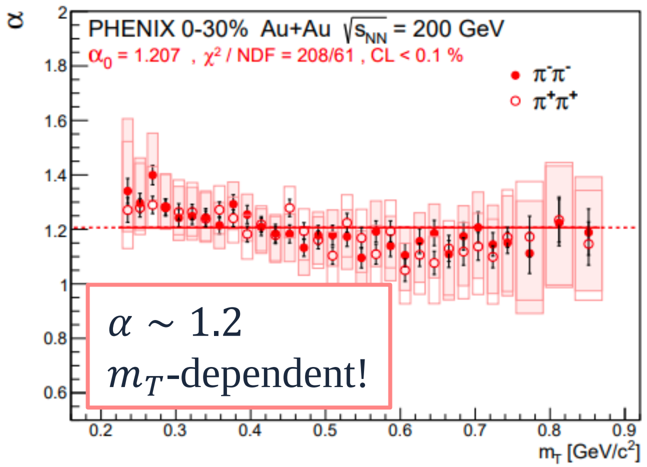
HBT-GGLP Interferometry Using Lévy Source

T. Csorgo *et al.*, EPJ C **36**, 67 (2004)

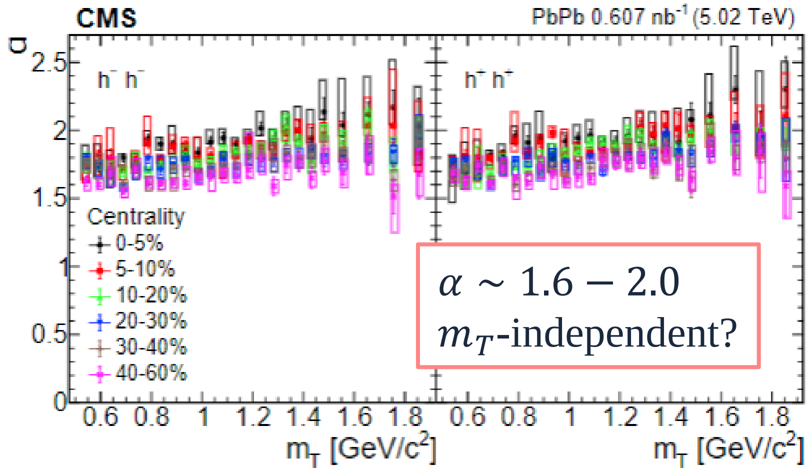
Lévy shape parameter α
- Symmetric **Gaussian-type source** $\rightarrow \alpha = 2$
- Symmetric **Cauchy-type source** $\rightarrow \alpha = 1$



NA61/SHINE, EPJ C **83**, 919 (2023)



PHENIX, PRC **97**, 064911 (2017)



CMS, PRC **109**, 024914 (2024)

Source function from AA collisions

- Longer-tail SF than Gaussian
- Not only size but also shape depends on m_T?

Cf. Lévy walk of pion in HIC in D. Kincses *et al.*, Commun. Phys. **8**, 55 (2025)